HEAT AND MASS TRANSFER ON ISOTHERMAL INCLINED POROUS PLATE IN THE PRESENCE OF CHEMICAL REACTION

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Abstract: Analytical solution of steady laminar flow of viscous electrically conducting incompressible fluid, over a semi-infinite inclined plate, which is at prescribed mass flux, in a porous medium with heat generation and chemical reaction is presented in the manuscript. The effects of various parameters like Schmidt number, chemical reaction, thermal Grashof number, solutal Grashof number, angle of inclination on the velocity, temperature and concentration are graphically presented are shown. The results obtained, show that these parameters are substantially influence.

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Key Words: heat transfer, mass transfer, heat generation, concentration, inclined wall, chemical reaction

1. Introduction

It is well known that natural convection heat transfer occurs as a result of temperature differences in an enclosure or near a heated or cooled flat plate. Natural convection along an inclined plate has received less attention than the cases of vertical and horizontal plates. However, this configuration is frequently encountered in engineering devices and in the natural environment.

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A number of researchers have considered an inclined, semi-infinite flat plate in their research because of its engineering applications. The study of natural convection flow of inclined plate is presented by the authors (Ganesan and Palani [1, 2] and Sparrow and Husar [3]). Said et al. [4] studied the problem of turbulent natural convection between inclined isothermal plates. Chen [5] presented the analysis to study natural convection flow over a permeable inclined surface with variable wall temperature and concentration. Hossain et al. [6] studied the free convection flow from an isothermal plate inclined at a small angle to the horizontal. Anghel et al. [7] presented a numerical solution of free convection flow past an inclined surface. Bhuvaneswari et al. [8] studied exact analysis of radiation convective flow heat and mass transfer over an inclined plate in a porous medium. Sivasankaran [9] presented a Lie group analysis of natural convection heat and mass transfer in an inclined surface. Shit and Halder [10] studied MHD flow, heat and mass transfer over an inclined permeable stretching sheet with thermal radiation and hall effect.

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Natural convection processes involving the combined mechanisms are also encountered in many natural processes, such as evaporation, condensation and agricultural drying, and in many industrial applications, such as the curing of plastics, cleaning and chemical processing of materials relevant to the manufacture of printed circuitry, manufacture of pulp-insulated cables, etc. Diffusion rates can be altered tremendously by chemical reactions. The Effect of a chemical reaction depends whether the reaction is homogeneous or heterogeneous. This depends on whether they occur in an interface or as a single phase volume reaction. In a well-mixed system, the reaction is heterogeneous if the reactants are in multiple phase, and homogeneous if the reactants are in the same phase. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is of the order $n$ if the reaction rate is proportional to the $n$th power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to the concentration itself. Kandasamy et al. [11] studied thermophoresis and variable viscosity effects on MHD mixed convective heat and mass transfer past a porous wedge in the presence of chemical reaction. Kandasamy and Devi [12] studied the effects of chemical reaction, heat and mass transfer on non-linear laminar boundary-layer flow over a wedge with suction or injection.
The study of the heat generation or absorption in moving fluids is important while dealing with chemical reactions and those concerned with dissociating fluids. Specifically, the effects of heat generation may alter the temperature distribution, consequently affecting the particle deposition rate in nuclear reactors, electronic chips and semiconductor wafers. In fact, the literature is replete with examples of heat transfer in the laminar flow of viscous fluids. For instance, Vajravelu and Hadjinicolaou [13] studied heat transfer characteristics in the laminar boundary-layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. Kumar [14] investigated heat transfer over a stretching porous sheet subjected to power law heat flux in the presence of a heat source, Askarizadeh and Ahmadikia [15] studied Analytical Analysis of the Dual-phase-lag Heat Transfer Equation in a Finite Slab with Periodic Surface Heat flux and Numerical Study of Natural Convection Heat Transfer in a Horizontal Wavy Absorber is presented by Ziapour and Rahimi [16]. A comprehensive report of unsteady Heat and Mass transfer near the stagnation-point on a vertical permeable surface is presented by Tamim and Abbassi [17].

In the present work, heat and mass transfer over an isothermal inclined porous plate in the presence of a chemical reaction and heat generation or absorption is studied when the wall is at prescribed mass flux. The effects of Schmidt number, Chemical reaction, Prandtl Number, Heat generation parameter, Magnetic field, Porous medium, Thermal Grashof number, Solutal Grashof number and angle of inclination on velocity, concentration and temperature field are presented.

2. Mathematical Analysis

Consider a steady laminar flow of an incompressible viscous electrically conducting fluid past a semi-infinite inclined wall with an acute angle $\phi$ from the vertical in the presence of chemical reaction, embedded in a porous medium. The wall is at a constant temperature $T_w$ which is higher than the ambient temperature $T_\infty$ and at prescribed mass flux. The flow is assumed to be in the $x$ - direction, which is taken along the semi-infinite inclined porous plate and $y$ - axis normal to it. A magnetic field of uniform strength $B_0$ is introduced normal to the direction of the flow. In the analysis, we assume that the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. It is also assumed that all fluid properties are constant. Then, under the usual Boussinesq’s and boundary
layer approximations, the governing equations of the mass, momentum, energy and concentration for the steady flow can be written as,

\[
\frac{\partial v}{\partial y} = 0, \quad (2.1)
\]

\[
v \frac{\partial u}{\partial y} = \vartheta \frac{\partial^2 u}{\partial y^2} + g \beta_T (T - T_\infty) \cos \varphi + g \beta_C (c - c_\infty) \cos \varphi - \frac{\vartheta}{K} u - \frac{\sigma B_0^2 u}{\rho}, \quad (2.2)
\]

\[
v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p} (T - T_\infty), \quad (2.3)
\]

\[
v \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2} - K_l (c - c_\infty), \quad (2.4)
\]

with boundary conditions

\[
u = 0, \quad v = -v_w, \quad T = T_w, \quad -D \frac{\partial c}{\partial y} = m_w, \text{ at } y = 0, \quad (2.5)
\]

\[
u \to 0, \quad T \to T_\infty, \quad c \to c_\infty, \text{ as } y \to \infty.
\]

The equation of continuity (1) with boundary condition (5) changes to

\[
v = -v_w. \quad (2.6)
\]

On assuming the non-dimensional variables as follows:

\[
Y = \frac{y v_w}{\vartheta}, \quad U = \frac{u}{u_w}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad C = \frac{c - c_\infty}{(m_w \vartheta / v_w D)}, \quad (2.7)
\]

\[
Gr_T = \frac{g \beta_T (T_w - T_\infty) \vartheta}{u_w v_w^2}, \quad Gr_C = \frac{g \beta_C m_w \vartheta^2}{u_w v_w^3 D}, \quad K = \frac{K' v_w^2}{\vartheta^2}, \quad M^2 = \frac{B_0^2 \vartheta \sigma}{v_w^2 \rho}, \quad (2.8)
\]

\[
Pr = \frac{\rho \vartheta c_p}{k} = \frac{\vartheta}{\alpha}, \quad S = \frac{Q \vartheta}{\rho c_p v_w^2}, \quad Sc = \frac{\vartheta}{D}, \quad K_c = \frac{\vartheta K_l}{v_w^2}.
\]

On using the above non-dimensional parameters, the equations (2) to (4) reduces to:

\[
\frac{d^2 U}{dY^2} + \frac{dU}{dY} + Gr_T \theta \cos \varphi + Gr_C C \cos \varphi - \frac{1}{K} U - M^2 U = 0 \quad (2.7)
\]

\[
\frac{d^2 \theta}{dY^2} + Pr \frac{d\theta}{dY} + Pr S \theta = 0 \quad (2.8)
\]
\[ \frac{d^2 C}{dY^2} + Sc \frac{dC}{dY} - K_c Sc C = 0 \]  
(2.9)

With the corresponding boundary conditions

\[ U = 0, \quad \theta = 1, \quad \frac{\partial C}{\partial Y} = -1, \quad \text{at} \quad Y = 0 \]
\[ U \to 0, \quad \theta \to 0, \quad C \to 0, \quad \text{as} \quad Y \to \infty \]  
(2.10)

Solution of equation (7) to (9) with boundary condition (10) is as follows:

\[ C = a e^{-bY} \]  
(2.11)
\[ \theta = e^{-dY} \]  
(2.12)
\[ U = p_1 e^{-fY} + p_2 e^{-dY} + p_3 e^{-bY} \]  
(2.13)

Here

\[ a = \frac{2}{Sc + \sqrt{Sc^2 + 4K_c Sc}}, \quad b = \frac{1}{a}, \]
\[ d = \frac{Pr + \sqrt{Pr^2 - 4 Pr S}}{2}, \quad N^2 = M^2 + \frac{1}{K}, \]
\[ f = \frac{1 + \sqrt{1 + 4 N^2}}{2}, \quad p_2 = \frac{-Gr \cos \phi}{d^2 - d - N^2}, \]
\[ p_3 = \frac{-Gr a \cos \phi}{b^2 - b - N^2}, \quad p_1 = -p_2 - p_3 \]

Wall shear stress

\[ \tau_f = \mu \left( \frac{du}{dy} \right)_{y=0} \]
and the skin friction coefficient is defined as

\[ C_f = \frac{\tau_f}{\rho u_w v_w} = \left( \frac{dU}{dY} \right)_{Y=0} = -fp_1 - dp_2 - bp_3 \]  
(2.14)

The effect of convection on the wall heat transfer rate

\[ q_w = -k \left( \frac{dT}{dy} \right)_{y=0} \]
and the Nusselt number is defined as

\[ Nu = \frac{q_w \theta}{(T_w - T_\infty) k v_w} = -\left( \frac{d\theta}{dY} \right)_{Y=0} = d \]  
(2.15)

The expression of dimensionless wall concentration is

\[ C(0) = a \]  
(2.16)
3. Result and Discussion

A study of the velocity field, temperature field, mass transfer, Nusselt number and skin friction of the steady laminar flow of an incompressible viscous electrically conducting fluid past a semi-infinite inclined wall, embedded in a porous medium has been carried out in the present research, governing equations (which are differential equations) (7), (8) and (9) are solved analytically by complementary functions and particular integral method. Then by using a FORTRAN program the obtained values are presented in various figures. Presented results are verified and found in good agreement as that of Kandasamy and Devi [12], Kumar [19, 20] and Ramadan and Chamkha [18].

The effects of Sc on velocity and concentration are presented in Fig. 2 and Fig. 3. Velocity and concentration decreases as Sc increases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the momentum and concentration boundary layer thickness. Physically, the increase of Sc means decrease of molecular diffusivity (D). That results in decrease of concentration boundary layer. Hence, the concentration of the species is higher for small values of Sc and lower for larger values of Sc.

The effects of chemical reaction on velocity and concentration are presented in Fig. 4 and Fig. 5. It is seen from the figures that the velocity and concentration of the fluid decreases with increase of chemical reaction. It is considered to be a homogeneous first-order chemical reaction. The diffusing species either can be destroyed or generated in the homogeneous reaction. The chemical reaction parameter can be adjusted to meet these circumstances if one takes $K_c > 0$ for a destructive reaction, $K_c < 0$ for a generative reaction and $K_c = 0$ for no reaction. The destructive chemical reaction is assumed here. Consequently, the concentration falls off for the increments of the chemical reaction parameter. This shows that the diffusion rates can be tremendously altered by the chemical reaction parameter.

Figure 6 shows the effects of Pr on temperature. The increase of Pr results in the decrease of temperature distribution, it is because of the fact that fluid has a thinner thermal boundary-layer with higher values of Pr.

Figure 7 and 8 show the effects of the heat generation parameter S on the velocity and temperature distribution. It is seen from this figure that when heat is generated the buoyancy force increases, which induces the flow rate to increase, giving rise to the increase in the velocity profiles. The effect of S on temperature profiles are shown in Fig. 8. It is clear that as more heat generated
the temperature also increases.

From Figure 9 it is noticed that, an increase in the magnetic field parameter $M$ leads to a decrease in the velocity. The application of a transverse magnetic field to an electrically conducting fluid rise to a resistive type force called Lorentz force. This force has the tendency to slow down the motion of the fluid. This trend is evident from Figure 9.

Figure 10 shows the effect of porosity on the velocity of the fluid. An increase in $K$ will reduce the resistance of the porous medium which leads to increasing of the velocity.

For various values of thermal Grashof number and solutal Grashof number, the velocity profiles are plotted in Fig. 11 and Fig. 12. The thermal Grashof number $Gr_T$ signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Also, as $Gr_T$ increases, the peak values of the velocity increase rapidly near the porous plate and then decays smoothly to the free stream velocity. The solutal Grashof number $Gr_c$ defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value. It is noticed that the velocity increases with increasing values of the solutal Grashof number.

The effect of inclination of the surface on the velocity is depicted in Fig. 13. The velocity decreases as the angle of inclination ($\phi$) increases. Velocity attains its high peak for vertical plate ($\phi = 0$), the fluid has higher velocity when the surface is vertical than when inclined because of the fact that the buoyancy effect decreases due to gravity components ($g \cos \phi$) as the plate is inclined. It is also interesting to note that the velocity has very good and clear effects of the range $0^\circ - 45^\circ$, (Refer. [18]).

4. Nomenclature

\begin{itemize}
  \item $y$ horizontal coordinate (m)
  \item $u$ axial velocity (m/s)
  \item $v$ transverse velocity (m/s)
  \item $T$ temperature of the fluid (K)
\end{itemize}
\( T_\infty \) far field temperature (K)
\( c \) species concentration (mol/m\(^3\))
\( c_\infty \) far field concentration (mol/m\(^3\))
\( c_w \) concentration on the surface (mol/m\(^3\))
\( g \) acceleration due to gravity (m/s\(^2\))
\( \beta_T \) coefficient of thermal expansion (K\(^{-1}\))
\( \beta_c \) coefficient of concentration expansion (m\(^3\)/mol)
\( \phi \) angle of inclination (degree)
\( \nu \) kinematic viscosity (m\(^2\)/s)
\( K \) permeability of porous medium (m\(^2\))
\( \sigma \) electrical conductivity (S/m)
\( B_0 \) magnetic field coefficient (T)
\( \alpha \) thermal diffusivity (m\(^2\)/s)
\( c_p \) specific heat (J Kg\(^{-1}\) K\(^{-1}\))
\( Q \) heat generation coefficient (W m\(^{-3}\)K\(^{-1}\))
\( \rho \) density (kg/m\(^3\))
\( D \) mass diffusion coefficient (m\(^2\)s\(^{-1}\))
\( K_l \) rate of chemical reaction (s\(^{-1}\))
\( m_w \) wall mass flux (mol/m\(^2\)s)
\( u_w \) surface velocity (m/s)
\( v_w \) suction velocity (m/s)
\( Gr_T \) thermal Grashof number
\( Gr_c \) solutal Grashof number
\( K \) dimensionless permeability parameter
\( M^2 \) magnetic field parameter
Pr Prandtl number,
\( N \) radiation parameter,
\( S \) heat generation parameter
\( Sc \) Schmidt number
\( K_c \) chemical reaction parameter
\( Y \) dimensionless horizontal coordinate
\( U \) dimensionless axial velocity
\( \theta \) dimensionless temperature
\( C \) dimensionless species concentration
Figure 1: Physical Model and Coordinate System

Figure 2: Schmidt number effect on velocity when $K_c = 0.2$, $Pr = 0.71$, $S = 0.1$, $M = 3.0$, $K = 7.0$, $Gr_T = 1.0$, $Gr_c = 0.5$ and $\phi = 30^\circ$
5. Conclusion

In the present research, the effects of Schmidt number, chemical reaction, Prandtl number, heat generation parameter, magnetic field parameter, porous medium, thermal Grashof number, solutal Grashof number and angle of inclination on velocity, concentration and temperature distributions are obtained which can be interpreted as follows for:

1. Velocity increases with the increase of K or S or $Gr_T$ or $Gr_c$ whereas velocity decreases with an increase in M or Sc or $K_c$ or $\phi$.

2. Concentration decreases with the increasing of Sc or $K_c$.

3. The Prandtl number reduces the thermal boundary layer while heat generation parameter enhances the thermal boundary layer.
Figure 4: Chemical reaction effect on velocity when $Sc = 0.78$, $Pr = 0.71$, $S = 0.1$, $M = 3.0$, $K = 7.0$, $Gr_T = 1.0$, $Gr_c = 0.5$ and $\phi = 30^\circ$

Figure 5: Chemical reaction effect on concentration when $Sc = 0.78$, $Pr = 0.71$, $S = 0.1$, $M = 3.0$, $K = 7.0$, $Gr_T = 1.0$, $Gr_c = 0.5$ and $\phi = 30^\circ$
Figure 6: Prandtl Number effect on temperature when $Sc = 0.78$, $K_c = 0.2$, $S=0.1$, $M= 3.0$, $K = 7.0$, $Gr_T= 1.0$, $Gr_c= 0.5$ and $\phi = 30^\circ$.

Figure 7: Heat generation parameter effect on velocity when $Sc = 0.78$, $K_c= 0.2$, $Pr=0.71$, $M = 3.0$, $K = 7.0$, $Gr_T= 1.0$, $Gr_c= 0.5$ and $\phi = 30^\circ$. 
Figure 8: The heat generation parameter effect on temperature when $Sc = 0.78$, $K_c = 0.2$, $Pr = 1.0$, $M = 3.0$, $K = 7.0$, $Gr_T = 1.0$, $Gr_c = 0.5$ and $\phi = 30^\circ$.

Figure 9: Magnetic effect on velocity when $Sc = 0.78$, $K_c = 0.2$, $Pr = 0.71$, $S = 0.1$, $K = 7.0$, $Gr_T = 1.0$, $Gr_c = 0.5$ and $\phi = 30^\circ$. 

Figure 10: Porosity effect on velocity when $Sc = 0.78$, $K_c = 0.2$, $Pr = 0.71$, $S = 0.1$, $M = 3.0$, $Gr_T = 1.0$, $Gr_c = 0.5$ and $\phi = 30^\circ$.

Figure 11: Thermal Grashof number effect on velocity when $Sc = 0.78$, $K_c = 0.2$, $Pr = 0.71$, $S = 0.1$, $M = 3.0$, $K = 7.0$, $Gr_c = 0.5$ and $\phi = 30^\circ$. 
Figure 12: Solutal Grashof number effect on velocity when $Sc = 0.78$, $K_c = 0.2$, $Pr = 0.71$, $S = 0.1$, $M = 3.0$, $K = 7.0$, $Gr_T = 1.0$ and $\phi = 30^\circ$

Figure 13: Effect of angle of inclination on velocity when $Sc = 0.78$, $K_c = 0.2$, $Pr = 0.71$, $S = 0.1$, $M = 3.0$, $K = 0.1$, $Gr_T = 1.0$ and $Gr_c = 0.5$
References


