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# INTEGRAL MODEL AND FORECASTING OF VECTOR DYNAMIC SERIES OF ECONOMIC PARAMETERS

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**Abstract:** In the assumption of existence of forecasting transformation and under condition of its continuity we give a way (algorithm) of construction of this transformation. This transformation is applied to forecasting of economic processes which are defined by a large number of factors. The offered model of forecasting most considers mutual influence of change of all quantitative indices in big system in the reporting period on result of each parameter in the perspective period. Universality of model allows to make easily its further modification for use at the solution of a wide range of economic, production, marketing and financial tasks, everywhere, where the effective forecast allows to rationalize administrative decisions and to receive qualitative results in the future. Also, this method can easily be parallelized.

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## 1. Introduction

The forecast of key macroeconomic indicators for the short and medium terms is a fundamentally significant issue in terms of forming economic policy: it is the forecast parameters that the basic characteristics of the future tax and monetary policies depend on.

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Economic forecast is based on hypothesis that future economy condition is predetermined by its past and present conditions. The matter is that management processes in industry, economy, and financial business are characterized by certain stability, inertness, existing structure and interrelations. Thus, forecast methods are based on extrapolation of behaviour or development of the objects in the future based on the tendency of their behaviour in the past and present. The main content of the economic forecast is qualitative and quantitative analysis of economical processes and detection of their development tendencies. A rather detailed analysis and comparison of the major classes of economic forecast models in use are given in the work [1].

Economical phenomena, as a rule, are defined by a large number of factors acting simultaneously and collectively. It is natural to suppose that economic forecasting of quantitative parameters of large systems having strong inner ties should be conducted jointly, taking into consideration their mutual influence on each other. Therefore, the choice of a single universal forecasting model, which considers former behaviour of the entire system of indicators as a whole, is becoming particularly important in modelling and forecasting of large systems.

In the present work, we suggest ***new econometric model***, which enriches the forecasting theory. It suggests ***new formula and instrument for accurate and flexible combined forecasting of quantitative parameters in large systems***.

This formula, considering the mutual influence of the elements of large systems in the accounting period, enables more correct forecast calculation for the perspective period. At the same time, our model is perfectly accurate when it is tested on any interval “within” the accounting period.

The results of this work were announced in [2]. In the work [3] similar problem is considered, with different approach to the construction of forecasting transformation.

## 2. Formalization of the Economic Problem

Let the statistical data recorded in a form of a table with $N$ rows and $T$ columns be given, i.e. let the values of data $S_n(k)$, $n = \overline{1, N}$ at points $k = 1, 2, \ldots, T$, where $T$ is an integer, be known. These data are the quantitative indicators of a certain economic parameter $S_n(t)$ at the moment of observation $t = k$. These economic parameters can be, for instance, such macroeconomic indicators as elements of the table of intersectoral balance and (or) systems of national accounts, indicators of social-economical progress estimates, as well as any groups of microeconomic indicators.

Building an econometric model is caused by a necessity to calculate the forecast value $S_n(T+1)$, $n = \overline{1, N}$:

$$\begin{pmatrix} S_1(1) & S_1(2) & \cdots & S_1(T) \\ S_2(1) & S_2(2) & \cdots & S_2(T) \\ \cdots & \cdots & \cdots & \cdots \\ S_N(1) & S_N(2) & \cdots & S_N(T) \end{pmatrix} \Rightarrow \begin{pmatrix} S_1(T+1) \\ S_2(T+1) \\ \cdots \\ S_N(T+1) \end{pmatrix}.$$

Firstly, let us move from studying the discrete case to the continuous one. To do so for every $n = \overline{1, N}$ we graph broken lines $S_n(t)$, connecting points $S_n(k)$, $k = \overline{1, T}$ with segments. This way we get continuous functions $S_n(t)$, $1 \leq t \leq T$ for each $1 \leq n \leq N$. Hence we have got vector-function $\vec{S}(t) = (S_1(t),\ S_2(t),\ \ldots,\ S_N(t))$, given and continuous on the interval $1 \leq t \leq T$, while the problem of forecasting parameters $S_n(T+1)$, $n = \overline{1, N}$ is reduced to the problem of forecasting values of the vector-function $\vec{S}(t)$ for the moment of time $t = T + 1$.

Due to the fact that there is some error or "noise" in the statistical material, it is practical to forecast not the function $\vec{S}(t)$ itself, but some average of it. This is consistent with the fact that, as we have mentioned earlier, the most important classical problem in studying non-stationary economic time series is the identification of the main tendency of the studied process development and building the forecast model based on it.

The following averaging technique is proposed. For an arbitrary non-negative function $q(t)$ defined for $t \leq T$ and every small number $\varepsilon > 0$, suppose

$$q_\varepsilon(t) = \frac{1}{\varepsilon} \int_{t-\varepsilon}^{t} q(\eta) d\eta, \tag{1}$$

$$\tilde{q}_\varepsilon(t) = \inf_{a>0} \left\{ a^{-2}\varepsilon : a^{-2}\varepsilon \geq \frac{1}{a} \int_{t-a}^{t} q(\eta) d\eta \right\}. \tag{2}$$

It should be noted here that the function $q_\varepsilon(t)$ is substantially simpler than function $\tilde{q}_\varepsilon(t)$. Nevertheless, it is more advisable to use averaging $\tilde{q}_\varepsilon(t)$, as it is mathematically "more flexible" averaging of the function $q(t)$ than averaging $q_\varepsilon(t)$. Averagings of type (2) were introduced by one of the authors of this work and were successfully applied to the solution of important problems of spectral theory of differential operators.

Hereinafter, we consider that $\varepsilon > 0$ is some small number.

$$\vec{S}_\varepsilon(t) = (S_{\varepsilon 1}(t), S_{\varepsilon 2}(t), \ldots, S_{\varepsilon N}(t))$$

will be used to denote the vector-function, elements $S_{\varepsilon n}(t)$ of which are formed from corresponding $S_n(t)$ elements of the vector-function $\vec{S}(t)$ based on one of the formulae (1) or (2).

## 3. Formulation of Main Forecasting Problem

We will consider the investigated time series as consisting of already averaged vector-functions $\vec{S}(t)$ (based on one of the formulae (1) or (2)) and for the convenience the index $\varepsilon$ will be omitted.

Let $T_0$ be an integer, $1 \le T_0 < T$. Interval $[T_0, T]$ we will call a *reference interval.*

**Problem (direct forecasting).** *Forecast the value of* $\vec{S}(t)$ *at the point* $t = T + 1$, *based on statistical data* $\vec{S}(t)$ *given on the reference interval* $[T_0,\ T]$.

Note that the problem is defined for forecasting one step ahead using the data only from the reference interval $[T_0,\ T]$. Other data, known on the interval $1 \le t < T_0$, will be used for building mathematical model and are not directly involved in the forecasting.

Let $N_0$ be an integer, so that $1 \le N_0 < N$.

In the current work, we will only consider the solution of this problem. Note that conditional and inverse forecasting problems can also be solved using the new model introduced in this work, and this will be described in detail in our subsequent works.

## 4. Building an Econometric Model

Let $G(t)$ be some matrix-function of $N \times N$ dimension, with elements $G_{ij}(t)$, for $1 \le i, j \le N$, given and continuous on the interval $[T_0,\ T]$:

$$G(t) = \begin{pmatrix} G_{11}(t) & G_{12}(t) & \cdots & G_{1N}(t) \\ G_{21}(t) & G_{22}(t) & \cdots & G_{2N}(t) \\ \cdots & \cdots & \cdots & \cdots \\ G_{N1}(t) & G_{N2}(t) & \cdots & G_{NN}(t) \end{pmatrix}. \tag{3}$$

Let us introduce an integral operator $F$, which acts according to the following formula:

$$F\left(\vec{S}(t)\right) = \int_{T_0}^{T} G(t)\vec{S}(t)dt \tag{4}$$

in the space of $N$-dimensional vector-functions $\vec{S}(t) = (S_1(t), S_2(t), \ldots, S_N(t))$ given and continuous on the segment $[T_0, T]$.

As all functions considered are continuous, the integral in (4) can be understood in the sense of Riemann.

The **main formula, which describes the econometric model of the time series** $\vec{S}(t)$ for $1 \leq t \leq T$, proposed by us, has the following form:

$$\vec{S}(k+1) = F\left(\vec{S}(t+k-T)\right) \tag{5}$$

for all integer $k: \ T - T_0 + 1 \leq k \leq T - 1$.

Taking (4) into consideration, it can be written down in the following form:

$$\vec{S}(k+1) = \int_{T_0}^{T} G(t)\vec{S}(t+k-T)dt, \quad T - T_0 + 1 \leq k \leq T - 1. \tag{6}$$

Thus, in order to build an econometric model it is necessary to select a matrix (3) in such a way that the formulae (5), (6) most adequately (in our case – mathematically accurate) describe the internal relation of the time series elements. In the mathematical part of this work (Section 5) we will justify the possibility of building such matrix $G(t)$ and will give an algorithm for building it (Section 7).

After building the model, assuming that the development pattern established in the past and present based on the formulae (5), (6) can be extrapolated for the future period, we get ***the main formula for forecasting the time series:***

$$\vec{S}(T+1) = F\left(\vec{S}(t)\right), \tag{7}$$

or in more detail:

$$\vec{S}(T+1) = \int_{T_0}^{T} G(t)\vec{S}(t)dt. \tag{8}$$

Operator $F$ will be further called ***a forecasting operator.*** Let us note that from the forecasting point of view, the equations (5), (6) imply that the forecasting operator continuously "learns" from statistical data of the past and present.

It should be noted that the model (5) is accurate "in the past and present". However, the accuracy requirement should not be reduced to an absurdity: thus, for any series of $T$ points $S(k)$, $k = 1, 2, \ldots, T$ it is always possible to find a polynomial of order $T - 1$, which goes through all the points, and, respectively, with a minimal zero sum of square deviations from the points. But in this case, obviously, it is not possible to talk about identifying main trend, since the total (interval) deviation of the polynomial of a high order from any "reasonable" trend will be significant. Issues mentioned above lead to a necessity to introduce additional conditions, which should be satisfied by the forecasting operator.

Firstly, in case when the values of the vector-function $\vec{S}(t)$ on the reference interval $[T_0, T]$ are constant $\vec{S}(t) = Const$, it would be more natural to expect that this value will be preserved for the forecasted period as well: $\vec{S}(T+1) = Const$; i.e. the requirement of "accuracy on constants" is natural. We will apply a more general condition — a requirement of "accuracy on polynomials", which means that if the behavior of the vector-function $\vec{S}(t)$ elements on the reference interval $[T_0, T]$ is described by polynomials, then the forecasted value $\vec{S}(T+1)$ is chosen as the value of these polynomials at point $T+1$. Let us formulate this condition.

**Condition 1.** *For some integer $L$ chosen in advance, if $\vec{P}_L(t)$ is an arbitrary vector-function, elements of which are polynomials of a variable $t$ of an order not higher than $L$, then*

$$F\left(\vec{P}_L(t)\right) = \vec{P}_L(T+1) \tag{9}$$

*or in a more detailed form*

$$\int_{T_0}^{T} G(t)\vec{P}_L(t)dt = \vec{P}_L(T+1). \tag{10}$$

Let us note that conditions (9) and (10) should be satisfied for any vector $\vec{P}_L(t) = (P_{L1}(t), P_{L2}(t), \dots, P_{LN}(t))$, elements of which are polynomials $P_{Ln}(t)$, $1 \le n \le N$, of an order not higher than $L$. In particular, when $L = 0$, Condition 1 coincides with the requirement of "accuracy on constants". The idea of the approximation method accuracy on polynomials (or, generally speaking, on some finite subspace) is quite often used in the section of mathematical theory of functions, which deals with the approximation problem; so we use this most important idea.

**Remark 1.** The requirement of accuracy on polynomials (9), (10) can be substituted by the conditions of accuracy on any linear independent system of vector-functions, for instance, on a system of trigonometric polynomials, etc.

## 5. Mathematical Justification

Let us directly move on to mathematical justification of the possibility to build an econometric model (5), (6), which meets the Condition 1. We will justify the possibility of building such $G(t)$ matrix and will give a short algorithm for its construction.

So, we need to build such a matrix $G(t)$ of form (3) so that the equations (6) and (10) would hold.

The check of condition (10) is significantly facilitated by the following result.

**Lemma 1.** *The Condition 1, i.e.* (9) *or* (10), *holds for any vector polynomial $\vec{P}_L(t)$ of a variable $t$ of an order not higher than $L$, if and only if it holds for all the vectors of form*

$$\vec{P}_{\ell n}(t) = (0, 0, \ldots, 0, \underbrace{t^{\ell}}_{n^{th}\, place}, 0, \ldots, 0), \quad 0 \le \ell \le L, \quad 1 \le n \le N, \tag{11}$$

*i.e. if and only if the matrix $G(t)$ satisfies the equations*

$$\int_{T_0}^{T} G(t)\vec{P}_{\ell\, n}(t)dt = \vec{P}_{\ell\, n}(T+1) \tag{12}$$

*for all integer values* $0 \le \ell \le L, \quad 1 \le n \le N.$

Proof of this lemma is trivial.

The polynomials of type (12) will be further called *test polynomials.*

Let us note that the sum of the vector equations of conditions (12) is equal to $(L+1)\cdot N$. We will denote this value by $M_0 = (L+1)\cdot N$.

Thus, we need to build such a matrix $G(t)$ of the form (3) so that the equations (6) and (12) would hold. The sum of these conditions is $(L+1)\cdot N + (T_0 - 1)$. We will denote this value by $M = M_0 + (T_0 - 1)$.

In order to write down the conditions (6) and (12) uniformly and with a single layout, we introduce the notation with continuous numbering $j = \overline{1, M}$:

$$\vec{a}_j(t) = \begin{cases} \vec{P}_{\ell n}(t), & j = (L+1)(n-1) + (\ell+1),\ 1 \le j \le M_0, \\ \vec{S}(t - T + k), & j = k + M - T + 1,\ M_0 + 1 \le j \le M. \end{cases} \tag{13}$$

Here indices $\ell, n$ and $k$ change in the following intervals:

$$0 \le \ell \le L, \quad 1 \le n \le N, \quad T - T_0 + 1 \le k \le T - 1$$

and are analytically written down through the index $j$ in the following way:

$$\begin{cases} n = \left[\frac{j-1}{L+1}\right] + 1,\ \ell = j - (L+1)\left[\frac{j-1}{L+1}\right] - 1, & \text{for } 1 \le j \le M_0, \\ k = j - M_0 + T - T_0, & \text{for } M_0 + 1 \le j \le M. \end{cases} \tag{14}$$

Hereinafter, symbol $[z]$ will denote an integer part of the number $z$.

Thus, relabeling has been introduced, $T_0 \le t \le T$:

$$\vec{a}_j(t) = \begin{cases} (0,0,\dots,0,\underbrace{t^{j-(L+1)\cdot\left[\frac{j-1}{L+1}\right]-1}}_{\left(\left[\frac{j-1}{L+1}\right]+1\right)\text{-th place}},0,\dots,0), & 1 \le j \le M_0, \\ \vec{S}(t+j-M-1), & M_0+1 \le j \le M. \end{cases}$$

It is obvious that the vectors $\{\vec{a}_i(t)\}_{i=1,M}$ built in such way are continuous on the segment $[T_0, T]$.
Similar to (13) we introduce the following notation:

$$\vec{b}_j = \begin{cases} \vec{P}_{\ell n}(T+1), \quad j=(L+1)\cdot(n-1)+(\ell+1), \quad 1 \le j \le M_0, \\ \vec{S}(k+1), \quad j = k+M-T+1, \quad M_0+1 \le j \le M. \end{cases} \tag{15}$$

Taking the ratio of indices (14) into consideration, formula (15) will be rewritten in the following form:

$$\vec{b}_j = \begin{cases} (0,0,\dots,0,\underbrace{(T+1)^{j-(L+1)\cdot\left[\frac{j-1}{L+1}\right]-1}}_{\left(\left[\frac{j-1}{L+1}\right]+1\right)\text{-th place}},0,\dots,0), & 1 \le j \le M_0, \\ \vec{S}(j-M+T), & M_0+1 \le j \le M. \end{cases} \tag{16}$$

Now taking the introduced notation into account, the conditions (6) and (12) will be written in the following form:

$$\int_{T_0}^{T} G(t)\vec{a}_j(t)dt = \vec{b}_j, \quad 1 \le j \le M. \tag{17}$$

***This is the main equation, based on which we will determine the matrix** $G(t)$**.**

Using $\vec{G}_n(t), \quad 1 \le n \le N$ let us denote the vector build from the coefficients of $n^{th}$ row of the $G(t)$ matrix:

$$\vec{G}_n(t) = (G_{n1}(t),\ G_{n2}(t),\dots,G_{nN}(t)). \tag{18}$$

Let $\Lambda$ be a matrix of $N \times M$ dimension with coefficients $\alpha_{nm}, \quad 1 \le n \le N, \quad 1 \le m \le M$ :

$$\Lambda = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1M} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2M} \\ \cdots & \cdots & \cdots & \cdots \\ \alpha_{N1} & \alpha_{N2} & \cdots & \alpha_{NM} \end{pmatrix}. \tag{19}$$

We will search for $\vec{G}_n(t)$ vectors in the following form:

$$\vec{G}_n(t) = \sum_{i=1}^{M} \alpha_{n\,i}\vec{a}_i(t), \quad 1 \le n \le N. \tag{20}$$

Using the notations (18), we rewrite the formula (17) in the following way:

$$\int_{T_0}^{T} \left(\vec{G}_n(t), \vec{a}_j(t)\right) dt = b_{jn}, \quad 1 \le n \le N, \quad 1 \le j \le M, \tag{21}$$

where $b_{jn}$ is an element of vector $\vec{b}_j = (b_{j1}, b_{j2}, \dots, b_{jN})$, while ( , ) denotes a scalar product of the vectors:

$$(\vec{f}, \vec{g}) = \sum_{n=1}^{N} f_n \cdot g_n, \quad \vec{f} = (f_1, f_2, \dots, f_N), \quad \vec{g} = (g_1, g_2, \dots, g_N).$$

Substituting (20) in (21), we get:

$$\sum_{i=1}^{M} \alpha_{ni} \cdot a_{ij} = b_{jn}, \quad 1 \le n \le N, \quad 1 \le j \le M \tag{22}$$

which denotes

$$a_{ij} = \int_{T_0}^{T} (\vec{a}_i(t), \vec{a}_j(t))dt \equiv \sum_{n=1}^{N} \int_{T_0}^{T} a_{in}(t) \cdot a_{jn}(t)dt. \tag{23}$$

Let us form $A$ and $B$ matrices of $M \times M$ and $M \times N$ dimensions correspondingly using the coefficients $a_{i\,j}$ and $b_{j\,n}$:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1M} \\ a_{21} & a_{22} & \cdots & a_{2M} \\ \cdots & \cdots & \cdots & \cdots \\ a_{M1} & a_{M2} & \cdots & a_{MM} \end{pmatrix}, B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1N} \\ b_{21} & b_{22} & \cdots & b_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ b_{M1} & b_{M2} & \cdots & b_{MN} \end{pmatrix}. \tag{24}$$

***An important stage is the check for nondegeneracy of the matrix*** $A$: $\det A \ne 0$. This implies a linear independence of the system of $\{\vec{a}_i(t)\}_{i=1,M}$ vector-functions, as elements of $L_2\,(T_0, T)$ vector space. We will cover the practical essence of this requirement in the next section.

Let $\det A \ne 0$. Then there is an inverse matrix $A^{-1}$. Taking (19) and (24) into consideration, the equation (22) can be represented in the matrix form:

$$\Lambda \cdot A = B^*, \tag{25}$$

where $B^*$ is a matrix (of $N \times M$ dimension), a transpose of the matrix $B$.

Determine matrix $\Lambda$ out of (25):

$$\Lambda = B^* \cdot A^{-1}. \tag{26}$$

Hence, the matrix $\Lambda$ is found completely and the matrix $G(t)$, given in the form (20), is fully determined. It is a continuous matrix-function in terms of its construction. Let us formulate the result obtained in a form of a theorem.

**Theorem 2.** *Let a system of vectors* $\{\vec{a}_i(t)\}_{i=\overline{1,M}}$ *be such, that matrix* $A$ *defined by formulae* (23) *and* (24) *is invertible, i.e.* $\det A \neq 0$. *Then for any* $\left\{\vec{b}_i\right\}_{i=\overline{1,M}}$ *vectors, there is such a matrix-function* $G(t)$ *continuous on the segment* $[T_0, T]$, *that requirements* (17) *are met.*

**Corollary 1.** *If a given time series* $\vec{S}(k)$, $k = \overline{1,T}$ *and* $T_0$, $L$ *parameters possess such inner property that a matrix* $A$ *built using formulae* (13), (23), (24) *is invertible, then the econometric model of the time series can be implemented according to formulae* (5), (6) *with a reference interval* $[T_0, T]$, *while the main formula of time series forecasting can be implemented according to formulae* (7), (8). *In such case, the formulae are accurate for a polynomial of an order not higher than L, i.e. converts the equations* (9), (10) *from Condition 1 into identity.*

Thus, under certain conditions (the essence of which we will describe later, in the next section), the econometric model of a vector time series introduced by us can be implemented. Additionally, while proving the Theorem we have also given an algorithm for building matrix $G(t)$ (for more details on the algorithm see the Section 7).

Unlike formulae previously used in modeling, as well as the model of vector autoregression, it is not possible to make any other conclusions out of explicit forms (5), (6)regarding relations between the elements of $S_n(k)$, $n = \overline{1,N}$, $k = \overline{1,T}$ time series, except for a conclusion that such dependency exists. Therefore, *we define the main value of such model as the ability to build a forecast of a time series* using formulae (7), (8).

Taking form (20) of matrix $G(t)$ into consideration, the main formula of the forecast (8) will be written in the following form:

$$S_n(T+1) = \int_{T_0}^{T} \left(\vec{G}_n(t), \vec{S}(t)\right) dt = \int_{T_0}^{T} \left(\sum_{m=1}^{M} \alpha_{nm} \vec{a}_i(t), \vec{S}(t)\right) dt$$
$$= \sum_{m=1}^{M} \alpha_{n\,m} \int_{T_0}^{T} \left(\vec{a}_m(t), \vec{S}(t)\right) dt.$$

Denoting here

$$d_m = \int_{T_0}^{T} \left(\vec{a}_m(t), \vec{S}(t)\right) dt, \tag{27}$$

we get

$$S_n(T+1) = \sum_{m=1}^{M} \alpha_{nm} \cdot d_m, \quad \text{for all} \quad n = \overline{1,N}. \tag{28}$$

And this is the final *result of the forecast in a form convenient for calculation.*

If we introduce a vector $\vec{d} = (d_1, d_2, \ldots, d_N)$, coefficients of which are determined by formula (28), then *the main formula of forecasting will be written in a compact (vector-matrix)* form:

$$\vec{S}(T+1) = \Lambda \cdot \vec{d}. \tag{29}$$

## 6. About Conditions of the Model Existence

In this section, we will analyze the theorem conditions, i.e. the conditions, under which the econometric model of a time series can be implemented by formulae (5), (6). In this work, we will not cover in detail the derivation of exact conditions (in terms of initial data) of the forecast operator existence, but will only analyze the possible situations and will give recommendations on their regularization.

According to the results of practical implementation of the models using real economic parameters and indicators, the matrix $A$ built during the process is invertible.

Let us recall that nondegeneracy of the matrix $A$ implies a linear independence of vectors $\{\vec{a}_i(t)\}_{i=1,M}$, i.e. $\det A \neq 0$. A part (for practical implementation of the model this part is significantly big) of these vectors are vector-polynomials of form (11) and, therefore, are linear independent from each other. The rest of the vectors are described by the elements of the given time series. And it is their values due to which problems with degeneracy of the matrix $A$ can arise.

Let us give an example of a dynamic system, for which an econometric model of a time series not always can be implemented by formulae (5), (6).

Let a vector time series of 6 levels be given:

| **t** | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|
| $S_1(t)$ | 2 | 2 | 5 | 2 | 2 | 7 |
| $S_2(t)$ | 3 | 3 | 5 | 3 | 3 | 7 |

Here $T = 6, \quad N = 2$.

We need to choose $T_0$, $L$ parameters for building a model. Let us first choose $T_0 = 5$, the value of $L$ it is not essential for us at this stage. For these values, the model returns the value of the column $k$ based on the values of columns $k-1$ and $k-2$ for all $k = 3, 4, 5, 6$. For instance, for $k = 3$ and $k = 6$ the operator of the main formula of the model for the same values of two column groups ($1^{st}$ and $2^{nd}$; $4^{th}$ and $5^{th}$) should have different values ($3^{rd}$ and $6^{th}$ columns correspondingly):

$$F\begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix} \Rightarrow \begin{pmatrix} 5 \\ 5 \end{pmatrix} \quad \text{and} \quad F\begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix} \Rightarrow \begin{pmatrix} 7 \\ 7 \end{pmatrix}. \tag{30}$$

It is obvious that such operator cannot exist.

From a practical point of view this can mean that the data system used for modeling is not complete and, may be, there is one more indicator, adding which would lead to the correctness of the problem. It is sufficient to add an indicator $S_3(t)$ in our example (this time $T = 6, \quad N = 3$):

| **t** | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|
| $S_1(t)$ | 2 | 2 | 5 | 2 | 2 | 7 |
| $S_2(t)$ | 3 | 3 | 5 | 3 | 3 | 7 |
| $S_3(t)$ | 0 | 0 | 0 | 0 | 1 | 0 |

For this time series the contradictions of form (30) do not arise, since the pairs of columns ($1^{st}$ and $2^{nd}$) and ($4^{th}$ and $5^{th}$) are no longer the same. In this case, matrix $A$ is invertible and the model can be implemented by formulae (5), (6).

Such approach (increasing the number of indicators involved in modeling) is sounder from a scientific point of view; however, it does not allow quick (within the computational algorithm) implementation of these changes without attracting additional experts.

To urgently address the identified problem (degeneracy of matrix $A$) within the computational algorithm it can be advised to introduce changes to the $T_0$, $L$ parameters selection. Thus, it is sufficient to choose $T_0$ parameter equal to 4 or 3, or 2, so that the contradiction of form (30) will not arise anymore. With such choice of $T_0$, the model of time series can be implemented by formulae (5), (6) without changing the amount of initial data.

In addition to that, changing the averaging parameter $\varepsilon$ could also be suggested as a recommendation for regularization of the problem identified when inverting matrix $A$.

## 7. Algorithm

While proving the Theorem, we have also given the algorithm for building the matrix $G(t)$. Let us describe in more detail the stages of the algorithm for building the forecast formula (29). We will outline only the main steps.

1. Input of statistical data in a form of a matrix consisting of $N$ rows and $T$ columns: $S_n(k)$, $n = \overline{1, N}$ for points $k = 1, 2, \ldots, T$.

2. Choose the reference interval $T_0$, the order $L$ of the test polynomials and the averaging parameter $\varepsilon$.

3. For each $n = \overline{1, N}$ construct broken lines $S_n(t)$, connecting the points $S_n(k)$, $k = \overline{1, T}$ with the segments, moving this way from a discrete to a continuous case.

4. For the broken lines $S_n(t)$, build averaging functions $S_{\varepsilon n}(t)$ using one of the formulae (1) or (2).

5. Represent the functions $S_{\varepsilon n}(t)$ and test polynomials $\vec{P}_{\ell n}(t)$ in a form convenient for further calculation, keeping in mind that we will need to calculate definite integrals.

6. Based on the formulae (23) calculate $a_{ij}$ coefficients of the matrix $A$.

7. Check if $\det A \neq 0$ and build an inverse matrix $A^{-1}$ (in case if $\det A = 0$, change $T_0$, $L$ or $\varepsilon$ parameters).

8. Based on formulae (16) calculate $b_{jn}$ coefficients of the matrix $B$.

9. Based on formula (26) find the matrix $\Lambda$.

10. Based on formula (27) build the vector $\vec{d}$.

11. Find the forecasted value $\vec{S}(T + 1)$ based on formula (29).

In general, the algorithm is not complicated. The most time- and labour-consuming calculations in the algorithm are the calculation of integrals and inversion of the big matrix $A$.

## 8. Conclusions

Suggested model of forecasting effectively considers the mutual influence of different economic parameters on each other when they are being forecasted simultaneously. Moreover, the forecasting operator actually "learns" from statistical material from the past. From this point of view, the forecasting model introduced by us is a neuron network.

The practical significance of the work results is that the suggested forecasting model takes into account the mutual influence of changes in quantitative indicators inside a large system in the accounting period on the result of every parameter in the perspective period. Being universal, this model can be further easily modified for applying it to solution of a wide range of economic, production, marketing and financial problems; everywhere, where accurate forecast enables rationalizing management decision and obtaining optimal results in the future.

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