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ON α^m-SYMMETRIC SPACES

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Abstract: In this paper, we introduce the space called α^m-symmetric by using α^m-closure of α^m-closed sets in topological spaces and study some of their properties.

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1. Introduction

In 1970, Levine generalized the concept of closed sets to generalized closed sets. After that there is a vast progress occurred in the field of generalized open sets (complement of respective closed sets). In topological spaces, it is well known that normality is preserved under closed continuous surjections. In 1967, A. Wilansky [1] has introduced the concept of US spaces. In 1968, C. E. Aull [2] studied some separation axioms between the T_1 and T_2 spaces, namely, S_1 and S_2. Next, in 1982, S. P. Arya et al.[3] have introduced and studied the concept of

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semi-US spaces and also they made study of s-convergence, sequentially semi-closed sets, sequentially s-compact notions. G. B. Navlagi studied P-Normal Almost-P-Normal and Mildly-P-Normal spaces. Closedness are basic concept for the study and investigation in general topological spaces. This concept has been generalized and studied by many authors from different points of views.

In 1978, Long and Herrington [4] used almost closedness due to Singal [5]. O. Njastad [6] introduced and defined an α-open and α-closed set. After the works of O.Njastad on α-open sets, various mathematicians turned their attention to the generalizations of various concepts in topology by considering semi-open, α-open sets. The concept of g-closed [7], s-open [8] and α-open sets has a significant role in the generalization of continuity in topological spaces. The modified form of these sets and generalized continuity were further developed by many mathematicians ([9], [10], [11], [12], [13]).

Very recently, we have introduced the notion of α^m-closed sets [14] which are strictly weaker than α-closed sets. We use α^m-open sets to define a spaces called α^m-symmetric in topological spaces. Also we derive the properties of α^m-symmetric spaces.

2. Preliminaries

Throughout this paper (X, τ) and (Y, σ) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned.

For a subset A of a space (X, τ), $cl(A)$, $int(A)$ and A^c denote the closure of A, the interior of A and the complement of A in X, respectively.

Definition 2.1. A subset A of a topological space (X, τ) is called

(a) a preopen set [15] if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$.

(b) a semiopen set [8] if $A \subseteq cl(int(A))$ and semi closed set if $int(cl(A)) \subseteq A$.

(c) an α-open set [6] if $A \subseteq int(cl(int(A)))$ and an α-closed set if $cl(int(cl(A))) \subseteq A$.

(d) a semi-preopen set [16] (β-open set) if $A \subseteq cl(int(cl(A)))$ and semi-preclosed set if $int(cl(int(A))) \subseteq A$.

(e) an α^m-closed set [14] if $int(cl(A)) \subseteq U$, whenever $A \subseteq U$ and U is α-open. The complement of α^m-closed set is called an α^m-open set.

Definition 2.2. [17] A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called

(a) an α^m-continuous if $f^{-1}(V)$ is α^m-closed in (X, τ) for every closed set V of (Y, σ).

(b) an α^m-irresolute if $f^{-1}(V)$ is α^m-closed in (X, τ) for every α^m-closed set V of (Y, σ).

Definition 2.3. [18] (a) Let X be a topological space and let $x \in X$. A subset N of X is said to be α^m-nbbd of x if there exists an α^m-open set G such that $x \in G \subset N$.

The collection of all α^m-nbhd of $x \in X$ is called an α^m-nbhd system at x and shall be denoted by $n^{\#_{\alpha^m}}(x)$.

(b) Let X be a topological space and A be a subset of X, A subset N of X is said to be α^m-nbhd of A if there exists an α^m-open set G such that $A \in G \subseteq N$.

(c) Let A be a subset of X. A point $x \in A$ is said to be an α^m-interior point of A, if A is an $n^{\#_{\alpha^m}}(x)$. The set of all α^m-interior points of A is called an α^m-interior of A and is denoted by $I_{\alpha^m}(A)$.

$I_{\alpha^m}(A) = \bigcup\{G : G \textit{ is } \alpha^m\textit{-open}, \ G \subset A\}.$

(d) Let A be a subset of X. A point $x \in A$ is said to be an α^m-closure of A. Then

$C_{\alpha^m}(A) = \bigcap\{F : A \subset F \in \alpha^m C(X, \tau_X)\}.$

Definition 2.4. [19] A subset A of a topological space X is called a α^m-Difference set (briefly, (α^m, D)-set) if there are $U, V \in \alpha^m O(X, \tau)$ such that $U \neq X$ and $A = U/V$.

It is true that every α^m-open set U different from X is a (α^m, D)-set if $A = U$ and $V = \phi$. So, we can observe the following.

Definition 2.5. A topological space (X, τ) is said to be

(a) $T_0\widetilde{\alpha^m}$ [20] if for each pair of distinct points $x,\ y$ in X, there exists a α^m-open set U such that either $x \in U$ and $y \notin U$ or $x \notin U$ and $y \in U$.

(b) $T_1\widetilde{\alpha^m}$ [20] if for each pair of distinct points $x,\ y$ in X, there exist two α^m-open sets U and V such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$.

(c) $T_2\widetilde{\alpha^m}$ [20] if for each distinct points $x,\ y$ in X, there exist two disjoint α^m-open sets U and V containing x and y respectively.

(a) $(\alpha^m,\ D^0)$ [19] if for any pair of distinct points x and y of X there exists an $(\alpha^m,\ D)$-set of X containing x but not y or $(\alpha^m,\ D)$-set of X containing y but not x.

(b) $(\alpha^m,\ D^1)$ [19] if for any pair of distinct points x and y of X there exists an $(\alpha^m,\ D)$-set of X containing x but not y and $(\alpha^m,\ D)$-set of X containing y but not x.

(c) $(\alpha^m,\ D^2)$ [19] if for any pair of distinct points x and y of X there exist disjoint $(\alpha^m,\ D)$-set G and E of X containing x and y, respectively.

3. On α^m-Symmetric Spaces

Definition 3.1. A topological space (X, τ) is said to be α^m-symmetric if for x and y in X, $x \in C_{\alpha^m}(\{y\})$ implies $y \in C_{\alpha^m}(\{x\})$.

Corollary 3.2. *If a topological space $(X,\ \tau)$ is a $T_1\widetilde{\alpha^m}$ space, then it is α^m-symmetric.*

Proof. In a $T_1\widetilde{\alpha^m}$ space, every singleton is α^m-closed, $(X,\ \tau)$ is α^m-symmetric. □

Corollary 3.3. *For a topological space (X, τ), the following statements are equivalent:*

(a) (X, τ) is α^m-symmetric and $T_0\widetilde{\alpha^m}$.

(b) (X,τ) *is* $T_1{\widetilde{\alpha^m}}$.

Proof. By the previous corollary, it suffices to prove only $(a) \Longrightarrow (b)$ Let $x \neq y$ and as (X,τ) is $T_0{\widetilde{\alpha^m}}$, we may assume that $x \in U \subseteq X/\{y\}$ for some $U \in \alpha^m O(X)$. Then $x \notin C_{\alpha^m}(\{y\})$ and hence $y \notin C_{\alpha^m}(\{x\})$. There exists a α^m-open set V such that $y \in V \subseteq X/\{x\}$ and thus (X,τ) is a $T_1{\widetilde{\alpha^m}}$-space. □

Proposition 3.4. *If* (X,τ) *is a* α^m*-symmetric space, then the following statements are equivalent:*

(a) (X,τ) *is a* $T_0{\widetilde{\alpha^m}}$*-space.*

(b) (X,τ) *is a* $T_1{\widetilde{\alpha^m}}$*-space.*

Proof. $(a) \Longleftrightarrow (b)$ Obvious from the previous corollary. □

Remark 3.5. [19] For a topological space $(X,\ \tau)$, the following properties hold:

(a) If $(X,\ \tau)$ is $T_k{\widetilde{\alpha^m}}$, then it is $(\alpha^m,\ D^k)$, for $k = 0, 1, 2$.

(b) If $(X,\ \tau)$ is $(\alpha^m,\ D^k)$, then it is $(\alpha^m,\ D^{k-1})$, for $k = 1, 2$.

Corollary 3.6. *[19] If* $(X,\ \tau)$ *is* $(\alpha^m,\ D^1)$*, then it is* $T_0{\widetilde{\alpha^m}}$.

Corollary 3.7. *For a* α^m*-symmetric space* (X,τ)*, the following are equivalent:*

(a) (X,τ) *is* $T_0{\widetilde{\alpha^m}}$.

(b) (X,τ) *is* $(\alpha^m,\ D^1)$.

(c) (X,τ) *is* $T_1{\widetilde{\alpha^m}}$.

Proof. $(a) \Longrightarrow (b)$ Follows from the previous corollary.

$(c) \Longrightarrow (b) \Longrightarrow (a)$ Follows from the previous remark and Corollary.

□

Proposition 3.8. *The following properties hold for the subsets A, B of a topological space (X, τ).*

(a) $A \subseteq K^{\alpha^m}(A)$.

(b) $A \subseteq B$ *implies that* $K^{\alpha^m}(A) \subseteq K^{\alpha^m}(B)$.

(c) *If A is α^m-open in (X, τ), then* $A = K^{\alpha^m}(A)$.

(d) $K^{\alpha^m}(K^{\alpha^m}(A)) = K^{\alpha^m}(A)$.

Proof. (a), (b) and (c) are immediate consequences of Definition. To prove (d), first observe that by (a) and (b), we have $K^{\alpha^m}(A) \subseteq K^{\alpha^m}(K^{\alpha^m}(A))$. If $x \notin K^{\alpha^m}(A)$, then there exists $U \in K^{\alpha^m}O(X, \tau)$ such that $A \subseteq U$ and $x \notin U$. Hence $K^{\alpha^m}(A) \subseteq U$, and so we have $x \notin K^{\alpha^m}(K^{\alpha^m}(A))$. Thus $K^{\alpha^m}(K^{\alpha^m}(A)) = K^{\alpha^m}(A)$.

□

Proposition 3.9. *If a singleton $\{x\}$ is a $(\alpha^m,\ D)$-set of (X, τ), then* $K^{\alpha^m}(\{x\}) \neq X$.

Proof. Since $\{x\}$ is a $(\alpha^m,\ D)$-set of (X, τ), then there exist two subsets $U_1,\ U_2 \in \alpha^m O(X, \tau)$ such that $\{x\} = U_1/U_2$, $\{x\} \subseteq U_1$ and $U_1 \neq X$. Thus, we have that $K^{\alpha^m}(\{x\}) \subseteq U_1 \neq X$ and so $K^{\alpha^m}(\{x\}) \neq X$.

□

Theorem 3.10. *If A and B are subsets of a space X, then*

(a) $C_{\alpha^m}(X) = X$ *and* $C_{\alpha^m}(\phi) = \phi$,

(b) $A \subset C_{\alpha^m}(A)$,

(c) *If B is any α^m-closed set containing A, then* $C_{\alpha^m}(A) \subset B$, *(d) If $A \subset B$, then $C_{\alpha^m}(A) \subset C_{\alpha^m}(B)$.*

Proof. (a) By the definition of an α^m-closure, X is the only α^m-closed set containing X. Therefore $C_{\alpha^m}(X) =$ Intersection of all the α^m-closed sets containing $X = \cap\{X\} = X$. That is $C_{\alpha^m}(X) = X$. By the definition of an α^m-closure, $C_{\alpha^m}(\phi) =$ Intersection of all the α^m-closed sets containing $\phi = \phi\cap$ any α^m-closed sets containing $\phi = \phi$. That is $C_{\alpha^m}(\phi) = \phi$.

(b) By the definition of an α^m-closure of A, It is obvious that $A \subset C_{\alpha^m}(A)$.

(c) Let B be any α^m-closed set containing A. Since $C_{\alpha^m}(A)$ is the intersection of all α^m-closed sets containing A, $C_{\alpha^m}(A)$ is contained in every α^m-closed set containing A. Hence in particular $C_{\alpha^m}(A) \subset B$.

(d) Let A and B be subsets of X such that $A \subset B$. By the definition of an α^m-closure, $C_{\alpha^m}(B) = \bigcap\{F : B \subset F \in \alpha^m C(X, \tau_X)\}$. If $B \subset F \in \alpha^m C(X, \tau_X)$, then $C_{\alpha^m}(B) \subset F$. Since $A \subset B$, $A \subset B \subset F \in \alpha^m C(X, \tau_X)$, we have $C_{\alpha^m}(A) \subset F$. Therefore $C_{\alpha^m}(A) \subset \bigcap\{F : B \subset F \in \alpha^m C(X, \tau_X)\} = C_{\alpha^m}(B)$. That is $C_{\alpha^m}(A) \subset C_{\alpha^m}(B)$.

□

Theorem 3.11. *If A and B are subsets of a space X, then $C_{\alpha^m}(A \cap B) \subset C_{\alpha^m}(A) \cap C_{\alpha^m}(B)$.*

Proof. Let A and B be subsets of X. Clearly $A \cap B \subset A$ and $A \cap B \subset B$, $C_{\alpha^m}(A \cap B) \subset C_{\alpha^m}(A)$ and $C_{\alpha^m}(A \cap B) \subset C_{\alpha^m}(B)$. Hence $C_{\alpha^m}(A \cap B) \subset C_{\alpha^m}(A) \cap C_{\alpha^m}(B)$.

□

Theorem 3.12. *If A and B are subsets of a space X, then $C_{\alpha^m}(A \cup B) = C_{\alpha^m}(A) \cup C_{\alpha^m}(B)$.*

Proof. Let A and B be subsets of X. Clearly $A \subset A \cup B$ and $B \subset A \cup B$. Hence $C_{\alpha^m}(A) \cup C_{\alpha^m}(B) \subset C_{\alpha^m}(A \cup B) \longrightarrow (i)$.

Now to prove $C_{\alpha^m}(A \cup B) \subset C_{\alpha^m}(A) \cup C_{\alpha^m}(B)$. Let $x \in C_{\alpha^m}(A \cup B)$ and suppose $x \notin C_{\alpha^m}(A) \cup C_{\alpha^m}(B)$. Then there exists an α^m-closed sets A_1 and B_1

with $A \subset A_1$, $B \subset B_1$ and $x \notin A_1 \cup B_1$. we have $A \cup B \subset A_1 \cup B_1$ and $A_1 \cup B_1$ is an α^m-closed set such that $x \notin A_1 \cup B_1$. Thus $x \notin C_{\alpha^m}(A \cup B)$ which is a contradiction to $x \in C_{\alpha^m}(A \cup B)$. Hence $C_{\alpha^m}(A \cup B) \subset C_{\alpha^m}(A) \cup C_{\alpha^m}(B) \longrightarrow (ii)$.

From (i) and (ii), we have $C_{\alpha^m}(A \cup B) = C_{\alpha^m}(A) \cup C_{\alpha^m}(B)$.

□

Definition 3.13. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called a α^m-open function if the image of every α^m-open set in (X, τ) is a Bc-open set in (Y, σ).

Proposition 3.14. *Suppose that $f : (X, \tau) \longrightarrow (Y, \sigma)$ is α^m-open and surjective. If (X, τ) is $T_k\widetilde{\alpha^m}$, then (Y, σ) is $T_k\widetilde{\alpha^m}$, for $k = 0, 1, 2$.*

Proof. We prove only the case for $T_1\widetilde{\alpha^m}$-space the others are similarly. Let (X, τ) be a $T_1\widetilde{\alpha^m}$-space and let $y_1,\ y_2 \in Y$ with $y_1 \neq y_2$. Since f is surjective, so there exist distinct points $x_1,\ x_2$ of (X, τ) such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since (X, τ) is a $T_1\widetilde{\alpha^m}$-space, there exist α^m-open sets G and H such that $x_1 \in G$ but $x_2 \notin G$ and $x_2 \in H$ but $x_1 \notin H$. Since f is a α^m-open function, $f(G)$ and $f(H)$ are α^m-open sets of (Y, σ) such that $y_1 = f(x_1) \in f(G)$ but $y_2 = f(x_2) \notin f(G)$, and $y_2 = f(x_2) \in f(H)$ but $y_1 = f(x_1) \notin f(H)$. Hence (Y, σ) is a $T_1\widetilde{\alpha^m}$-space.

□

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