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ALGORITHMIC APPROACH TO UNBALANCED FUZZY TRANSPORTATION PROBLEM

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Abstract: The purpose of this article is to present an optimal solution for solving an unbalanced fuzzy transportation problem(UFTP) in which the cost coefficients, demand and supply quantities are triangular fuzzy numbers. An efficient computational procedure is given for solving such model. An optimal fuzzy transportation cost is also kept to the minimum. The degeneracy problem is also avoided by this method. To illustrate the proposed method, a numerical example is solved and the obtained result is compared with the results of other existing approaches. The proposed method provides an appropriate solution to the decision makers for taking best decision when they are handling various types of logistic problems having imprecise parameters.

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1. Introduction

In today's highly competitive market, the pressure on organization is to

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find better ways to create and deliver value added service to the customers in order to become to stronger. The transportation models provide a powerful framework to meet the challenge. They ensure the efficient movement and timely availability of raw materials and finished goods.

The basic transportation problem was originally developed by Hitchcock [6]. We can find initial basic feasible solution by using Vogel's Approximation Method VAM [2]. Many workers describe modifications to Vogel's Approximation method for obtaining initial solutions to the unbalanced transportation problem. Shimshak [12] propose a modification (SVAM) which ignores any penalty that involves a dummy row/column. Goyal [5] suggests another modification in (GVAM) where the cost of transporting goods to or from a dummy point is set equal to the highest transportation cost in the problem, rather than to zero. The method proposed by Ramakrishnan [11] consists of four steps of reduction and one step of VAM. Nagaraj Balakrishnan [9] suggests further modification in SVAM. Jaikumar [7], TOC (Total Opportunity Cost Matrix) using the VAM procedure. They coupled VAM with total opportunity cost and achieved by initial solutions (MNNMPM). Several sorts of methods have been established for finding the optimal solution. Among them some methods have been introduced which directly attain the optimal solution namely zero suffix method [13], ASM-method [1] etc. But these two methods for finding optimal solution of a transportation problem do not reflect optimal solution proved by Mohammed [8]. In general the transportations problems are solved with the assumptions that the coefficients or cost parameters are specified in precise way i.e., in crisp environment.

In real life, there are many diverse situations due to uncertainty in judgments, lack of evidence etc. Sometimes it is not possible to get relevant precise data for the cost parameter. Zimmerman [14] showed that solutions obtained fuzzy linear programming method is always efficient. Subsequently, Zimmermann's fuzzy linear programming has developed into several fuzzy optimization methods for solving the transportation problems. Edward Samuel [4] showed improved zero point method (IZPM), is used for solving unbalanced fuzzy transportation problems. Nagoor Gani and Mohamed Assarudeen [10], Proposed a New operation on triangular fuzzy number for solving linear programming problem.

In this study, basic idea is to get an optimal solution for an unbalanced fuzzy transportation problem. This paper presents a new method, which is simple and easy to understand. This can be an alternative to the modification distribution method (MODI) [2]. No path tracing is required in this approach. The algorithm of the approach is detailed with suitable numerical examples.

Further comparative studies of the new technique with other existing algorithms are established by means of sample problems.

2. Preliminaries

In this section, some basic preliminaries are given below

2.1. Definition

A fuzzy number $\tilde{A}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3) and it's membership function is given below

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ \dfrac{x - a_3}{a_2 - a_3}, & a_2 \leq x \leq a_3, \\ 0, & \text{otherwise.} \end{cases}$$

2.2. Arithmetic operations[10]

In this section, arithmetic operations between two triangular fuzzy numbers, defined on the universal set of real numbers $\Re$, are presented.

If $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers, then the following is obtained:

(i)$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.

(ii)$\tilde{A} - \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$.

(iii)$\tilde{A}$ x $\tilde{B}$ =(c_1, c_2, c_3), where T = $(a_1b_1, a_1b_3, a_3b_1, a_3b_3)$; $c_1 = min(T)$, $c_2 = a_2b_2$, $c_3 = max(T)$.

2.3. Graded Mean Integration Representation Method, see [3]

The graded mean integration representation method is used to defuzzify the triangular fuzzy number. If $\tilde{A}$=(a_1, a_2, a_3) is a triangular fuzzy number, then the graded mean integration representation of $\tilde{A}$ is $\frac{a_1+4a_2+a_3}{6}$.

3. Proposed Method

In this section, a new method is proposed for finding a fuzzy optimal solution for a given unbalanced fuzzy transportation problem, in which transportation costs, demand and supply are represented as triangular fuzzy numbers.

Step 1. Initialization. Construct the fuzzy transportation table for the given unbalanced fuzzy transportation problem and then, convert it into a balanced one.

Step 2. Develop the cost table. (a) Subtract each row entries of the fuzzy transportation table from each row maximum.

(b) In the reduced matrix obtained from 2(a), locate the maximum element in each column and then subtract that from each element of that column.

Step 3. Find the opportunity cost table. Calculate penalties for each column by taking the difference between the smallest and next smallest unit fuzzy transportation cost in the same column.

Step 4. Optimality criterion. Select the column with the largest penalty and allocate as much as possible in the cell having the least cost in the selected column satisfying the rim conditions. If there is a tie in the values of penalties then calculate their corresponding column cost value and select the minimum cost column.

Step 5. Revise the opportunity cost table. Adjust the supply and demand and cross out the satisfied the row or column.

Step 6. Determination of cell for allocation, repeat step 3 to step 4 until and unless all the demands are satisfied and all the supplies are Exhausted.

Step 7. Compute total fuzzy transportation cost for the feasible allocation from the original fuzzy cost table.

3.1. Important Remarks

If there is a tie in the minimum fuzzy transportation cost, then select the smallest cell for allocation from the original fuzzy cost table.

4. Numerical example

To illustrate the proposed method, the following Unbalanced Fuzzy Transportation Problem is solved.

4.1. Problem 1

Table1 gives the uncertain availability of the product available at four sources and their demand at three destinations, and the approximate cost for transporting one unit quantity of product from each source to each destination is represented by triangular fuzzy number. Determine the fuzzy optimal transportation of products such that the total fuzzy transportation cost is minimum.

Table 1

	D1	D2	D3	Availability(ai)
S1	(1,2,3)	(5,7,9)	(12,14,16)	(4,5,6)
S2	(1,3,5)	(1,3,5)	(0,1,2)	(6,8,10)
S3	(3,5,7)	(2,4,6)	(5,7,9)	(6,7,8)
S4	(0,1,2)	(4,6,8)	(1,2,3)	(13,15,17)
Demand(bj)	(5,7,9)	(8,9,10)	(16,18,20)	

Since $\sum_{i=1}^{4} a_i = (29, 35, 41)$; $\sum_{j=1}^{3} b_j = (29, 34, 39)$, so the chosen problem is an unbalanced fuzzy transportation problem. By applying the above proposed method, allocations for the given fuzzy transportation problem are obtained as follows.

Table 2: Using step 1, we get

	D1	D2	D3	D4	Availability (ai)
S1	(1,2,3)	(5,7,9)	(12,14,16)	(0,0,0)	(4,5,6)
S2	(1,3,5)	(1,3,5)	(0,1,2)	(0,0,0)	(6,8,10)
S3	(3,5,7)	(2,4,6)	(5,7,9)	(0,0,0)	(6,7,8)
S4	(0,1,2)	(4,6,8)	(1,2,3)	(0,0,0)	(13,15,17)
Demand (bj)	(5,7,9)	(8,9,10)	(16,18,20)	(0,1,2)	

5. Results and discussion

From the investigations and the results given above, it is clear that the proposed method is better than VAM, SVAM [12], GVAM[5], BVAM[9], RVAM[11],

Table 3: Using step 2, we get

	D1	D2	D3	D4	Availability (ai)
S1	(0,0,0)	(0,0,0)	(3,4,5)	(0,0,0)	(4,5,6)
S2	(11,12,13)	(7,7,7)	(2,2,2)	(11,11,11)	(6,8,10)
S3	(9,10,11)	(4,4,4)	(3,4,5)	(7,7,7)	(6,7,8)
S4	(7,7,7)	(7,7,7)	(0,0,0)	(8,8,8)	(13,15,17)
Demand (bj)	(5,7,9)	(8,9,10)	(16,18,20)	(0,1,2)	

Table 4: Using step 3, we get

	D1	D2	D3	D4	Availability (ai)
S1	(0,0,0)	(0,0,0)	(3,4,5)	(0,0,0)	(4,5,6)
S2	(11,12,13)	(7,7,7)	(2,2,2)	(11,11,11)	(6,8,10)
S3	(9,10,11)	(4,4,4)	(3,4,5)	(7,7,7)	(6,7,8)
S4	(7,7,7)	(7,7,7)	(0,0,0)	(8,8,8)	(13,15,17)
Demand (bj)	(5,7,9)	(8,9,10)	(16,18,20)	(0,1,2)	
Column Penalty	(7,7,7)	(4,4,4)	(2,2,2)	(7,7,7)	

ASM[1], ZSM[13] and MNNMPM[7], for solving unbalanced fuzzy transportation problems and the advantage is to get an optimal solution.

6. Conclusion

In this paper a new method is proposed, which is simple and easy to understand that is better than the existing methods, it is efficient and gives an optimal solution. The advantage of the proposed method is detailed with suitable numerical examples. Further comparative studies of the proposed method with other existing fuzzy transportation algorithms are established by means of sample problems and also can be applied for solving transportation problems occurring in real life situations.

Table 5: Using step 4 and 5, we get

	D1	D2	D3	D4	Availability (ai)
S1	(0,0,0)	(0,0,0)	(3,4,5)	**(0,1,2)**(0,0,0)	(4,5,6)
S2	(11,12,13)	(7,7,7)	(2,2,2)	*	(6,8,10)
S3	(9,10,11)	(4,4,4)	(3,4,5)	*	(6,7,8)
S4	(7,7,7)	(7,7,7)	(0,0,0)	*	(13,15,17)
Demand (bj)	(5,7,9)	(8,9,10)	(16,18,20)	*	
Column Penalty	(7,7,7)	(4,4,4)	(0,0,0)	*	

Table 6: Using step 6, finally we get

	D1	D2	D3	D4	Availability (ai)
S1	**(4,4,4)**(0,0,0)	*	*	**(0,1,2)**(0,0,0)	*
S2	*	**(2,2,2)**(7,7,7)	**(4,6,8)**(2,2,2)	*	*
S3	*	**(6,7,8)**(4,4,4)	*	*	*
S4	**(3,3,3)**(7,7,7)	*	**(10,12,14)**(0,0,0)	*	*
Demand (bj)	*	*	*	*	

Table 7: Using step 7, we get

Table 8

	D1	D2	D3	Availability (ai)
S1	**(4,4,4)**(1,2,3)	(5,7,9)	(12,14,16)	(4,5,6)
S2	(1,3,5)	**(2,2,2)**(1,3,5)	**(4,6,8)**(0,1,2)	(6,8,10)
S3	(3,5,7)	**(6,7,8)**(2,4,6)	(5,7,9)	(6,7,8)
S4	**(3,3,3)**(0,1,2)	(4,6,8)	**(10,12,14)**(1,2,3)	(13,15,17)
Demand (bj)	(5,7,9)	(8,9,10)	(16,18,20)	

The minimum fuzzy transportation cost is (4,4,4)(1,2,3) + (2,2,2)(1,3,5) +(4,6,8)(0,1,2)+(6,7,8)(2,4,6)+(3,3,3)(0,1,2)+(10,12,14)(1,2,3) = (28,75,134)
Defuzzified transportation cost is **77.00**

To illustrate our procedure, the solutions obtained by VAM [2], SVAM [12], GVAM[5], BVAM[9], RVAM[11], ASM[1], ZSM[13], MNNMPM[7], IZPM[4], MODI[2] and our procedure on twelve randomly generated unbalanced fuzzy transportation problems are shown in the following table. The complete detailed results of these twelve problems are not shown here for space considerations, and are available from the author.

Table 9: Solutions obtained by all procedures

Pr.	VAM	[12]	[5]	[9]	[11]	[1]	[13]	[7]	[4]	[2]	Proposed
1	84	103.3	81.3	81	95	95	81	81.3	77	77	77
2	2427	2427	2753.7	2467	2427	2467	2427	2753.7	2427	2427	2427
3	516.3	516.3	516.3	516.3	516.3	516.3	516.3	536.3	516.3	516.3	516.3
4	145	144	144	144	144	145	144	144	144	144	144
5	57.3	55.3	51.3	57.3	51.3	55.3	55.3	61.3	51.3	51.3	51.3
6	1114	924	944	1124	982.7	924	924	1172.7	924	924	924
7	488.3	488.3	488.3	698.3	488.3	488.3	488.3	488.3	488.3	488.3	488.3
8	92.7	92.7	92.7	116.7	92.7	92.7	92.7	116.7	92.7	92.7	92.7 77
9	782	772.7	772.7	802	797	772.7	792.7	782	772.7	772.7	772.7
10	2827.3	2451.3	2451.3	2952.7	2451.3	2451.3	2451.3	2952.7	2451.3	2451.3	2451.3
11	1745.2	1697	1667.7	1652.7	1652.7	1652.7	1652.7	1697	1652.7	1652.7	1652.7
12	9203	9203	9203	9802.3	9203	9203	9203	9802.3	9203	9203	9203

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