SEPARATION AXIOMS IN BITOPOLOGICAL ORDERED SPACES

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Abstract: In this paper, we introduce and study some new type of separation axioms in bitopological ordered spaces via (i, j) -semiopen sets.

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1. Introduction

The concept of bitopological spaces was first introduced by Kelly [1]. After the introduction of the definition of a bitopological space by Kelly, a large number of topologists have turned their attention to the generalization of different concepts of a single topological space in this space. A bitopological ordered space $(X, \tau_1, \tau_2, \leq)$ is a bitopological space (X, τ_1, τ_2) equipped with a partial order $\leq$ (that is, reflexive, transitive and antisymmetric). A subset A of $(X, \tau_1, \tau_2, \leq)$ is said to be increasing (resp. decreasing) if $A = \{x \in X : a \leq x \text{ for } a \in A\}$ (resp. $A = \{x \in X : x \leq a \text{ for some } a \in A\}$), that is, if $A = \bigcup_{a \in A} [a, \rightarrow]$ (resp. $A = \bigcup_{a \in A} [\rightarrow, a]$), where $[a, \rightarrow] = \{x \in X : a \leq x\}$ (resp.

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$[\rightarrow, a] = \{x \in X : x \leq a\}$). In this paper, we introduce and study some new type of separation axioms in bitopological ordered spaces via (i, j) -semiopen sets.

2. Preliminaries

For a subset A of a bitopological space (X, τ_1, τ_2) , $i\text{Cl}(A)$ and $i\text{Int}(A)$ denote the closure of A and the interior of A , respectively with respect to (X, τ_i) .

Definition 1. [5] A subset S of a bitopological space (X, τ_1, τ_2) is said to be (i, j) -semiopen if $S \subset j\text{Cl}(i\text{Int}(S))$.

Definition 2. [5] The intersection of all (i, j) -semiclosed sets containing $S \subset X$ is called the (i, j) -semiclosure of S and is denoted by $(i, j)\text{-}s\text{Cl}(S)$. The family of all (i, j) -semiopen (resp. (i, j) -semiclosed) sets of (X, τ_1, τ_2) is denoted by $(i, j)\text{-}SO(X)$ (resp. $(i, j)\text{-}SC(X)$). The family of all (i, j) -semiopen (resp. (i, j) -semiclosed) sets of (X, τ_1, τ_2) containing a point $x \in X$ is denoted by $(i, j)\text{-}SO(X, x)$ (resp. $(i, j)\text{-}SC(X, x)$).

Definition 3. A subset $M(x)$ of a bitopological space (X, τ_1, τ_2) is called an (i, j) -semineighbourhood of a point $x \in X$ if there exists an (i, j) -semiopen set S such that $x \in S \subset M(x)$.

Definition 4. [4] A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be (i, j) -irresolute if $f^{-1}(U)$ is (i, j) -semiopen in X for every (i, j) -semiopen set U of Y .

Definition 5. A bitopological space (X, τ_1, τ_2) is said to be:

1. (i, j) -semi- T_1 if for every $x, y \in X$, $x \neq y$, there exist $U, V \in (i, j)\text{-}SO$ such that $x \in U$, $y \notin V$ and $y \in V$, $x \notin U$.
2. (i, j) -semi- T_2 if for every $x, y \in X$, $x \neq y$, there exist $U, V \in (i, j)\text{-}SO(X)$ such that $x \in U$, $y \notin V$ and $y \in V$, $x \notin U$ and $U \cap V = \emptyset$.
3. (i, j) -semiregular if for any closed set F in X and $a \in X \setminus F$, there exist disjoint (i, j) -semiopen sets U and V in X containing a and F , respectively.

Definition 6. [2] A bitopological ordered space $(X, \tau_1, \tau_2, \leq)$ is said to be upper (resp. lower) T_1 -ordered if for each pair of elements $a \not\leq b$ (that is, a is not related to b) in X there exists a decreasing (resp. increasing) open set U containing b (resp. a) such that $a \notin U$ (resp. $b \notin U$). $(X, \tau_1, \tau_2, \leq)$ is said to be T_1 -ordered if it is both lower and upper T_1 -ordered.

Definition 7. [2] A bitopological ordered space $(X, \tau_1, \tau_2, \leq)$ is said to be T_2 -ordered if for each pair of elements $a \not\leq b$ in x there exist disjoint open sets U and V in X containing a and b respectively, U is increasing and V is decreasing.

3. On (i, j) -Semi- T_1 and (i, j) -Semi- T_2 Ordered Spaces

Definition 8. Let $(X, \tau_1, \tau_2, \leq)$ be a bitopological ordered space and A a subset of X . Define:

$I^{(i,j)-s}(A) = \cap \{F : F \text{ is an increasing } (i, j)\text{-semiclosed subset of } X \text{ containing } A\},$

$D^{(i,j)-s}(A) = \cap \{K : K \text{ is a decreasing } (i, j)\text{-semiclosed subset of } X \text{ containing } A\},$

$I^{o(i,j)-s}(A) = \cup \{G : G \text{ is an increasing } (i, j)\text{-semiopen subset of } X \text{ contained in } A\},$

$D^{o(i,j)-s}(A) = \cup \{H : H \text{ is an decreasing } (i, j)\text{-semiopen subset of } X \text{ contained in } A\},$

Clearly, $I^{(i,j)-s}(A)$ (resp. $D^{(i,j)-s}(A)$) is the smallest increasing (resp. decreasing) (i, j) -semiclosed subset of X containing A and $I^{o(i,j)-s}(A)$ (resp. $D^{o(i,j)-s}(A)$) is the largest increasing (resp. decreasing) (i, j) -semiopen subset of X contained in A .

Proposition 9. For any subset A of a bitopological ordered space $(X, \tau_1, \tau_2, \leq)$, the following hold:

1. $X \setminus I^{(i,j)-s}(A) = D^{o(i,j)-s}(X \setminus A).$
2. $X \setminus D^{(i,j)-s}(A) = I^{o(i,j)-s}(X \setminus A).$
3. $X \setminus I^{o(i,j)-s}(A) = D^{(i,j)-s}(X \setminus A).$
4. $X \setminus D^{o(i,j)-s}(A) = I^{(i,j)-s}(X \setminus A).$

Proof. We shall prove (1) only, (2), (3) and (4) can be proved in a similar manner.

(1). Since $I^{(i,j)-s}(A)$ is an (i, j) -semiclosed increasing set containing A , $X \setminus I^{(i,j)-s}(A)$ is an (i, j) -semiopen decreasing set such that $X \setminus I^{(i,j)-s}(A) \subset X \setminus A$. Let U be an another (i, j) -semiopen decreasing set such that $U \subset X \setminus A$. Then $X \setminus U$ is an (i, j) -semiclosed increasing set such that $X \setminus U \supset A$. It follows that $I^{o(i,j)-s}(A) \subset X \setminus U$. That is $U \subset X \setminus I^{(i,j)-s}(A)$. Thus, $X \setminus I^{(i,j)-s}(A)$ is

the largest (i, j) -semiopen decreasing set such that $X \setminus I^{(i,j)s}(A) \subset X \setminus A$. That is $X \setminus I^{(i,j)s}(A) = D^{o(i,j)-s}(X \setminus A)$. $\square$

Lemma 10. Let $(X, \tau_1, \tau_2, \leq)$ be a bitopological ordered space and A a subset of X . Then $x \in I^{(i,j)s}(A)$ (resp. $x \in D^{o(i,j)-s}(A)$) if and only if for every decreasing (resp. increasing) (i, j) -semiopen subset U of X containing x , $U \cap A \neq \emptyset$.

Proof. Let U be a decreasing (i, j) -semiopen subset of X containing x such that $U \cap A = \emptyset$. Then $X \setminus U$ is an increasing (i, j) -semiclosed subset of X containing A . Therefore, $I^{(i,j)s}(A) \subset X \setminus U$. Thus $x \notin I^{(i,j)s}(A)$. Conversely, if $x \notin I^{(i,j)s}(A)$. Then $x \setminus I^{(i,j)s}(A)$ is a decreasing (i, j) -semiopen subset of X containing x , but disjoint from A . The case of $D^{o(i,j)-s}(A)$ can be dealt similarly. $\square$

Definition 11. A bitopological ordered space $(X, \tau_1, \tau_2, \leq)$ is said to be upper (resp. lower) (i, j) -semi- T_1 -ordered if for each pair of elements $a \not\leq b$ (that is, a is not related to b) in X there exists a decreasing (resp. increasing) (i, j) -semiopen set U containing b (resp. a) such that $a \notin U$ (resp. $b \notin U$). $(X, \tau_1, \tau_2, \leq)$ is said to be (i, j) -semi- T_1 -ordered if it is both lower and upper (i, j) -semi- T_1 -ordered.

Clearly every T_1 -ordered space is (i, j) -semi- T_1 -ordered and every (i, j) -semi- T_1 -ordered space is (i, j) -semi- T_1 but the converses are not true, in general.

Example 12. Let $X = \{a, b, c\}$ be equipped with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\emptyset, X, \{a, b\}\}$ and with the partial order $\leq$ defined as $a \leq a, a \leq b, a \leq c, b \leq b, c \leq b, c \leq c$. Then $(X, \tau_1, \tau_2, \leq)$ is an (i, j) -semi- T_1 space but not τ_1 - T_1 -ordered.

Example 13. Let $X = \{a, b, c\}$ be equipped with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\emptyset, X, \{a, b\}\}$ and with the partial order $\leq$ defined as $a \leq a, a \leq b, a \leq c, b \leq b, c \leq b, c \leq c$. Then $(X, \tau_1, \tau_2, \leq)$ is an (i, j) -semi- T_1 space but not (i, j) -semi- T_1 ordered.

Theorem 14. For a bitopological ordered space $(X, \tau_1, \tau_2, \leq)$, the following statements are equivalent:

1. $(X, \tau_1, \tau_2, \leq)$ is lower (resp. upper) (i, j) -semi- T_1 -ordered,
2. for each pair $a \not\leq b$ of X , there exists an (i, j) -semiopen set U containing a (resp. b) such that $x \not\leq b$ (resp. $a \not\leq x$) for all $x \in U$,
3. for each $x \in X$, $[\leftarrow, x]$ (resp. $[x, \rightarrow]$) is (i, j) -semiclosed,

4. when the net $\{x_\alpha\}_{\alpha \in A}$ (i, j) -semiconverges to a and $x_\alpha \leq b$ (resp. $b \leq x_\alpha$) for all $\alpha \in A$, then $a \leq b$ (resp. $b \leq a$).

Proof. We shall prove the theorem for lower (i, j) -semi- T_1 -ordered spaces only. (1) $\Rightarrow$ (2): Let $a \not\leq b$. Then by hypothesis, there exists an increasing (i, j) -semiopen set U containing a such that $b \notin U$. If $x \in U$ and $x \leq b$, then $b \in U$, a contradiction.

(2) $\Rightarrow$ (3): Let $y \in X \setminus [\leftarrow, x]$. Then $y \not\leq x$. Then there exists an (i, j) -semiopen set U containing y such that $u \not\leq x$ for all $u \in U$. That is, $y \in U \subset X \setminus [\leftarrow, x]$. Hence $[\leftarrow, x]$ is (i, j) -semiclosed.

(3) $\Rightarrow$ (1): Obvious.

(1) $\Rightarrow$ (4): Let $\{x_\alpha\}_{\alpha \in A}$ be a net in X (i, j) -semiconverges to a and $x_\alpha \leq b$ for all $\alpha \in A$. If possible $a \not\leq b$, then by hypothesis, $X \setminus [\leftarrow, x]$ is an (i, j) -semiopen set containing a . Then there exists $\lambda \in A$ such that $x_\alpha \in X \setminus [\leftarrow, x]$ for all $\alpha \leq \lambda$. That is $x_\alpha \not\leq b$ for all $\alpha \geq \lambda$, which is a contradiction.

(4) $\Rightarrow$ (1): Let X be not lower (i, j) -semi- T_1 -ordered. Then there exists a pair $a \not\leq b$ in X such that for every (i, j) -semiopen set U containing a , there exists $x_u \in U$ with $x_u \leq b$. Let $\mathcal{U}$ be the collection of all (i, j) -semiopen sets containing a , then $\mathcal{U}$ is directed by inclusion and the net $\{x_u\}_{u \in \mathcal{U}}$ (i, j) -semiconverges to a . By hypothesis $a \leq b$, which is a contradiction. $\square$

Corollary 15. If $(X, \tau_1, \tau_2, \leq)$ is lower (upper) (i, j) -semi- T_1 -ordered and $\tau_i \leq \tau_i^*$, then $(X, \tau_1^*, \tau_2^*, \leq)$ is also lower (upper) (i, j) -semi- T_1 -ordered.

Theorem 16. Every biopen subspace of an (i, j) -semi- T_1 -ordered space is (i, j) -semi- T_1 -ordered.

Proof. Let $(X, \tau_1, \tau_2, \leq)$ be an (i, j) -semi- T_1 -ordered space and A a biopen subspace of X . Let $x \in X$. Then by Theorem 9 (3), $X \setminus [\leftarrow, x]$ and $X \setminus [x, \rightarrow]$ are (i, j) -semiopen sets in X . Since A is biopen, $A \cap (X \setminus [\leftarrow, x])$ and $A \cap (X \setminus [x, \rightarrow])$ are (i, j) -semiopen in A . But $A \cap (X \setminus [\leftarrow, x]) = A \setminus [\leftarrow, x]$ and $A \cap (X \setminus [x, \rightarrow]) = A \setminus [x, \rightarrow]$. Hence, again by Theorem 9 (3) A is (i, j) -semi- T_1 -ordered. $\square$

Theorem 17. A bitopological space $(X, \tau_1, \tau_2, \leq)$ is (i, j) -semi- T_1 -ordered if and only if for each $x \in X$, there exists an (i, j) -semi- T_1 -ordered (i, j) -semiopen set in X which is both increasing and decreasing containing x .

Proof. If X is (i, j) -semi- T_1 -ordered, then X is the required set for each $x \in X$. Conversely, let $a \not\leq b$ in X . Then, by hypothesis, there exist (i, j) -semi- T_1 -ordered (i, j) -semiopen sets U_1 and U_2 in X containing a and b , respectively, where U_1 and U_2 are both increasing and decreasing sets. If $b \notin U_1$ and $a \notin U_2$,

then there is nothing to prove. But if $b \in U_1$ (resp. $a \in U_2$), then there exist (i, j) -semiopen sets V and W in U_1 (resp. U_2) containing a and b , respectively, V is increasing and W is decreasing in U_1 (resp. U_2) also $b \notin V$ and $a \notin W$. Since U_1 (resp. U_2) is both increasing and decreasing and it also an (i, j) -semiopen subset of X , V is an increasing and W is a decreasing (i, j) -semiopen subsets of X . Hence $(X, \tau_1, \tau_2, \leq)$ is (i, j) -semi- T_1 -ordered. $\square$

Theorem 18. *Let f be an order preserving (that is, $x \leq y$ in X if and only if $f(x) \leq^* f(y)$ in X^*) (i, j) -irresolute function from a bitopological ordered space $(X, \tau_1, \tau_2, \leq)$ to a bitopological ordered space $(X^*, \tau^*, \leq^*)$. If $(X^*, \tau^*, \leq^*)$ is (i, j) -semi- T_1 -ordered, then $(X, \tau_1, \tau_2, \leq)$ is also (i, j) -semi- T_1 -ordered.*

Proof. Let $a \not\leq b$ in X . Then $f(a) \not\leq^* f(b)$ in X . Then there exists an increasing (i, j) -semiopen set U^* in X^* containing $f(a)$ but $f(b) \notin U^*$. Since f is order preserving and (i, j) -irresolute, $f^{-1}(U^*) = U$ is an increasing (i, j) -semiopen subset of X . Also, $a \in U$ and $b \notin U$. Hence $(X, \tau_1, \tau_2, \leq)$ is lower (i, j) -semi- T_1 -ordered. Similarly, we can prove $(X, \tau_1, \tau_2, \leq)$ is upper (i, j) -semi- T_1 -ordered. $\square$

Theorem 19. *The product of a family of (i, j) -semi- T_1 -ordered spaces is also (i, j) -semi- T_1 -ordered.*

Proof. Let $\{(X_\lambda, \tau_\lambda, \leq_\lambda) : \lambda \in \Lambda\}$ be a family of (i, j) -semi- T_1 -ordered space and $(X, \tau_1, \tau_2, \leq)$ be the product ordered space. Let $a = (a_\lambda)$ and $b = (b_\lambda)$ be two elements of X such that $a \not\leq b$. Then there exists $\mu \in \Lambda$ such that $a_\mu \not\leq_\mu b_\mu$. Since $(X, \tau_1, \tau_2, \leq)$ is an (i, j) -semi- T_1 -ordered space, there exists an (i, j) -semiopen set U_μ such that $a_\mu \in U_\mu$ but $b_\mu \notin U_\mu$. Define $U = \prod U_\mu$, where $U_\lambda = X_\lambda$ if $\lambda \neq \mu$. Then U is an increasing (i, j) -semiopen subset of X containing a but not b . Therefore, $(X, \tau_1, \tau_2, \leq)$ is lower (i, j) -semi- T_1 -ordered. Similarly, we can prove $(X, \tau_1, \tau_2, \leq)$ is upper (i, j) -semi- T_1 -ordered. $\square$

Definition 20. A bitopological ordered space $(X, \tau_1, \tau_2, \leq)$ is said to be (i, j) -semi- T_2 -ordered if for each pair of elements $a \not\leq b$ in x there exist disjoint (i, j) -semiopen sets U and V in X containing a and b , respectively, U is increasing and V is decreasing.

Example 21. Let X be an infinite set, $\tau_1 = \text{tau}_2 = \text{a finite complement topology with discrete order}$. Then $(X, \tau_1, \tau_2, \leq)$ is an (i, j) -semi- T_1 ordered but not (i, j) -semi- T_2 -ordered.

Example 22. Let $X = \{a, b, c\}$ be equipped with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\emptyset, X, \{a, b\}\}$ and with the partial order $\leq$ defined

as $a \leq a$, $a \leq b$, $a \leq c$, $b \leq b$, $c \leq b$, $c \leq c$. Then $(X, \tau_1, \tau_2, \leq)$ is an (i, j) -semi- T_2 space but not T_2 -ordered.

Example 23. Let $X = \{a, b, c\}$ be equipped with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\emptyset, X, \{a, b\}\}$ and with the partial order $\leq$ defined as $a \leq a$, $a \leq b$, $a \leq c$, $b \leq b$, $c \leq b$, $c \leq c$. Then $(X, \tau_1, \tau_2, \leq)$ is an (i, j) -semi- T_2 space but not (i, j) -semi- T_2 ordered.

Example 24. Let $X = \{a, b, c\}$ be equipped with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$, $\tau_2 = \mathcal{P}(X)$ and with the partial order $\leq$ defined as $a \leq a$, $a \leq b$, $a \leq c$, $b \leq b$, $c \leq b$, $c \leq c$. Then $(X, \tau_1, \tau_2, \leq)$ is an (i, j) -semi- T_1 ordered but not (i, j) -semi- T_2 ordered.

Theorem 25. A bitopological ordered space $(X, \tau_1, \tau_2, \leq)$ is (i, j) -semi- T_2 -ordered if and only if for each $x \in X$, there exists an increasing (resp. decreasing) (i, j) -semiclopen subset of X containing x which is (i, j) -semi- T_2 -ordered.

Proof. If X is (i, j) -semi- T_2 -ordered, then X is the required increasing (resp. decreasing) (i, j) -semiclopen subset of X for all $x \in X$. Conversely, let $x \not\leq y$ in X . By hypothesis, there exists an increasing (i, j) -semiclopen set V in X containing x . If $y \in V$, then nothing to prove. If $y \notin V$. Then $X \setminus V$ is a decreasing (i, j) -semiclopen subset of X containing y . Hence $(X, \tau_1, \tau_2, \leq)$ is (i, j) -semi- T_2 -ordered. Dually, we can prove the theorem for decreasing (i, j) -semiclopen subsets of X . $\square$

Theorem 26. Let $\{(X_\alpha, \tau_1^\alpha, \tau_2^\alpha \leq_\alpha) : \alpha \in \Lambda\}$ be an arbitrary family of topological order spaces. Then the product space $X = \prod_{\alpha \in \Lambda} X_\alpha$ equipped with the product topology τ and with the partial order $\leq$, defined as $(x_\alpha)_{\alpha \in \Lambda}$. Then, in the product ordered space,

$$\begin{aligned} I^{(i,j)s}(\prod_{\alpha \in \Lambda} A_\alpha) &\subseteq \prod_{\alpha \in \Lambda} I_X^{(i,j)s}(A_\alpha) \\ D^{(i,j)s}(\prod_{\alpha \in \Lambda} A_\alpha) &\subseteq \prod_{\alpha \in \Lambda} D_X^{(i,j)s}(A_\alpha) \end{aligned}$$

Proof. Let $x = (x_\alpha) \notin I^{(i,j)s}(\prod_{\alpha \in \Lambda} A_\alpha)$ and U_β be a decreasing (i, j) -semiopen subset of X_β containing x_β for some $\beta \in \Lambda$. Define, $U = \prod_{\alpha \in \Lambda} U_\alpha$, where $U_\alpha = X_\alpha$ if $\alpha \neq \beta$. Then, U is a decreasing (i, j) -semiopen subset of $\prod_{\alpha \in \Lambda} X_\alpha$ containing x . Therefore, by Lemma 10, $U \cap \prod_{\alpha \in \Lambda} A_\alpha \neq \emptyset$ that is, $(U_\beta \cap A_\beta) \times \prod_{\alpha \neq \beta} A_\alpha \neq \emptyset$.

Thus, $U_\beta \cap A_\beta \neq \emptyset$. Again, by Lemma 10, $x_\beta \in I_X^{(i,j)s}(A_\beta)$. Similarly we can prove that $D^{(i,j)s}(\prod_{\alpha \in \Lambda} A_\alpha) \subset \prod_{\alpha \in \Lambda} D_X^{(i,j)s}(A_\alpha)$ $\square$

Theorem 27. *The product of a family of (i, j) -semi- T_2 -ordered spaces is also (i, j) -semi- T_2 -ordered.*

Proof. Let $\{(X_\lambda, \tau_\lambda, \leq_\lambda) : \lambda \in \Lambda\}$ be a family of (i, j) -semi- T_2 -ordered space and $(X, \tau_1, \tau_2, \leq)$ be the product ordered space. If $(x_\lambda), (y_\lambda) \in X$ such that $(x_\lambda) \not\leq (y_\lambda)$. Then, there exists $\mu \in \Lambda$ such that $x_\mu \not\leq_\mu y_\mu$. Thus, there exist disjoint (i, j) -semiopen sets U_μ and V_μ in X_μ containing x_μ and y_μ respectively, where U_μ is increasing and V_μ is decreasing. Define, $U = \prod_{\lambda \in \Lambda} U_\lambda$ such that $U_\mu = X_\lambda$ if $\lambda \neq \mu$ and $U = \prod_{\lambda \in \Lambda} V_\lambda$ such that $V_\mu = X_\lambda$ if $\lambda \neq \mu$. Then U is an increasing (i, j) -semiopen subset of X containing (x_λ) and V is a decreasing (i, j) -semiopen subset of X containing (y_λ) . Clearly, U and V are disjoint. Hence $(X, \tau_1, \tau_2, \leq)$ is (i, j) -semi- T_2 -ordered. $\square$

Theorem 28. *If $(X, \tau_1, \tau_2, \leq)$ is (i, j) -semi- T_2 -ordered, then the graph of the order of X is $\{(a, b) \in X \times X : a \leq b\}$ an (i, j) -semiclosed subset of $X \times X$.*

Proof. Let G be the graph of the order of X and $(x, y) \in (X \times X) \setminus G$. Then $x \not\leq y$. By hypothesis, there exist disjoint (i, j) -semiopen sets U and V such that $x \in U$, U is increasing and $y \in V$, V is decreasing. Therefore, $U \times V$ is an (i, j) -semiopen subset of $X \times X$ containing (x, y) . Also $U \times V \cap G = \emptyset$. Thus, $(x, y) \in U \times V \subset (X \times X) \setminus G$. Hence F is (i, j) -semiclosed. $\square$

Theorem 29. *A bitopological ordered space $(X, \tau_1, \tau_2, \leq)$ is (i, j) -semi- T_2 if and only if for each $x \in X$ the intersection of all increasing (resp. decreasing) (i, j) -semiclosed i -(i, j)-semineighbourhoods (resp. d -(i, j)-semineighbourhoods) of x is $[x^*, \rightarrow]$ (resp. $[\leftarrow, x]$).*

Proof. Let $(X, \tau_1, \tau_2, \leq)$ be an (i, j) -semi- T_2 -ordered space and $x \in X$. If $G^* = \cap\{F : F \text{ is an increasing } (i, j)\text{-semiclosed } i\text{-(}i, j\text{)-semineighbourhood of } x\}$. Clearly, $[x^*, \rightarrow] \subset G^*$. Let $y \notin [x^*, \rightarrow]$. Then $x \not\leq y$. Then there exist disjoint (i, j) -semiopen sets U and V containing x and y , respectively such that U is increasing and V is decreasing. Hence $x \in U \subset X \setminus V$. Therefore, $X \setminus V$ is an increasing (i, j) -semiclosed i -(i, j)-semineighbourhood of x and $y \notin X \setminus V$. Hence $G^* = [x^*, \rightarrow]$. Similarly, we can show that intersection of all decreasing (i, j) -semiclosed i -(i, j)-semineighbourhoods of x is $[\leftarrow, x]$. Conversely, let $x \leq y$

in X . Then $y \notin [x^*, \rightarrow]$. By hypothesis, there exists an increasing (i, j) -semiclosed i -(i, j)-semineighbourhood F of x such that $y \notin F$. Then $y \in X \setminus F$, $X \setminus F$ is a decreasing (i, j) -semiopen set. Also, there exists an increasing (i, j) -semiopen set U such that $x \in U \subset F$. Clearly, $U \cap (X \setminus F) = \emptyset$. Hence $(X, \tau_1, \tau_2, \leq)$ is (i, j) -semi- T_2 -ordered. By dual argument we can prove that (i, j) -semi- T_2 -ordered if intersection of all decreasing (i, j) -semiclosed d -(i, j)-semineighbourhoods of x is $[x^*, \rightarrow]$ for all $x \in X$. $\square$

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