International Journal of Pure and Applied Mathematics
Volume 113 No. 2 2017, 369-376
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v113i2.14

ijpam.eu

SOME RESULTS ON FUZZY GRAPHS

P.K. Kishore Kumar[1 §], S. Lavanya[2]
[1]Bharathiar University
Coimbatore, INDIA
[2]Department of Mathematics
Bharathi Women's College
Chennai, INDIA

Abstract: In this paper we analyse the concepts of fuzzy graph coloring identifying the concept of k-fuzzy coloring in a new approach. Also we discuss some concepts on strong arcs in fuzzy graphs and fuzzy line graphs.Also we identify the problem of traffic congestion in round abouts using the strength of connectedness with arcs connected such as α strong arcs, β strong arcs, and δ arcs. $G : \langle V, E\rangle$ is obtained using concept of strong adjacency.

AMS Subject Classification: 05C72
Key Words: k-fuzzy coloring,Strength of connectedness,αstrong arc,βstrong arc and δ arc.

1. Introduction

The concept of fuzzy graph evolved from the motive of uncertainty and vagueness. This lead to the ambiguity in certain situations. Lofti.A.Zadeh has found the importance of fuzziness in different environments and introduced the concept of fuzzy logic. Later many research have been made to test the real time scenario based on fuzzy logic and its use. This paper is based on the applications of fuzzy graph involving the arcs defined and its strength of connectedness between each arcs using maximum of its strength.

Received: January 15, 2017
Revised: February 27, 2017
Published: March 25, 2017

§Correspondence author

Several applications have been into study and useful for real life situations. Some of them are represented graphically using fuzzy graphs. [1]Mordeson et al., proposed the concept of fuzzy graphs and fuzzy hypergraphs. Rakesh et al.,[2] in their paper analysed the concept of fuzzy graph coloring in Traffic Light Problem. [3]Lavanya et al., has analysed the concept of Fuzzy total coloring and its application to job scheduling.[4]Eslahchi et al., identified the vertex strength of fuzzy graphs.[5]Sunil Mathew et al., analysed the concept of classification of strong arcs and its workings on fuzzy graphs. In this paper we review the concept of strength of connectedness of arcs based on its classification we apply in some real circumstances to ensure the effectiveness of these arcs. Traffic coloring problem are discussed by some authors on applications of fuzzy graphs and graphs. Here we take the Traffic path and analyse how to avoid congestions in round abouts in peak time. In peak time the traffic level of cars in round abouts is more in western countries. Because of that the rate of flow is slower in the roads and leads to traffic congestion. In this paper we apply the concept of strong arcs and identify the path which has the higher flow and sustain the congestion level to normal.

2. Main Definitions and Results

Definition 1. A fuzzy graph G=(V,σ,μ) where V is the vertex set, σ is a fuzzy subset of V and μ is a membership value on σ such that μ(u,v) $\leq \sigma$(u) $\wedge$ σ(v) for every u,v $\in$ V.

Definition 2. **Strongly adjacent vertices** An edge is strongly adjacent then $\frac{1}{2}min(\sigma(x), \sigma(y)) \leq \mu(xy)$

In this paper we modify the definition of strongly adjacent vertices according to the value of k.

Definition 3. **Modified definition of strong adjacent edges** An edge is strongly k-adjacent in fuzzy graph if $\frac{1}{k}min(\sigma(x), \sigma(y)) \leq \mu(xy)$ for $k = 1, 2, 3, ...$

Definition 4. **k-fuzzy coloring of** $G = (X, \sigma, \mu)$ The properties which a fuzzy coloring should satisfy are below (a) $\vee\Gamma = \sigma$ (b) $\gamma_i \wedge \gamma_j = 0$ (c)For every strong edge xy of G, $min\gamma_i(x), \gamma_i(y) = 0(1 \leq i \leq k)$

Remark: For what values of k, the edges ina fuzzy graph becomes strongly adjacent.

Example: Consider the fuzzy graph with vertices v_1, V_2, V_3, V_4 having the membership values 1,0.6,0.8,0.4 and the edge set $v_1v_2, v_1v_4, v_4v_3, v_2v_3, v_4v_2$ as 0.4,0.7,0.3,0.1,0.35 . In the edge v_2v_3, according to our proposed definition we define the value of k. We will make the edge e_4 as strongly adjacent by defining the value of k. In the considered edge, for k = 4, $\frac{1}{4}min(0.6, 0.4) = \frac{0.4}{4} = 0.1 \leq 0.1 = \mu(v_2v_3)$.

Therefore v_2v_3 becomes strong. For k=4, all the edges of fuzzy graph are strongly adjacent. Thus if we define the coloring of fuzzy graph then by usual definition of fuzzy graph coloring $\chi(G^F) = 3$.

If we consider the value of k as k=1 in our definition then the chromatic number of the graph becomes incremented. Hence we get $\chi(G^F) = 4$

Theorem 5. *For a finite value of k, the edges in fuzzy graph can be made strongly adjacent.*

It can be easily verified that any coloring of a fuzzy graph is made equivalent to a crisp graph coloring by defining the valueof k, where k is finite.

Theorem 6. *A fuzzy graph can be colored properly if all its edges are strongly adjacent.*

Proof. Consider a fuzzy graph with n vertices. The edges of fuzzy graph is strongly adjacent if $\frac{1}{k}min(\sigma(x), \sigma(y)) \leq \mu(xy)$.

Suppose if an edge is not strongly adjacent. Then by method of induction.

For $K \geq 2$, we define the values of k to identify the strong edges.

We consider the edge is strongly adjacent for $k = N - 1$ and prove that it is true for $K = N$ where N is a finite number$(N > 1)$.

Since the edge is strongly adjacent for K=N-1. It is satisfied for K=N. Hence the theorem. □

Definition 7. Strength of connectedness in fuzzy graphs The strength of connectedness between two nodes u and v is defined as the maximum of the strengths of all the paths between u and v. It is denoted by CONN(u,v). $CONN_{(G-(x,y))}$(x,y) is the strength of connectedness between x and y in the fuzzy graph obtained from G by deleting the arc(x,y).

Definition 8. An arc (x,y) in G is called α-strong if

$$\mu(x, y) > CONN_{(G-(x,y))}(x, y).$$

An arc (x,y) in G is called β-strong if μ(x,y)= $CONN_{(G-(x,y))}$(x,y) and δ-arc if μ(x,y) < $CONN_{(G-(x,y))}$(x,y).

An δ-arc is called a $\delta^{\square}$-arc if μ(x,y)> μ(u,v) where (u,v) is a weakest arc of G.

The following optimal algorithm is used to check the degree of connectedness in an undirected edge weighted graph.

Obtain G-e. Finding the strength of connectedness between u and v.

Let $CONN_{G-e}$(x,y) = t' and t" be the strength of a weakest arc in G. Also t= μ(u,v).

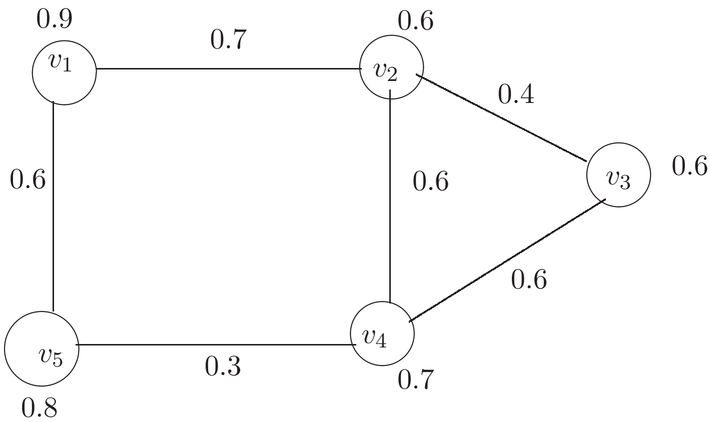
(a) If t > t' then (u,v) is α-strong

(b) If t = t' then (u,v) is β-strong

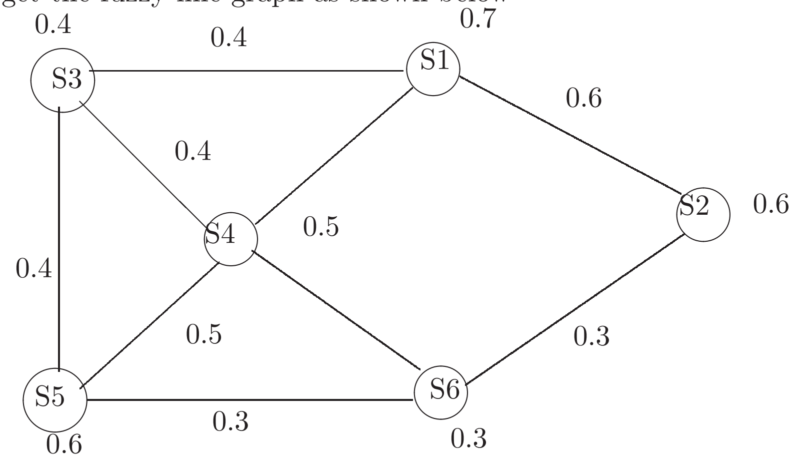
(c) If t < t' then (u,v) is δ-arc and (u,v) is a δ*-arc if t > t".

Definition 9. Fuzzy line graph A Line graph is a graph L(G) satisfying the following conditions(i) each vertex of L(G) represents an edge of G (ii) two vertices of L(G) are adjacent if and only if their corresponding edges share a common endpoint in G.It is also known as fuzzy intersection graph defined on the edge set E.

Let G =(V,E) be a graph where V=$\{v_1, v_2, , v_n\}$. Let $S_i = \{v_i, x_{i_1}, x_{i_2}, x_{i_m}\}$ where $x_{ij} \in E$ and x_{i_j} has v_i as a vertex, j=1,2,m; i=1,2,,n.
Let $S = \{S_1, S_2, , S_n\}$ and $T = \{(S_i, S_j)/S_i, S_j$\$$\in$ S,S_i∩S_j≠∅,i≠j}. Then I(S)=(S,T) is an intersection graph. If G = (σ,μ) is a fuzzy graph and I(S,τ,ν) is the fuzzy intersection graph then the fuzzy subsets τ and ν of S and T are defined as follows ∀ S_i, τ $(S_i) = \sigma(v_i)$ and ∀ S_i, S_j∈T $\nu(S_i, S_j) = \mu$ (v_i, v_j) For the graph G = (V,E) , Line graph L(G) is the intersection graph defined on the edge set E. i.e., I(E). The line graph L(G) = (Z,W) together with the fuzzy sets λ,ω is the fuzzy line graph L_F(G)=(λ,ω) where Z=$\{S_1, S_2, , S_n\}$, S_i={{x_i}∪{u,v}/x_i∈E}, x_i ={u,v},u,v∈V}i=1,2,E and W={(S_i, S_j)/ S_i∩S_j≠ ∅ ,i≠j}. The fuzzy sets λ,ω of Z and W are defined as follows ∀ S_i∈Z $\lambda(S_i)$ =$\mu(x_i)$ ∀S_i, S_j∈W, $\omega(S_i, S_j) = \mu(x_i) \wedge \mu(x_j)$. Let us consider the example of fuzzy graph shown below

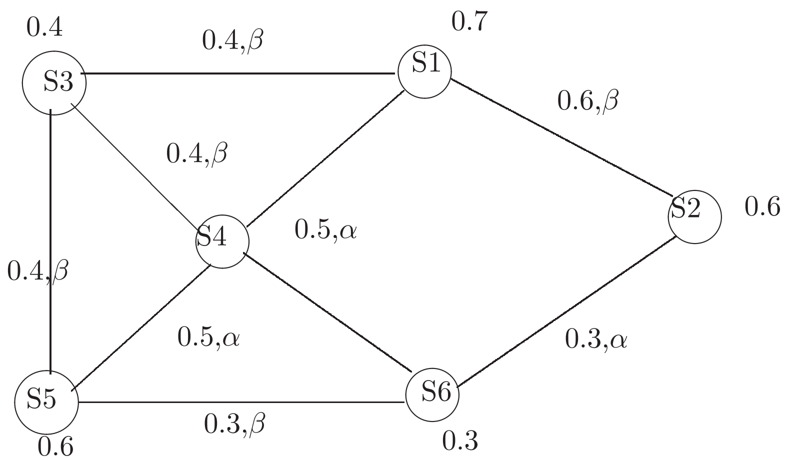


We find the fuzzy line graph for this fuzzy graph using the definition of fuzzy intersection graphs as follows.
S_1=$\{x_{12}\cup\{v_1, v_2\}\}$
S_2=$\{x_{15}\cup\{v_1, v_5\}\}$
S_3=$\{x_{23}\cup\{v_2, v_3\}\}$
S_4=$\{x_{24}\cup\{v_2, v_4\}\}$
S_5=$\{x_{34}\cup\{v_3, v_4\}\}$
S_6=$\{x_{45}\cup\{v_4, v_5\}\}$
W=$\{\{S_1, S_2\}$,$\{S_1, S_3\}$,$\{S_1, S_4\}$,$\{S_2, S_6\}$,$\{S_3, S_4\}$,$\{S_3, S_5\}$,$\{S_4, S_5\}$,$\{S_4, S_6\}\}$
Thus we get the fuzzy line graph as shown below



3. Strong Arcs in Fuzzy Line Graphs

In this part we consider the fuzzy line graph and classify the types of strong arcs accordingly. Consider the fuzzy line graph constructed above For edge $(S_1, S_2) = e_1$, t'= $CONN_{G-e_1}(S_1, S_2)$=max{0.3,0.3,0.3,0.3,0.3} $= 0.3$ t=μ $(S_1, S_2) = 0.6$ $\because$ t>t' It is α-strong arc. For edge $(S_2, S_6) = e_2$, t'= $CONN_{G-e_2}(S_2, S_6)$=max{0.3,0.3,0.3,0.3} $= 0.3$ t=μ $(S_2, S_6) = 0.3$ $\because$ t=t' It is β-strong arc. For edge $(S_1, S_4) = e_3$, t'= $CONN_{G-e_3}(S_1, S_4) =$ max{0.3,0.3, 0.4,0.4} $= 0.4$ t=μ $(S_1, S_4) = 0.5$ $\because$ t¿t' It is α-strong arc. For edge $(S_4, S_6) = e_4$, t'= $CONN_{G-e_4}(S_4, S_6) =$ max{0.3,0.3,0.3,0.3} $= 0.3$ t=μ $(S_4, S_6) = 0.3$ $\because$ t=t' It is β-strong arc. For edge$(S_3, S_1) = e_5$, t'= $CONN_{G-e_5}(S_3, S_4) =$ max{0.4,0.3,0.4,0.3} $= 0.4$ t=μ $(S_3, S_1) = 0.4$ $\because$ t=t' It is β-strong arc. For edge $(S_5, S_6) = e_6$, t'= $CONN_{G-e_6}(S_5, S_6) =$ max{0.3,0.3,0.3,0.3} $=$ 0.3 t=μ $(S_5, S_6) = 0.3$ $\because$ t=t' It is β-strong arc. For edge $(S_3, S_4) = e_7$, t'= $CONN_{G-e_7}(S_3, S_4) =$ max{0.4,0.3,0.3,0.3} $= 0.4$ t=μ $(S_3, S_4) = 0.4$ $\because$ t=t' It is β-strong arc. For edge $(S_5, S_4) = e_8$, t'=$CONN_{G-e_8}(S_5, S_4)$= max{0.3,0.4,0.3,0.3,0.4} $= 0.4$ t=μ $(S_5, S_4) = 0.5$ $\because$ t¿t' It is α-strong arc. For edge$(S_3, S_5) = e_9$, t'=$CONN_{G-e_9}(S_3, S_5) =$ max{0.3,0.3,0.4,0.4} $= 0.4$ t=μ $(S_3, S_1) = 0.4$ $\because$ t=t' It is β-strong arc. Hence we classify the types of arc for the fuzzy line graph constructed above as follows



The above fuzzy line graph does not constitute a weakest arc called δ*-arc since there is no δ-arc present in this example. Hence the above graph has a strong path based on their membership functions for each edges. f-bridge of fuzzy line graph: An arc is called an f-bridge of $G_L$if its removal reduces the strength of connectedness between some pair of nodes in G_L. f-cutnode: An f-cutnode w is a node in G_L whose removal from G_L reduces the strength of connectedness between some pair of nodes other than w. In figure 1 the fuzzy line graph G_L constitutes the f-bridge $(S_1, S_2), (S_5, S_4)$ and (S_1, S_4) since an arc (x,y) is an f-bridge iff it is α-strong in G_L and there are no f-cutnodes since there is no node common to two α-strong arc in G_L. Theorem: A fuzzy graph G:(V,σ,μ) is a block iff for every pair of nodes u,v such that either (u,v) is a δ-arc or μ(u,v) = 0 there exists atleast two internally disjoint strongest u-v paths. By above theorem, It is clear that the above graph G_L is not a block since it does not contain δ-arc .

4. Application of strong arcs in Roundabout

The problem of roundabout traffic congestion in western countries is a common issue in day to day life. In this paper we analyse the concept of strong arcs using fuzzy graphs and obtain a better solution for this problem. Consider the roundabout which has four two way roads leading to all the four directions for a car driver to traverse the way. The roundabout is a complete graph in structure which has four nodes A,B,C,D namely. We consider the level of car flow in each sides as a vertex membership values for each nodes and the ways in which each cars flow as a edge membership values of the graph. With respect to each sides of the flow of cars we identify the nature of arc classified into α strong, β strong and δ strong arcs. Consider the complete fuzzy graph with four vertices and six edges respectively. For nodes A to C concerned to the membership values of the graph we have CONN(A,C) = 0.3 For nodes A to B we have CONN(A,B) = 0.2 For nodes A to D, CONN(A,D) = 0.3 For nodes B to C, CONN(B,C) = 0.3 For nodes B to D, CONN(B,D) = 0.2 We classify each links of nodes according to the rules of strong arcs. In that view we see that Nodes A to C, A to D is an δ arc. Nodes A to B, B to C and B to D are α arcs. There is no possibility of a β arc in this example we discuss. The results of the arc classification shows that there is a heavy congestion with an α strong arcs, a medium flow in β strong arcs and a normal flow in δ arcs. In our problem nodes A to B, B to C and B to D are the lanes with high traffic flow represents the α strong arcs means more number of vehicles pass through the same lanes. Hence these lanes could be diverted to other lanes with normal or medium flow so that the traffic congestion could be reduced in roundabouts.

5. Conclusion and future work

Thus we analyse the concepts of k-fuzzy coloring in fuzzy graphs in modified approach. Also the concepts of strong arcs in fuzzy graph, fuzzy line graphs are discussed. We understand that the knowledge of strong arcs in fuzzy graphs gives a great importance in any real time applications. Application of strong arcs in traffic flow problem gives an idea of reducing the rate of traffic flow in peak time and avoid congestion in roundabouts. In future we plan to work on applying the strong arcs concept in various fuzzy graphs and model a real time application on it.

References

[1] J.N. Mordeson, C.S. Peng, Operations on fuzzy graphs, *Information Sciences*, (1994), 159-170, Article ID: 1016.

[2] Rakesh Jaiswal and Shweta Rai, Application of fuzzy graph coloring in traffic light problem, *International Journal of Innovative Research in Science, Engineering and Technology*, **5** (2016), 6950-6955, Article ID: 15680.

[3] S. Lavanya and R. Sattanathan, Fuzzy total coloring of fuzzy graphs, *International Journal of Information Technology and Knowledge Management*, **2**, No. 1 (1998), 37-39.

[4] Ch. Eslahchi and B.N. Onagh, Vertex strength of fuzzy graph, Hindawi Publishing Corporation (2006), Article ID: 43614.

[5] M. Sunil, M.S. Sunitha, Types of arcs in a fuzzy graph, *Information Sciences* (2009), 1760-1768, Article ID: 1016.