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SOLVING MULTI-OBJECTIVE INTERVAL TRANSPORTATION PROBLEM USING GREY SITUATION DECISION-MAKING THEORY BASED ON GREY NUMBERS

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1. Introduction

In mathematical interval programming models deal with uncertainty and interval coefficients [8]. An interval transportation problem construct the data of supply, demand and objective functions such as cost, time etc in some intervals. This problem can be converted into a classical MOTP by using the concept of right limit, half-width, left limit, and center of an interval [1].

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Das at al. (1999) used fuzzy programming technique to solve MOITP in which cost coefficient, destination and source parameters are in interval form. Sengupta, Pal, and Chakraborty (2001) also provide solution of linear interval number programming problems in which their coefficients are in interval form. For better solution of MOITP Sengupta and Pal (2003) developed fuzzy techniques based solution in which mid pont and width of the interval they consider and provide efficient solution. In 2006 they developed algorithm for find the solution of shortest path problem of transportation network in which coefficients are in interval numbers. For interval valued objective function problem Wu (2009) derived KarushKuhnTucker conditions and by using this condition he find the solution of problem. P. Pandian and G. Natarajan (2010) utilized separation method and obtained compromise solution of integer transportation problems in which transportation cost, supply and demand are in uncertain form. Sudhakar, V.J. and Navaneethakumar, V. (2010) developed innovative approach to find an efficient solution for IITP (integer interval transportation problems). Dutta and A. Satyanarayana Murthy (2010) solve a fuzzy TP with additional restrictions by introducing a linear fractional programming method and they consider transportation costs in interval form. P. Pandian and D. Anuradha (2011) achieve an optimal solution for fully integer interval transportation problems by using split and bound method. In this approach they have not consider midpoint and width of the intervals but based on floating point method they find the solution. S.K. Roy and D. R. Mahapatra (2011) developed a method based on weighted sum and solved multi-objective stochastic transportation problem in which parameters are lognormal random variables and the coefficients of the objectives are interval numbers with inequality type of constraints. Arpita Panda and Chandan Bikash Das (2013) [2] focused on the solution of Cost Varying Interval Transportation Problem under Two Vehicles by applying fuzzy programming technique. Abdusalam mohmed khalifa, E. E. Ammar (2014) have used the concept of UUIT, ULIT, LUIT and lower interval LLIT to solve Rough Interval Multi-objective Transportation Problems (RITP). In 2015 Vincent F. Yu, Kuo-Jen Hu & An-Yuan Chang obtained compromise solution for the MOITP by applying an interactive approach. Dalbinder Kaur, Sathi Mukherjee and Kajla Basu (2015) [5] present the solution of a multi-objective and multi-index real-life transportation problem by applying an exponential membership function in fuzzy programming technique.

2. General Form of Interval Transportation Problem With Multiple Objectives

The formulation of MITP is the problem of minimizing k interval valued objective functions with interval supply and interval destination parameters is given and an efficient algorithm is presented to find the optimal solution of MITP.
The mathematical model of MITP when all the cost coefficient, supply and demand are interval-valued is given by:
Minimize $Z^k = \sum_{i=1}^{m} \sum_{j=1}^{n} \left[C_{L_{ij}}^k, C_{R_{ij}}^k\right] x_{ij}$ where $k = 1, 2, ..., K$,
Subject to
$$\sum_{j=1}^{n} x_{ij} = [a_{L_i}, a_{R_i}], \ i = 1, 2, ..., m,$$
$$\sum_{i=1}^{m} x_{ij} = [b_{L_j}, b_{R_j}], \ j = 1, 2, ..., n,$$
$x_{ij} \geq 0$, $i = 1, 2, ..., m$, $j = 1, 2, ..., n$,
With
$\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j}$ and $\sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j}$
Where the source parameter lies between left limit a_{L_i} and right limit a_{R_i}. Similarly, destination parameter lies between left limit b_{L_j} and right limit b_{R_j} and $\left[C_{L_{ij}}^k, C_{R_{ij}}^k\right]$, $(k = 1, 2, ..., K)$is an interval indicating the uncertain cost for the transportation problem; it can exemplify delivery time, quantity of goods delivered, under used capacity, etc. [4].

3. A Method to Solve Multi-objective Interval Transportation Problem

Consider some notations to define the variables and the sets in multi-objective interval transportation problem.
Let $A = \{A_1, A_2, ..., A_m\}$ be the set of m-origins having a_i $(i = 1, 2, ...m)$ units of supply respectively.Let $B = \{B_1, B_2, ..., B_n\}$ be the set of n-destinations with b_j $(j = 1, 2, ..., n)$ units of requirement respectively. There is a penalty c_{ij} such as cost or delivery time or safety of delivery etc. associated with transporting a commodity from i^{th} source to j^{th} destination. A variable x_{ij} represent the unknown quantity to be shipped from i^{th} source to j^{th} destination.

The problem is to determine the transportation schedule when multiple objectives with interval parameters exist. Grey situation decision making theory is used to minimize or maximize the total transportation penalty according to the problem which satisfying supply and demand conditions.
Assume that the set of m-origins $A = \{a_1, a_2, ..., a_m\}$ as the set of events, the set of n-destination $B = \{b_1, b_2, ..., b_n\}$ as the set of countermeasures, the penalty c_{ij} as the situation set denoted by $C = \{c_{ij} = (a_i, b_j)\,|a_i \in A,\, b_j \in B\}$. First of all confirm the decision making goals (objectives), try to find the corresponding effect measure matrix $\otimes U^{(k)}$ as

$$\otimes\tilde{U}^{(k)} = \left[\otimes\tilde{u}_{ij}^{(k)}\right] = \begin{bmatrix} \otimes\tilde{u}_{11}^{(k)} & \otimes\tilde{u}_{12}^{(k)} & .... & \otimes\tilde{u}_{1m}^{(k)} \\ \otimes\tilde{u}_{21}^{(k)} & \otimes\tilde{u}_{22}^{(k)} & .... & \otimes\tilde{u}_{2m}^{(k)} \\ .... & .... & .... & .... \\ \otimes\tilde{u}_{n1}^{(k)} & \otimes\tilde{u}_{n2}^{(k)} & .... & \otimes\tilde{u}_{nm}^{(k)} \end{bmatrix}$$

Here the data of decision making goals for transporting a product is the effect sample value $\otimes\tilde{u}_{ij}^{(k)}$ $(i = 1, 2, ..., n;\ j = 1, 2, ..., m)$ of situation $c_{ij} \in C$ with objective $k = 1, 2, ..., s$. due to that the decision information is a grey number but not an exact number, the effect sample value $\otimes\tilde{u}_{ij}^{(k)}$ of the situation c_{ij} with objective k is a grey number. Let $\otimes\tilde{u}_{ij}^{(k)} = \left[u_{ij}^{(k)L}, u_{ij}^{(k)U}\right]$ where $u_{ij}^{(k)L}$ and $u_{ij}^{(k)U}$ are respectively the upper and the lower limit of effect value of situation c_{ij} with objective k.
Now, find the upper effect measure and lower effect measure using formula [7]:
1) For objectives which involve the effect sample value fulfilling "the bigger, the better" and "the more, the better", we establish the effect value to measure the upper effect measure for upper and lower limit respectively and the upper effect measure try to find the maximum variation data such as speed, safety etc.

$$\otimes\tilde{r}_{ij}^{(k)} = \frac{\otimes\tilde{u}_{ij}^{(k)}}{\max\limits_{i}\max\limits_{j}\left\{\otimes\tilde{u}_{ij}^{(k)}\right\}}$$

2) For objectives that necessitate the effect sample value fulfilling "the less, the better", we establish the effect value to measure the lower effect measure for upper and lower limit respectively and the lower effect measure seek out to find the minimum variation data such as travel time, cost etc.

$$\otimes\tilde{r}_{ij}^{(k)} = \frac{\min\limits_{i}\min\limits_{j}\left\{\otimes\tilde{u}_{ij}^{(k)}\right\}}{\otimes\tilde{u}_{ij}^{(k)}}$$

Due to that the above two effective measures $\otimes\tilde{r}_{ij}^{(k)} = \left[r_{ij}^{(k)L}, r_{ij}^{(k)U}\right]$ can satisfy
(1) Dimensionless $r_{ij}^{(k)L}, r_{ij}^{(k)U}$
(2) $r_{ij}^{(k)L}, r_{ij}^{(k)U} \in [0,1]$;
(3) The more ideal the effect is, the bigger the $\otimes\tilde{r}_{ij}^{(k)}$ is.
and achieve the consistent effect measure matrix $\otimes\tilde{R}^{(k)}$ of situation C with objective k using upper effect measure and lower effect measure as

$$\otimes\tilde{R}^{(k)} = \left(\otimes\tilde{r}_{ij}^{(k)}\right) = \begin{bmatrix} \otimes\tilde{r}_{11}^{(k)} & \otimes\tilde{r}_{12}^{(k)} & \cdots & \otimes\tilde{r}_{1m}^{(k)} \\ \otimes\tilde{r}_{21}^{(k)} & \otimes\tilde{r}_{22}^{(k)} & \cdots & \otimes\tilde{r}_{2m}^{(k)} \\ \cdots & \cdots & \cdots & \cdots \\ \otimes\tilde{r}_{n1}^{(k)} & \otimes\tilde{r}_{n2}^{(k)} & \cdots & \otimes\tilde{r}_{nm}^{(k)} \end{bmatrix}$$

Subtract each data from 1 of the consistent matrix of effect measure.
There are two different way to obtain objective weights $\eta_1, \eta_2, ..., \eta_s$. 1) Decision maker give a weight to the objectives according to their first preference. 2) Decision maker apply some methods to obtain the objective weights.
Here, for the objective weights $\eta_1, \eta_2, ..., \eta_s$ the optimization model of objective weight is applied by constructing lagrange function as

$$\begin{aligned} L(\eta_i, \lambda) = \mu \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{s} d^{+} \left(\otimes\tilde{r}_{ij}^{(k)}, \otimes\tilde{v}_{i}^{(k)+}\right) \eta_k \\ - \mu \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{s} d^{-} \left(\otimes\tilde{r}_{ij}^{(k)}, \otimes\tilde{v}_{i}^{(k)-}\right) \eta_k \\ + (1-2\mu) \sum_{i=1}^{s} \eta_k \ln \eta_k - \lambda \left(\sum_{k=1}^{s} \eta_k - 1\right) \end{aligned} \tag{1}$$

After simplification we obtaing the formula to find the objective weight which is as follow

$$\eta_k = \frac{\exp\left\{\frac{\left[\mu \sum_{i=1}^{n} \sum_{j=1}^{m} d^{-}\left(\otimes\tilde{r}_{ij}^{(k)}, \otimes\tilde{v}_{i}^{(k)-}\right) - \mu \sum_{i=1}^{n} \sum_{j=1}^{m} d^{+}\left(\otimes\tilde{r}_{ij}^{(k)}, \otimes\tilde{v}_{i}^{(k)+}\right)\right]}{(1-2\mu)} - 1\right\}}{\sum_{k=1}^{s} \exp\left\{\frac{\left[\mu \sum_{i=1}^{n} \sum_{j=1}^{m} d^{-}\left(\otimes\tilde{r}_{ij}^{(k)}, \otimes\tilde{v}_{i}^{(k)-}\right) - \mu \sum_{i=1}^{n} \sum_{j=1}^{m} d^{+}\left(\otimes\tilde{r}_{ij}^{(k)}, \otimes\tilde{v}_{i}^{(k)+}\right)\right]}{(1-2\mu)} - 1\right\}}$$

(2)

From the objective weights and consistent matrices of effect measure acquire the comprehensive effect measure of situation c_{ij} is $\otimes\tilde{r}_{i\,j} = \sum_{k=1}^{s} \eta_k \otimes \tilde{r}_{i\,j}^{(k)}$ and achieve the comprehensive matrix of effect measure [3].

$$\otimes\tilde{R} = (\otimes\tilde{r}_{i\,j}) = \begin{bmatrix} \otimes\tilde{r}_{11} & \otimes\tilde{r}_{12} & \cdots & \otimes\tilde{r}_{1m} \\ \otimes\tilde{r}_{21} & \otimes\tilde{r}_{22} & \cdots & \otimes\tilde{r}_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ \otimes\tilde{r}_{n1} & \otimes\tilde{r}_{n2} & \cdots & \otimes\tilde{r}_{nm} \end{bmatrix}$$

Find solutions for multi-objective transportation problem from comprehensive matrix $\otimes\tilde{r}_{i\,j}$ of effect measure using modified distribution method in LINGO package and here supply and demand shipped from origins to destinations is already given.

4. Developed Algorithm

Input
Effect measure matrix $\otimes\tilde{U}^{(k)} = \left(\otimes\tilde{U}^{(1)}, \otimes\tilde{U}^{(2)}, ..., \otimes\tilde{U}^{(s)}; n \times m\right)$
Output
Solution of MITP
Compute the efficient solution of MITP using the optimization model of objective weight.
Solve MITP
begin

Step 1 **Read**: Example
while example = MITP do
for k=1 to s do
enter effect measure matrix $\otimes\tilde{U}^{(k)}$
end

Step 2 Find the lower effect measure $\otimes\tilde{r}_{i\,j}^{(k)}$ and upper effect measure $\otimes\tilde{r}_{i\,j}^{(k)}$ for both upper and lower limit and accomplish the consistent matrix of effect measure $\otimes\tilde{R}^{(k)} = \left[\otimes\tilde{r}_{i\,j}^{(k)}\right]$.
For k=1 to s do

$$\otimes\tilde{r}_{i\,j}^{(k)} = \frac{\otimes\tilde{u}_{i\,j}^{(k)}}{\max\limits_{i}\max\limits_{j}\left\{\otimes\tilde{u}_{i\,j}^{(k)}\right\}} \text{ and } \otimes\tilde{r}_{i\,j}^{(k)} = \frac{\min\limits_{i}\min\limits_{j}\left\{\otimes\tilde{u}_{i\,j}^{(k)}\right\}}{\otimes\tilde{u}_{i\,j}^{(k)}}$$

$$\otimes\tilde{R}^{(k)} = \left(\otimes\tilde{r}_{i\,j}^{(k)}\right) = \begin{bmatrix} \otimes\tilde{r}_{11}^{(k)} & \otimes\tilde{r}_{12}^{(k)} & \cdots & \otimes\tilde{r}_{1m}^{(k)} \\ \otimes\tilde{r}_{21}^{(k)} & \otimes\tilde{r}_{22}^{(k)} & \cdots & \otimes\tilde{r}_{2m}^{(k)} \\ \cdots & \cdots & \cdots & \cdots \\ \otimes\tilde{r}_{n1}^{(k)} & \otimes\tilde{r}_{n2}^{(k)} & \cdots & \otimes\tilde{r}_{nm}^{(k)} \end{bmatrix}$$

end

Step 3 Subtract each value from 1 of consistent matrix of effect measure.
for k=1 to s do
$\otimes\tilde{R}^{(k)} = 1 - \otimes\tilde{R}^{(k)}$
end

Step 4 Find the positive and negative ideal vector of effect measure $\otimes\tilde{v}_i^{(k)+}$ and $\otimes\tilde{v}_i^{(k)-}$ for both upper and lower limit respectively.
for k=1 to s do
$\otimes\tilde{v}_i^{(k)+} = \max\limits_{j}\otimes\tilde{r}_{i\,j}^{(k)}$ and $\otimes\tilde{v}_i^{(k)-} = \min\limits_{j}\otimes\tilde{r}_{i\,j}^{(k)}$
end

Step 5 Find the deviation $d^{+}\left(\otimes\tilde{r}_{i\,j}^{(k)} - \otimes\tilde{v}_i^{(k)+}\right)$ and $d^{-}\left(\otimes\tilde{r}_{i\,j}^{(k)} - \otimes\tilde{v}_i^{(k)-}\right)$ of upper and lower limit for objective $k = 1, 2, , s$ after that find the total deviation under all targets.
for k=1 to s do
$d^{+}\left(\otimes\tilde{r}_{i\,j}^{(k)}, \otimes\tilde{v}_i^{(k)+}\right) = \left|\otimes\tilde{r}_{i\,j}^{(k)} - \otimes\tilde{v}_i^{(k)+}\right|$
and
$d^{-}\left(\otimes\tilde{r}_{i\,j}^{(k)}, \otimes\tilde{v}_i^{(k)-}\right) = \left|\otimes\tilde{r}_{i\,j}^{(k)} - \otimes\tilde{v}_i^{(k)-}\right|$
end
for k=1 to s do
$D^{+} = \sum\limits_{i=1}^{n}\sum\limits_{j=1}^{m} d^{+}\left(\otimes\tilde{r}_{i\,j}^{(k)}, \otimes\tilde{v}_i^{(k)+}\right)$ and $D^{-} = \sum\limits_{i=1}^{n}\sum\limits_{j=1}^{m} d^{-}\left(\otimes\tilde{r}_{i\,j}^{(k)}, \otimes\tilde{v}_i^{(k)-}\right)$
end

Step 6 Select the balance coefficient μ $(1 \prec \mu \prec 1/2)$ between the objectives and compute the objectives weight $\eta_1, \eta_2, ..., \eta_s$. $\mu = 1/3$
for k=1 to s do
η_k

end

Step 7 Get the comprehensive effect measure matrix for situation s_{ij} is $\otimes\tilde{R} = [\otimes\tilde{r}_{ij}]$.
for k=1 to s do
$\otimes\tilde{R} = [\otimes\tilde{r}_{ij}] = \sum_{k=1}^{s} \otimes\tilde{r}_{ij}^{(k)} \cdot \eta_k$
end

Step 8 Find solutions for multi-objective interval transportation problem from comprehensive matrix $\otimes\tilde{R} = [\otimes\tilde{r}_{ij}]$ of effect measure using modified distribution method in LINGO package.
end

5. Illustration Examples

To illustrate the above method, consider the following examples of multi-objective transportation problem.
Numerical illustration 1: A company has three production facilities (origins) A_1, A_2 and A_3 with production capacity of 8, 19 and 17 units of a product respectively. These units are to be shipped to four warehouses B_1, B_2, B_3 and B_4 with requirement of 11, 3, 14 and 16 units respectively. The transportation cost and transportation time between companies to warehouses are given below [6].

$$\otimes\tilde{U}^{(1)} = \begin{bmatrix} [1,2] & [1,3] & [5,9] & [4,8] \\ [1,2] & [7,10] & [2,6] & [3,5] \\ [7,9] & [7,11] & [3,5] & [5,7] \end{bmatrix}$$

$$\otimes\tilde{U}^{(2)} = \begin{bmatrix} [3,5] & [2,6] & [2,4] & [1,5] \\ [4,6] & [7,9] & [7,10] & [9,11] \\ [4,8] & [1,3] & [3,6] & [1,2] \end{bmatrix}$$

Solution: Consider case set, counter set and situation set. Production facilities of company are the case. Let $A = \{A_1, A_2, A_3\}$ is the case set and A_1, A_2 and A_3 on the behalf of three production facilities of company (origins). Destination is the counter. $B = \{B_1, B_2, B_3, B_4\}$ is the counter set and B_1, B_2, B_3 and B_4 on the behalf of four destinations.
Situation set $C = \{c_{ij} = (A_i, B_j) \mid A_i \in A, B_j \in B\}$ is structured by A and B.
1) Authenticate the decision-making goals time and cost and after that consider the upper limit of intervals of the matrices as a one matrix and similarly

consider the matrices for the lower limits. So the effect measure matrices under two goals are given below.

$$U^{(1)L} = \begin{bmatrix} 1 & 1 & 5 & 4 \\ 1 & 7 & 2 & 3 \\ 7 & 7 & 3 & 5 \end{bmatrix} \quad U^{(1)U} = \begin{bmatrix} 2 & 3 & 9 & 8 \\ 2 & 10 & 6 & 5 \\ 9 & 11 & 5 & 7 \end{bmatrix}$$

$$U^{(2)L} = \begin{bmatrix} 3 & 2 & 2 & 1 \\ 4 & 7 & 7 & 9 \\ 4 & 1 & 3 & 1 \end{bmatrix} \quad U^{(2)U} = \begin{bmatrix} 5 & 6 & 4 & 5 \\ 6 & 9 & 10 & 11 \\ 8 & 3 & 6 & 2 \end{bmatrix}$$

2) For transporting a product, time and cost are "the less, the batter", so use lower effect measure. Here, the lower effect measure for first data $r_{11}^{(1)L} = \frac{\min\limits_{1}\min\limits_{1}\{u_{11}^{L}\}}{u_{11}^{L}} = \frac{1}{1} = 1$. Similarly obtain lower effect measure for each data of the matrices. Therefore the consistent matrices of effect measure are given below.

$$U^{(1)L} = \begin{bmatrix} 1 & 1 & 0.2 & 0.25 \\ 1 & 0.14286 & 0.5 & 0.33333 \\ 0.14286 & 0.14286 & 0.66667 & 0.6 \end{bmatrix}$$
$$U^{(1)U} = \begin{bmatrix} 1 & 0.66667 & 0.22222 & 0.25 \\ 1 & 0.2 & 0.33333 & 0.4 \\ 0.22222 & 0.27273 & 1 & 0.71429 \end{bmatrix}$$
$$U^{(2)L} = \begin{bmatrix} 0.33333 & 0.5 & 0.5 & 1 \\ 0.75 & 0.14286 & 0.28571 & 0.11111 \\ 0.25 & 1 & 0.33333 & 1 \end{bmatrix}$$
$$U^{(2)U} = \begin{bmatrix} 0.8 & 0.5 & 1 & 0.4 \\ 0.83333 & 0.33333 & 0.4 & 0.18182 \\ 0.25 & 0.66667 & 0.33333 & 1 \end{bmatrix}$$

3) Subtract each value from 1 of consistent effect measure matrix under target $k = 1, 2$.

$$U^{(1)L} = \begin{bmatrix} 0 & 0 & 0.8 & 0.75 \\ 0 & 0.85714 & 0.5 & 0.66667 \\ 0.85714 & 0.85714 & 0.33333 & 0.4 \end{bmatrix}$$
$$U^{(1)U} = \begin{bmatrix} 0 & 0.33333 & 0.77778 & 0.75 \\ 0 & 0.8 & 0.66667 & 0.6 \\ 0.77778 & 0.72727 & 0 & 0.28571 \end{bmatrix}$$
$$U^{(2)L} = \begin{bmatrix} 0.66667 & 0.5 & 0.5 & 0 \\ 0.25 & 0.85714 & 0.71429 & 0.88889 \\ 0.75 & 0 & 0.66667 & 0 \end{bmatrix}$$
$$U^{(2)U} = \begin{bmatrix} 0.2 & 0.5 & 0 & 0.6 \\ 0.16667 & 0.66667 & 0.6 & 0.81818 \\ 0.75 & 0.33333 & 0.66667 & 0 \end{bmatrix}$$

4) The positive and the negative ideal vector of effect measure for both objectives are given below:

The positive ideal vector of effect measure for two objectives	The negative ideal vector of effect measure for two objectives
For first objective k=1	
$v_1^{(1)L+} = \max\limits_j \left\{ r_{1j}^{(1)L} \right\} = 0.8$	$v_1^{(1)L-} = \min\limits_j \left\{ r_{1j}^{(1)L} \right\} = 0$
$v_2^{(1)L+} = \max\limits_j \left\{ r_{2j}^{(1)L} \right\} = 0.85714$	$v_2^{(1)L-} = \min\limits_j \left\{ r_{2j}^{(1)L} \right\} = 0$
$v_3^{(1)L+} = \max\limits_j \left\{ r_{3j}^{(1)L} \right\} = 0.85714$	$v_3^{(1)L-} = \min\limits_j \left\{ r_{3j}^{(1)L} \right\} = 0.33333$
$v_1^{(1)U+} = \max\limits_j \left\{ r_{1j}^{(1)U} \right\} = 0.77778$	$v_1^{(1)U-} = \min\limits_j \left\{ r_{1j}^{(1)U} \right\} = 0$
$v_2^{(1)U+} = \max\limits_j \left\{ r_{2j}^{(1)U} \right\} = 0.8$	$v_2^{(1)U-} = \min\limits_j \left\{ r_{2j}^{(1)U} \right\} = 0$
$v_3^{(1)U+} = \max\limits_j \left\{ r_{3j}^{(1)U} \right\} = 0.77778$	$v_3^{(1)U-} = \min\limits_j \left\{ r_{3j}^{(1)U} \right\} = 0$
For first objective k=2	
$v_1^{(2)L+} = \max\limits_j \left\{ r_{1j}^{(2)L} \right\} = 0.66667$	$v_1^{(2)L-} = \min\limits_j \left\{ r_{1j}^{(2)L} \right\} = 0$
$v_2^{(2)L+} = \max\limits_j \left\{ r_{2j}^{(2)L} \right\} = 0.88889$	$v_2^{(2)L-} = \min\limits_j \left\{ r_{2j}^{(2)L} \right\} = 0.25$
$v_3^{(2)L+} = \max\limits_j \left\{ r_{3j}^{(2)L} \right\} = 0.75$	$v_3^{(2)L-} = \min\limits_j \left\{ r_{3j}^{(2)L} \right\} = 0$
$v_1^{(2)U+} = \max\limits_j \left\{ r_{1j}^{(2)U} \right\} = 0.6$	$v_1^{(2)U-} = \min\limits_j \left\{ r_{1j}^{(2)U} \right\} = 0$
$v_2^{(2)U+} = \max\limits_j \left\{ r_{2j}^{(2)U} \right\} = 0.81818$	$v_2^{(2)U-} = \min\limits_j \left\{ r_{2j}^{(2)U} \right\} = 0.16667$
$v_3^{(2)U+} = \max\limits_j \left\{ r_{3j}^{(2)U} \right\} = 0.75$	$v_3^{(2)U-} = \min\limits_j \left\{ r_{3j}^{(2)U} \right\} = 0$

5)

$$\sum_{i=1}^{3}\sum_{j=1}^{4} d^-\left(r_{ij}^{(1)L} - v_i^{(1)L-}\right) = 4.688095,\ \sum_{i=1}^{3}\sum_{j=1}^{4} d^+\left(r_{ij}^{(1)L} - v_i^{(1)L+}\right) = 4.035714$$

$$\sum_{i=1}^{3}\sum_{j=1}^{4} d^-\left(r_{ij}^{(1)U} - v_i^{(1)U-}\right) = 5.718543,\ \sum_{i=1}^{3}\sum_{j=1}^{4} d^+\left(r_{ij}^{(1)U} - v_i^{(1)U+}\right) = 3.70368$$

$$\sum_{i=1}^{3}\sum_{j=1}^{4} d^-\left(r_{ij}^{(2)L} - v_i^{(2)L}\right) = 4.79365,\ \sum_{i=1}^{3}\sum_{j=1}^{4} d^+\left(r_{ij}^{(2)L} - v_i^{(2)L+}\right) = 3.42857$$

$$\sum_{i=1}^{3}\sum_{j=1}^{4} d^-\left(r_{ij}^{(2)U} - v_i^{(2)U-}\right) = 4.63485,\ \sum_{i=1}^{3}\sum_{j=1}^{4} d^+\left(r_{ij}^{(2)U} - v_i^{(2)U+}\right) = 3.37121$$

6) Give the equilibrium coefficient $\mu = 1/3$ and calculate the weight of the objectives η_1, η_2,....,η_s using equation (2). The weights of the objective are $\eta_1^L = 0.329003$, $\eta_2^L = 0.670997$, $\eta_1^U = 0.679446, \eta_2^U = 0.320554$.

7) The comprehensive matrix of effect measure is got according to $\otimes\tilde{r}_{i\,j} = \sum_{k=1}^{s} \eta_k \otimes \tilde{r}_{i\,j}^{(k)}$;

$$\otimes\tilde{r}_{i\,j} = \begin{bmatrix} [0.44733, 0.06412] & [0.3355, 0.38677] & [0.5987, 0.52842] & [0.24675, 0.70191] \\ [0.16775, 0.05343] & [0.85714, 0.75725] & [0.64379, 0.64529] & [0.81578, 0.66995] \\ [0.78525, 0.76887] & [0.282, 0.60009] & [0.557, 0.21373] & [0.1316, 0.19411] \end{bmatrix}$$

8) Solution of comprehensive matrix of effect measure using modified distribution method in LINGO package.

1. Solution of comprehensive matrix of effect measure for lower limit
   $x_{12} = 2, x_{13} = 6, x_{21} = 11, x_{23} = 8, x_{32} = 1, x_{34} = 16$

2. Solution of comprehensive matrix of effect measure for upper limit
   $x_{12} = 3, x_{13} = 5, x_{21} = 11, x_{23} = 8, x_{33} = 1, x_{34} = 16$

$$\begin{aligned} U^{(1)L} &= 2 \times 1 + 6 \times 5 + 11 \times 1 + 8 \times 2 + 1 \times 7 + 16 \times 5 = 146 \\ U^{(2)L} &= 2 \times 2 + 6 \times 2 + 11 \times 4 + 8 \times 7 + 1 \times 1 + 16 \times 1 = 133 \\ U^{(1)U} &= 3 \times 3 + 5 \times 9 + 11 \times 2 + 8 \times 6 + 1 \times 5 + 16 \times 7 = 241 \\ U^{(2)U} &= 3 \times 6 + 5 \times 4 + 11 \times 6 + 8 \times 10 + 1 \times 6 + 16 \times 2 = 222 \end{aligned}$$

Comparison

Deepika Rani [4] (Linear membership function)	Deepika Rani [4] (Exponential membership function)	Deepika Rani [4] (Hyperbolic membership function)	S. K. Das, A. Goswami and S. S. Alam [6]	Developed Method
[172.2,222.55]	[171.50,221.63]	[172.2,222.55]	[119.14,214.42]	[146,241]
[206.1,252.75]	[207.54,254.36]	[206.1,252.75]	[180.64,241.1]	[133,222]

Numerical illustration 2: A company has three production facilities (origins) A_1, A_2 and A_3 with production capacity of [7, 9], [17, 21] and [16, 18] units of a product respectively. These units are to be shipped to four warehouses B_1, B_2, B_3 and B_4 with requirement of [10, 12], [2, 4], [13, 15] and [15, 17] units respectively. The transportation cost and transportation time between companies to warehouses are given below [4].

$$\otimes\tilde{U}^{(1)} = \begin{bmatrix} [1,2] & [1,3] & [5,9] & [4,8] \\ [1,2] & [7,10] & [2,6] & [3,5] \\ [7,9] & [7,11] & [3,5] & [5,7] \end{bmatrix}$$

$$\otimes\tilde{U}^{(2)} = \begin{bmatrix} [3,5] & [2,6] & [2,4] & [1,5] \\ [4,6] & [7,9] & [7,10] & [9,11] \\ [4,8] & [1,3] & [3,6] & [1,2] \end{bmatrix}$$

Solution: Solutions of comprehensive matrix of effect measure using LINGO Software.

1. Solution of comprehensive matrix of effect measure for lower limit
   $x_{12} = 1$, $x_{13} = 6$, $x_{21} = 10$, $x_{23} = 1$, $x_{32} = 1$, $x_{34} = 1$
2. Solution of comprehensive matrix of effect measure for upper limit
   $x_{12} = 2$, $x_{13} = 5$, $x_{21} = 12$, $x_{23} = 5$, $x_{33} = 3$, $x_{34} = 15$

$$U^{(1)L} = 1 \times 1 + 6 \times 5 + 10 \times 1 + 7 \times 2 + 1 \times 7 + 15 \times 5 = 137$$
$$U^{(2)L} = 1 \times 2 + 6 \times 2 + 10 \times 4 + 7 \times 7 + 1 \times 1 + 15 \times 1 = 119$$
$$U^{(1)U} = 2 \times 3 + 5 \times 9 + 12 \times 2 + 5 \times 6 + 3 \times 5 + 15 \times 7 = 225$$
$$U^{(2)U} = 2 \times 6 + 5 \times 4 + 12 \times 6 + 5 \times 10 + 3 \times 6 + 15 \times 2 = 202$$

Comparison

Deepika Rani [4] (Linear membership function)	Deepika Rani [4] (Exponential membership function)	Deepika Rani [4] (Hyperbolic membership function)	Developed Method
[159.02, 205.03]	[158.95,204.90]	[159.02,205.03]	[137,225]
[176.54, 227.95]	[186.48,227.95]	[176.54,227.95]	[119,202]

Numerical illustration 3: A company has three production facilities (origins) A_1, A_2 and A_3 with production capacity of [7, 9], [17, 21] and [16, 18] units of a product respectively. These units are to be shipped to four warehouses B_1, B_2, B_3 and B_4 with requirement of [10, 12], [2, 4], [13, 15] and [15, 17] units respectively. The transportation cost and transportation time between companies to warehouses are given below [4].

$$\otimes\tilde{U}^{(1)} = \begin{bmatrix} 1 & 2 & 7 & 7 \\ 1 & 9 & 3 & 4 \\ 8 & 9 & 4 & 6 \end{bmatrix}, \qquad \otimes\tilde{U}^{(2)} = \begin{bmatrix} 4 & 4 & 3 & 3 \\ 5 & 8 & 9 & 10 \\ 6 & 2 & 5 & 1 \end{bmatrix}$$

Solution: Solutions of comprehensive matrix of effect measure using LINGO Software.
$x_{12} = 2$, $x_{13} = 5$, $x_{21} = 12$, $x_{23} = 5$, $x_{33} = 3$, $x_{34} = 15$

$$U^{(1)} = 2 \times 2 + 5 \times 7 + 12 \times 1 + 5 \times 3 + 3 \times 4 + 15 \times 6 = 168$$
$$U^{(2)} = 2 \times 4 + 5 \times 3 + 12 \times 5 + 5 \times 9 + 3 \times 5 + 15 \times 1 = 158$$

Comparison

Numerical illustration 4: A company has three production facilities (origins) A_1, A_2 and A_3 with production capacity of 12, 16 and 20 units of

Deepika Rani [4] (Linear membership function)	Deepika Rani [4] (Exponential membership function)	Deepika Rani [4] (Hyperbolic membership function)	Developed Method
149.6	149.59	149.6	168
174	173.99	174	158

a product respectively. These units are to be shipped to four warehouses B_1, B_2, B_3 and B_4 with requirement of 15, 12, 10 and 11 units respectively. The transportation cost and transportation time between companies to warehouses are given below.

$$\otimes\tilde{U}^{(1)} = \begin{bmatrix} [0.5, 1.1] & [0.8, 1.2] & [1.3, 1.7] & [1.8, 2.2] \\ [1.2, 1.6] & [1.3, 1.5] & [1.5, 1.9] & [2.0, 2.4] \\ [1.6, 2.0] & [1.8, 2.2] & [2.2, 2.4] & [1.2, 1.6] \end{bmatrix}$$

$$\otimes\tilde{U}^{(2)} = \begin{bmatrix} [0.4, 0.9] & [0.7, 1.1] & [1.2, 1.5] & [1.6, 2.0] \\ [1.1, 1.8] & [0.9, 1.4] & [1.3, 1.8] & [2.1, 2.5] \\ [1.9, 2.2] & [1.5, 2.0] & [2.3, 2.7] & [1.4, 2.1] \end{bmatrix}$$

Solution: Solutions of comprehensive matrix of effect measure using LINGO Software.

1. Solution of comprehensive matrix of effect measure for lower limit
   $x_{11} = 12,\ x_{22} = 6,\ x_{23} = 10,\ x_{31} = 3,\ x_{32} = 6,\ x_{34} = 11$
2. Solution of comprehensive matrix of effect measure for upper limit
   $x_{11} = 12,\ x_{22} = 12,\ x_{23} = 4,\ x_{31} = 3,\ x_{33} = 6,\ x_{34} = 11$

$$\begin{aligned} U^{(1)L} &= 12 \times 0.5 + 6 \times 1.3 + 10 \times 1.5 + 3 \times 1.6 + 6 \times 1.8 + 11 \times 1.2 = 57.6 \\ U^{(2)L} &= 12 \times 0.4 + 6 \times 0.9 + 10 \times 1.3 + 3 \times 1.9 + 6 \times 2.3 + 11 \times 1.4 = 53.3 \\ U^{(1)U} &= 12 \times 1.1 + 12 \times 1.5 + 4 \times 1.9 + 3 \times 2.0 + 6 \times 2.4 + 11 \times 1.6 = 76.8 \\ U^{(2)U} &= 12 \times 0.9 + 12 \times 1.4 + 4 \times 1.8 + 3 \times 2.2 + 6 \times 2.7 + 11 \times 2.1 = 80.7 \end{aligned}$$

Numerical illustration 5: A company has three production facilities (origins) A_1, A_2 and A_3 with production capacity of 18, 10 and 22 units of a product respectively. These units are to be shipped to four warehouses B_1, B_2, B_3 and B_4 with requirement of 16, 13, 12 and 9 units respectively. The transportation cost and transportation time between companies to warehouses are given below.

$$\otimes\tilde{U}^{(1)} = \begin{bmatrix} [4, 6] & [5, 8] & [7, 9] & [8, 12] \\ [6, 8] & [7, 13] & [10, 14] & [15, 19] \\ [8, 12] & [10, 14] & [16, 20] & [8, 10] \end{bmatrix}$$

$$\otimes\tilde{U}^{(2)} = \begin{bmatrix} [2, 5] & [4, 7] & [6, 8] & [7, 11] \\ [6, 10] & [8, 11] & [9, 12] & [12, 16] \\ [10, 14] & [12, 15] & [14, 18] & [18, 20] \end{bmatrix}$$

Solution: Solutions of comprehensive matrix of effect measure using LINGO Software.

1. Solution of comprehensive matrix of effect measure for lower limit
   $x_{11} = 16,\ x_{12} = 2,\ x_{23} = 10,\ x_{32} = 11,\ x_{33} = 2,\ x_{34} = 9$

2. Solution of comprehensive matrix of effect measure for upper limit
   $x_{11} = 16,\ x_{13} = 2,\ x_{23} = 10,\ x_{32} = 13,\ x_{34} = 9$

$$U^{(1)L} = 16 \times 4 + 2 \times 5 + 10 \times 10 + 11 \times 10 + 2 \times 16 + 9 \times 8 = 388$$
$$U^{(2)L} = 16 \times 2 + 2 \times 4 + 10 \times 9 + 11 \times 12 + 2 \times 14 + 9 \times 18 = 452$$
$$U^{(1)U} = 16 \times 6 + 2 \times 9 + 10 \times 14 + 13 \times 14 + 9 \times 10 = 526$$
$$U^{(2)U} = 16 \times 5 + 2 \times 8 + 10 \times 12 + 13 \times 15 + 9 \times 20 = 591$$

6. Conclusion

This paper present the compromise solution of multi-objective interval transportation problem obtained using grey situation decision making theory based method with objective weights. The comparison shows that the compromise solution is better nd acceptable in real life situation when more than one objective available in transporting a product.

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