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A UNIFIED BEHOVIOR OF 2-D AND 3-D NONINVERTIBLE MAP

M. Mammeri
Department of Mathematics
University of Kasdi Merbah
Ouargla, ALGERIA

Abstract: In this paper, the unified behavior for maps with different dimensions is possible from some cases of dynamical systems. This short paper proposes a 2-D noninvertible discrete chaotic map with one bifurcation parameter, and that had only one nonlinear term, and a new 3-D noninvertible discrete chaotic map with twelve bifurcation parameters, and six-nonlinear terms. The two maps they have unified dynamical behavior and can displays same chaotic attractors form same bifurcation parameters and from identical bifurcation route to chaos. This interesting phenomenon is justified by numerical simulation.

AMS Subject Classification: 37C29, 37D45, 37G35
Key Words: noninvertible map, dynamical behavior, 2-D and 3-D discrete maps, chaotic attractors

1. Introduction

Of course, unifying dynamical behaviors for different dimensions of maps are probably rare in dynamical systems, the theoretical research of chaotic behaviors of maps with quadratic inverse and constant Jacobian is a very interesting topic in dynamical systems. One of these models in 2-D is the most famous map, has been intensively studied over past few decades [1, 2, 3, 4]. In recent years, there are many documents have studied 3-D chaotic maps such as with quadratic inverse and constant Jacobian [5, 6, 7, 8, 9]. On the other hand, the

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2-D or 3-D noninvertible map is defined as a map with no-constant Jacobian, a few such example are well known, many papers have studied 2-D and 3-D maps with no-constant Jacobian [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22]. So the study of the noninvertible map has important value since, large number of physics, engineering and economics systems have been widely studied found to exhibit a class of noninvertible maps [20, 21, 22]. This short paper investigate a new two simplest 2-D and 3-D noninvertible discrete chaotic maps. The two maps they have an unified dynamical behavior and can exhibits same type of chaotic attractors form identical bifurcation route to chaos. This very interesting phenomenon in dynamical systems is justified by some numerical simulations.

2. The Proposed 2-D and 3-D Maps

We consider the two-dimensional polynomial noninvertible map is given as follows:

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} x - ay \\ x - axy \end{pmatrix} \tag{1}$$

Where $(x, y) \in \mathbb{R}^2$ and $a \in \mathbb{R}$ is the bifurcation parameter. For $a = 0$ the map (1) reduces to two-dimensional linear map, the map (1) is of class $C^\infty(\mathbb{R}^2)$, and is one-to-one, the map (1) has only one fixed point is $(0, 0)$. Furthermore, this model permits the construction of a new family of attractors dependent on the bifurcation parameter a and initial conditions, as shown in Figures.3. The 2-D map (1) can be extended to a 3-D map.

We consider the new simple three-dimensional polynomial noninvertible quadratic system described by:

$$\begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \begin{pmatrix} x - ay \\ x - bxy \\ f(x, y, z) \end{pmatrix} \tag{2}$$

Where $f(x, y, z) = a_0 + a_1x + a_2y + a_3z + a_4x^2 + a_5y^2 - cz^2 + a_6xy + a_7xz + a_8yz$ and $(a, b, c, a_i)_{0 \leq i \leq 8} \in \mathbb{R}^{12}$ are bifurcation parameters and $(x, y, z) \in \mathbb{R}^3$ are the state variables. Doubtless, map (2) is an extended version of the two-dimensional noninvertible discrete chaotic system (1). Furthermore, the map (2) has at most two fixed points are $(0, 0, \alpha)$, where $\alpha = \frac{-(a_3-1) \pm \sqrt{(a_3-1)^2 - 4ca_0}}{2c}$, if $a_0 = 0$, the origin $(0, 0, 0)$ is fixed point of the map (2). Figures.4 shows a new family of attractors and chaotic attractors from some cases of the map (2).

3. Numerical Investigation

In this section, we choose eight forms of the map (2) in which the dynamical behaviors of the map (1) and map (2) are identical. Moreover, there are some other cases of the map (2) in which this property holds. The dynamical behaviors of the map (1) and for the map (2) in the cases (2.1) till (2.8) are investigated numerically.

We take the same initial conditions $x_0 = y_0 = z_0 = 0.01$, and bifurcation parameters $b = 0.8$ and $c = 0.1$, let a vary between 0.5 and 1.4. Figures.1(a) and Figures.2(a) shows respectively the identical bifurcation diagrams route to chaos for the state variable x plotted versus increasing the bifurcation parameter a of the map (1) and for the map (2) in the cases (2.1) till (2.8). Also, Figures.1(b) and Figures.2(b) shows respectively the identical diagrams of the largest Lyapunov exponent of the proposed map (1) and for the map (2) in the cases (2.1) till (2.8). Furthermore, Figures.3 and Figures.4 display respectively the same phase portraits of the map (1) and for the map (2) in the cases (2.1) till (2.8).

$$f(x, y, z) = -cz^2, \tag{2.1}$$

$$f(x, y, z) = y - cz^2, \tag{2.2}$$

$$f(x, y, z) = y^2 - cz^2, \tag{2.3}$$

$$f(x, y, z) = x + y - cz^2, \tag{2.4}$$

$$f(x, y, z) = x + y^2 - cz^2, \tag{2.5}$$

$$f(x, y, z) = x + y + y^2 - cz^2, \tag{2.6}$$

$$f(x, y, z) = x + y + x^2 + y^2 - cz^2, \tag{2.7}$$

$$f(x, y, z) = 1 + x + y + xy + y^2 - cz^2. \tag{2.8}$$

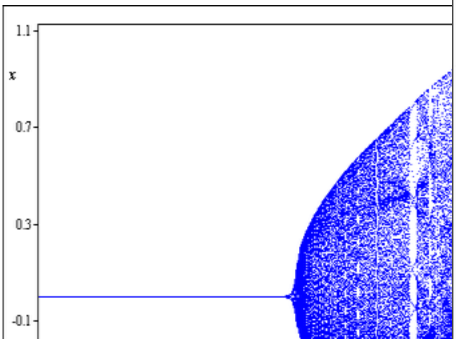

Figure 1(a): Bifurcation diagram of the map (1) of state x versus $0.5 \leq a \leq 1.4$.

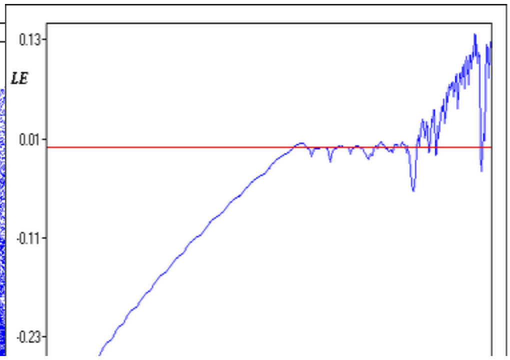

Figure 1(b): Lyapunov exponents of the map (1) versus $0.5 \leq a \leq 1.4$.

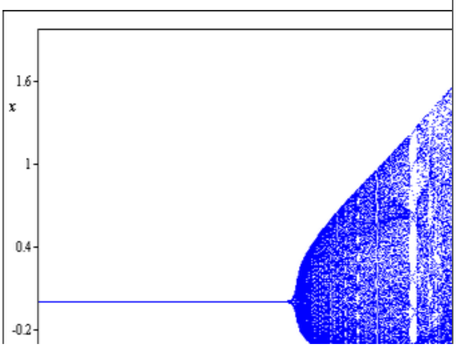

Figure 2(a): Bifurcation diagram of the map (2) of state x versus $0.5 \leq a \leq 1.4$ and $b = 0.8, c = 0.1$.

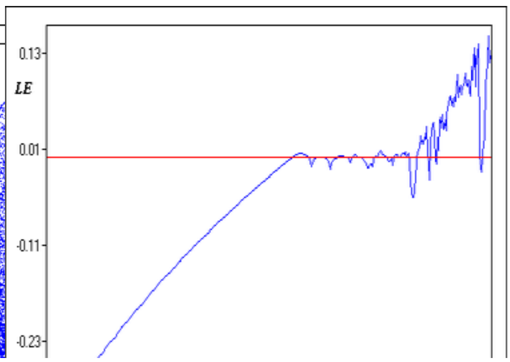

Figure 2(b): Lyapunov exponents of the map (2) versus $0.5 \leq a \leq 1.4$ and $b = 0.8, c = 0.1$.

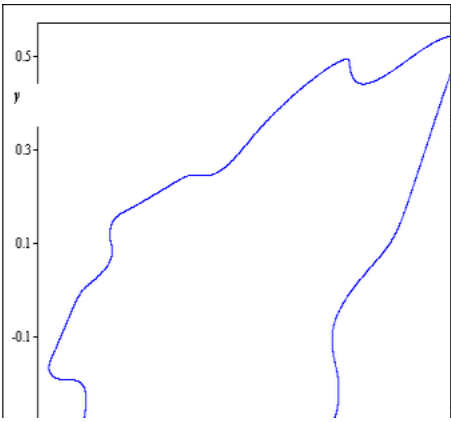

Figure 3(a): Periodic attractor of the map (1) ($a = 1.218$).

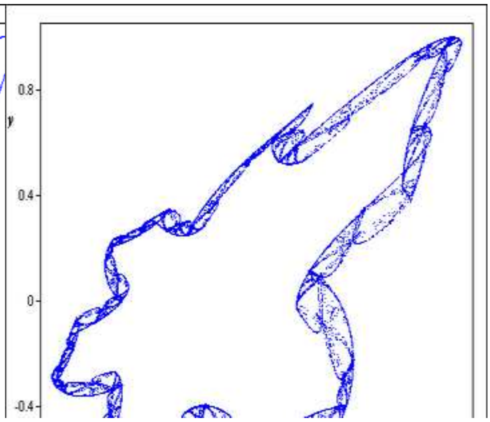

Figure 3(b): Chaotic attractor of the map (1) ($a = 1.26$).

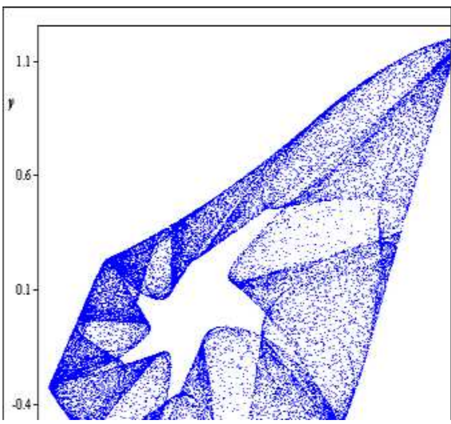

Figure 3(c): Chaotic attractor of the map (1) ($a = 1.302$).

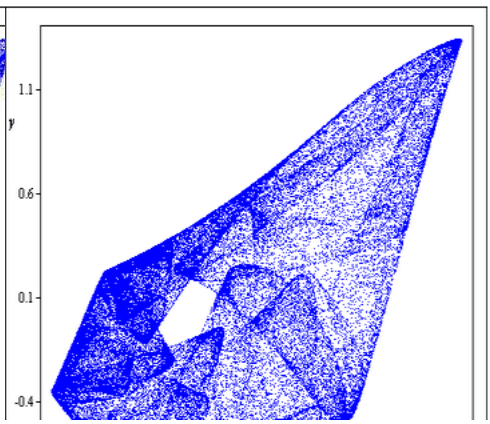

Figure 3(d): Chaotic attractor of the map (1) ($a = 1.33$).

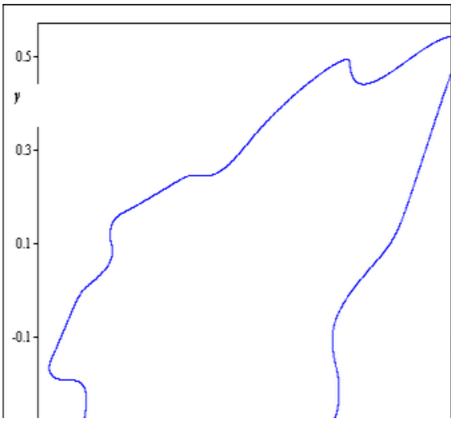

Figure 4(a): Periodic attractor of the map (2) ($a = 1.218, b = 0.8$ and $c = 0.1$).

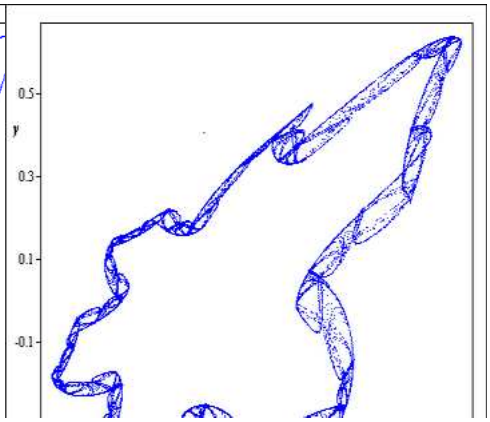

Figure 4(b): Chaotic attractor of the map (2) ($a = 1.26, b = 0.8$ and $c = 0.1$).

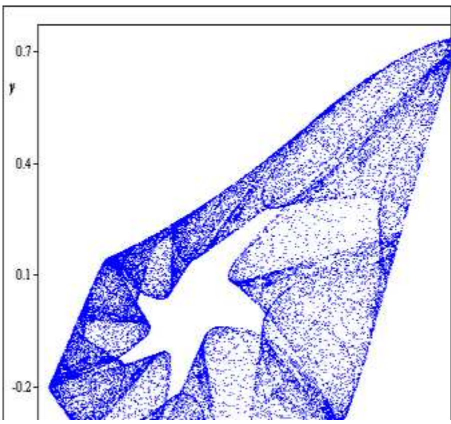

Figure 4(c): Chaotic attractor of the map (2) ($a = 1.302, b = 0.8$ and $c = 0.1$).

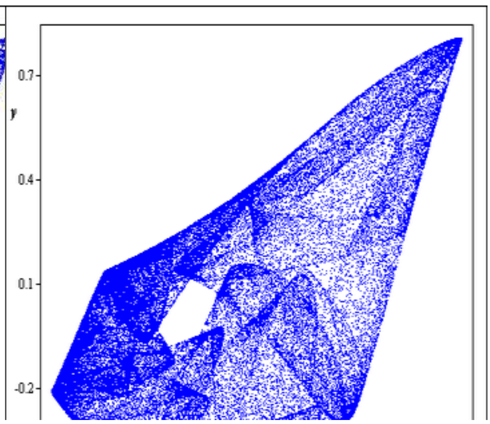

Figure 4(d): Chaotic attractor of the map (2) ($a = 1.33, b = 0.8$ and $c = 0.1$).

4. Conclusion

This short paper investigate a new two simplest 2-D and 3-D noninvertible discrete maps that realizes a new phenomenon in which the dynamical behaviors for our proposed map (1) and map (2) in the cases (2.1) till (2.8) are unified and they have same chaotic attractors form same bifurcation parameters and from identical bifurcation route to chaos, this important phenomenon is obtained by numerical investigation.

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