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COST-BENEFIT ANALYSIS OF TWO IDENTICAL WARM STANDBY SYSTEM SUBJECT TO UNDER HEAVY RAIN WITH PARTIALLY OPERATIVE AFTER REPAIR

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Abstract: The aim of this paper is to improve the reliability through regenerative point technique. In this paper two identical units in the warm standby system are considered. The standby component is referred as 'warm standby' when the failure rate is nonzero and is less than the failure rate of a primary component. Each unit of the system has three modes via; operative, failed and partially operative. The failure and repair times follow exponential and general time distributions respectively. We transform the basic equations of the proposed model into an integro-differential equations and solve it by semi-Markov processes. Using the regeneration point technique several reliability characteristics have been obtained. Various reliability parameters such as mean time to system failure, availability, busy period, number of visit periods by repairman and profit have been computed and analyzed by graphical illustrations.

1. Introduction

To improve the reliability of the systems many researchers have been worked on

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the reliability modeling. Osaki [4] and Taneja et al. [7] investigated reliability models of such systems with different failure rate and different repair facilities. Tuteja and Taneja [6] worked on warm standby system. Singh and Taneja [5], and Malhotra and Taneja [3] studied comparative study of the systems. Environmental conditions cannot be control which may fluctuate due to changing climate. Therefore, Goel and Sharma [1], Gupta and Goel [2] have obtained reliability measures of cold standby repairable systems operating under different weather conditions. Unprecedented heavy rain that hits Chennai city and other part of Tamilnadu during December 2015. All flight operations at Chennai airport had been partially suspended until further review and a number of flights have been disrupted as waters inundated the runway and the tarmac. SpiceJet and Indigo said the airfield had been closed due to flooding. While considering above facts and practical situations in mind, here reliability measures of a system of identical units operating under heavy rain obtained using semi-Markov process and regenerative point technique. In such situation we can improve the reliability of the system to first repair the system become partially operative after the repair of partially operative unit the system become operative.

Briefly mentioned all sections and subsections.

Model and transition probabilities and mean sojourn times have been developed and they are given below:

- Mean Times to system failure

- availability analysis

- busy period analysis of the repairman

- expected number of visits by the repairman

- profit analysis

- particular cases

The assumptions for the model are given below:

- both units are identical

- Starting the unit is not partially fail but after repair the unit becomes partially operative

- Only one repairman with the system

2. Notations

λ_1 :   Failure rate from operative unit

λ_2 :   Failure rate from warm stand by unit

o   :   up state

□   :   failed state

$G_1(t), g_1(t)$:   C.d.f. and P.d.f. of the repair time of failure unit to partially operative

$G_2(t), g_2(t)$:   C.d.f. and P.d.f. of the repair time of partially operative to operative unit

Op   :   operative unit

ws   :   warn standby unit

Fur_1 :   failure unit is under repair from the stage 1

Fur_2 :   failure unit is under repair from the stage 2

PouR_1 :   partial operative unit is under repair is continuing from the stage 1

PouR_2 :   partial operative repair is under repair is continuing from the stage 2

FuR_1 :   repair is continuing on failed unit from the stage 1

FuR_2 :   repair is continuing on failed unit from the stage 2

Fwr   :   waiting for repair

3. Transition Probabilities and Mean Sojourn Times

The epochs of entry into states 0, 1 and 2 regenerative points and thus these are regenerative states. States 3, 4, 5, 6, 7 and 8 are non regenerative states.

The non zero elements p_{ij} of the transition probability for the system are found out as

$$p_{ij} = \lim_{s \to 0} q_{ij}^*(s)$$

$$p_{01} = \lambda_1/(\lambda_2 + \lambda_1), \ p_{02} = \lambda_2/(\lambda_2 + \lambda_1), \ p_{18} = g_1^*(\lambda_1), \ p_{13} = (1 - g_1^*(\lambda_1))$$
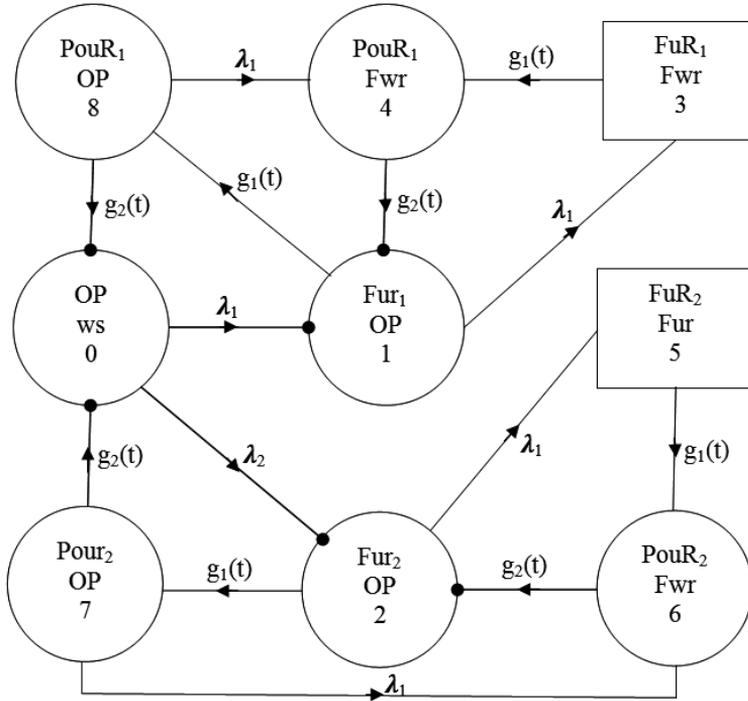
Figure 1: State Transition Diagram

$$p_{34} = g_1^*(0), \ p_{41} = g_2^*(0), \ p_{27} = g_1^*(\lambda_1), \ p_{25} = (1 - g_1^*(\lambda_1))$$
$$p_{70} = g_2^*(\lambda_1), \ p_{76} = (1 - g_2^*(\lambda_1)), \ p_{80} = g_2^*(\lambda_1), \ p_{84} = (1 - g_2^*(\lambda_1))$$
$$p_{56} = g_1^*(0), \ p_{62} = g_2^*(0)$$
$$p_{10}^8 = g_1^*(\lambda_1), \ g_2^*(\lambda_1), \ p_{11}^{84} = g_1^*(\lambda_1)(1 - g_2^*(\lambda_1)), \ p_{11}^{34} = (1 - g_1^*(\lambda_1))$$
$$p_{20}^7 = g_1^*(\lambda_1)g_2^*(\lambda_1), \ p_{22}^{76} = g_1^*(\lambda_1)(1 - g_2^*(\lambda_1)), \ p_{22}^5 6 = (1 - g_1^*(\lambda_2))$$

For the transition probabilities, it can be verified that

$$p_{01} + p_{02} = 1$$
$$p_{10}^8 + p_{11}^{84} + p_{11}^{34} = 1$$
$$p_{20}^7 + p_{22}^{56} + p_{22}^{76} = 1$$

The mean sojourn times μ_i in state 'i' are given by

$$\mu_0 = 1/(\lambda_2 + \lambda_1), \ \mu_1 = 1/\lambda_1(1 - g_1^*(\lambda_1)), \ \mu_2 = 1/\lambda_1(1 - g_1^*(\lambda_1)),$$

the unconditional mean time taken by the system to transit from any state 'i' when time is counted from epoch at entrance into state 'i' is stated as:

$$m_{ij} = \int_0^\infty t q_{ij}(t)dt = -q_{ij}^{*'}(0)$$

Thus,

$$m_{01} + m_{02} = \mu_0, \ m_{13} = g_1^{*1}(\lambda_1) + 1/\lambda_1(1 - g_1^*(\lambda_1)),$$
$$m_{18} = -g_1^{*1}(\lambda_1), \ m_{27} = -g_1^{*1}(\lambda_1),$$
$$m_{25} = g_1^{*1}(\lambda_1) + 1/\lambda_1(1 - g_1^*(\lambda_1)), \ m_{34} = -g_1^{*1}(0),$$
$$m_{41} = -g_2^{*1}(0), \ m_{56} = -g_1^{*1}(0), \ m_{62} = -g_2^{*1}(0),$$
$$m_{76} = g_1^{*1}(\lambda_1) + 1/\lambda_1(1 - g_1^*(\lambda_1)), \ m_{70} = -g_2^{*1}(\lambda_1),$$
$$m_{84} = g_1^{*1}(\lambda_1) + 1/\lambda_1(1 - g_1^*(\lambda_1)), \ m_{80} = -g_2^{*1}(\lambda_1),$$
$$m_{10}^8 + m_{11}^{84} + m_{13} = K$$
$$m_{10}^8 + m_{11}^{84} + m_{11}^{34} = K_1$$

4. Analysis of Mean Time to System Failure

Regarding the failed states as absorbing states and applying the arguments used for regenerative processes, the following recursive relation for $\phi_i(t)$ are obtained

$$\phi_0(t) = Q_{01}(t) \text{ⓢ} \phi_1(t) + Q_{02}(t) \text{ⓢ} \phi_2(t)$$
$$\phi_1(t) = Q_{13}(t) + Q_{10}^{(8)}(t) \text{ⓢ} \phi_0(t) + Q_{11}^{(84)}(t) \text{ⓢ} \phi_1(t)$$
$$\phi_2(t) = Q_{25}(t) + Q_{20}^{(7)}(t) \text{ⓢ} \phi_0(t) + Q_{22}^{(76)}(t) \text{ⓢ} \phi_2(t)$$

Taking Laplace-Stieltjes Transforms (L.S.T) of these relations and solving them by Cramer's rule for $\phi_0^{**}(s)$.

Now, the mean time to system failure (MTSF) when the system starts from the state '0' is

$$T_0 = \lim_{s \to 0} \frac{1 - Q_0^{**}(s)}{s} = \lim_{s \to 0} \frac{1 - \frac{N(S)}{D(S)}}{s} = \lim_{s \to 0} \frac{D(s) - N(s)}{sD(s)}$$
$$= \frac{D'(0) - N'(0)}{(D(0)} = \frac{N}{D}$$

where $N = (1 - p_{11}^{84})(K + \mu_0(p_{10}^8 + p_{13}))$ and $D = (1 - p_{11}^{84})p_{11}^{34}$.

5. Availability Analysis

The availability of a system is defined as the probability that the system is operating and provides service when requested.

Using the probabilistic argument and $A_i(t)$ as the probability of unit entering into up state at time t, given that the unit entered is regenerative state i at $t = 0$, the following recursive relation are obtained.

$$A_0(t) = M_0(t) + q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t)$$
$$A_1(t) = M_1(t) + q_{10}^{(8)}(t) © A_0(t) + (q_{11}^{(84)}(t) + q_{11}^{(34)}(t)) © A_1(t)$$
$$A_2(t) = M_2(t) + q_{20}^{(7)}(t) © A_0(t) + (q_{22}^{(76)}(t) + q_{22}^{(56)}(t)) © A_2(t)$$

where $M_0(t) = e^{-(\lambda_1+\lambda_2)t}dt$, $M_1(t) = e^{-\lambda_1 t}(\bar{G}_1)(t)dt$, $M_2(t) = e^{-\lambda_1 t}(\bar{G}_1)(t)dt$.

Taking Laplace transforms (L.T.) of these relations and solving them by Crammer rule for $A_0^*(s)$, we obtain

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)}$$

where,

$$
\begin{aligned}
N_1(s) = {} & M_0^*(s) - M_0^*(s)q_{22}^{*(76)}(s) - M_0^*(s)q_{22}^{*(56)}(s) \\
& + M_0^*(s)q_{11}^{*(84)}(s)q_{22}^{*(76)}(s) + M_0^*(s)q_{11}^{*(34)}(s)q_{22}^{*(76)}(s) \\
& + M_0^*(s)q_{11}^{*(84)}(s)q_{22}^{*(56)}(s) + M_0^*(s)q_{11}^{*(34)}(s)q_{22}^{*(56)}(s) \\
& + M_1^*(s)q_{01}^*(s) - M_1^*(s)q_{01}^*(s)q_{22}^{*(76)}(s) + M_2^*(s)q_{02}^*(s) \\
& - M_2^*(s)q_{02}^*(s)q_{11}^{*(84)}(s) - M_2^*(s)q_{02}^*(s)q_{11}^{*(34)}(s) \\
& - M_1^*(s)q_{01}^*(s)q_{22}^{*(56)}(s) \\
D_1(s) = {} & 1 - q_{22}^{*(76)} - q_{22}^{*(56)}(s) - q_{11}^{*(84)}(s) - q_{11}^{*(34)}(s) \\
& + q_{11}^{*(84)}(s)q_{22}^{*(76)}(s) + q_{11}^{*84}(s)q_{22}^{*(56)}(s) \\
& + q_{11}^{*(34)}(s)q_{22}^{*(76)}(s) + q_{11}^{*(34)}(s)q_{22}^{*(56)}(s) \\
& - q_{01}^*(s)q_{10}^{*(8)}(s) + q_{01}^*(s)q_{10}^{*(8)}(s)q_{22}^{*(76)}(s) \\
& + q_{01}^*(s)q_{10}^{*(8)}(s)q_{22}^{*(56)}(s) - q_{02}^*(s)q_{10}^{*(7)}(s) \\
& + q_{02}^*(s)q_{10}^{*(7)}(s)q_{11}^{*(84)}(s) + q_{02}^*(s)q_{20}^{*(7)}(s)q_{11}^{*(34)}(s)
\end{aligned}
$$

The steady state availability of the system is given by

$$A_0 = \lim_{s \to 0}(sA_0^*(s)) = \lim_{s \to 0}\left(s\frac{N_1(s)}{D_1(s)}\right) = \frac{N_1(0)}{D_1'(0)} = \frac{N_1}{D_1}$$

$$N_1 = (M_0 + M_0)p_{10}^{(8)} + M_0(1 - p_{10}^{(8)})(1 - p_{10}^{(8)})$$

$$D_1 = p_{10}^{(8)}(K_1 + \mu_0 p_{10}^{(8)})$$

where, K_1 is already specified.

6. Busy Period Analysis of the Repair Man

$B_i(t)$ = Probability that the repair man is busy at instant t, given that the system entered regenerative state i at $t = 0$,

$$B_0(t) = q_{01}(t) © B_1(t) + q_{02}(t) © B_2(t)$$

$$B_1(t) = W_1(t) + q_{10}^{(8)}(t) © B_0(t) + (q_{11}^{(84)}(t) + q_{11}^{(34)}(t)) © B_1(t)$$

$$B_2(t) = W_2(t) + q_{20}^{(7)}(t) © B_0(t) + (q_{22}^{(76)}(t) + q_{22}^{(56)}(t)) © B_2(t)$$

where

$$W_1(t) = e^{-\lambda_1 t}(\bar{G}_1)(t)dt + (\lambda_2 e^{-\lambda_1 t} © 1)(\bar{G}_1)(t)dt$$

$$W_2(t) = e^{-\lambda_1 t}(\bar{G}_1)(t)dt + (\lambda_1 e^{-\lambda_1 t} © 1)(\bar{G}_1)(t)dt$$

Taking Laplace transforms (L.T.) of these relations and solving them by applying crammer's rule for $B_0^*(s)$, we obtain

$$B_0^*(s) = \frac{N_2(s)}{D_1(s)}$$

where

$$N_2(s) = q_{01}^*(s)W_1^*(s)(1 + q_{01}^*(s)W_1^*(s) - q_{01}^*(s)W_1^*(s)q_{22}^{*(76)}(s)$$
$$- q_{01}^*(s)W_1^*(s)q_{22}^{*(56)}(s) + q_{01}^*(s)W_2^*(s) - q_{02}^*(s)W_2^*(s)q_{11}^{*(84)}(s)$$
$$- q_{02}^*(s)W_2^*(s)q_{11}^{*(34)}(s)$$

and $D_1(s)$ is already specified.

In steady state, the total fraction of time for which the system is under repair 1 is given by

$$B_0 = \lim_{s \to 0}(sB_0^*(s)) = \lim_{s \to 0}\left(\frac{s(N_2(s))}{D_1(s)}\right) = \frac{N_2(0)}{D_1'(0)} = \frac{N_2}{D_1}$$

where, $N_2 = W_5 p_1 0^{(8)}$, $W_1 = W_1^*(0) = W_2^*(0)$ and D_1 is already specified.

7. Expected Number of Visits by the Repair Man

We define $V_0(t)$ = expected number of visits by the repair man in $(0, t]$, given that the system started from the regenerative state i at $t = 0$

$$V_0(t) = Q_{01}(t) \, \text{Ⓢ} \, (1 + V_1(t)) + Q_{02}(t) \, \text{Ⓢ} \, (1 + V_1(t))$$
$$V_1(t) = Q_{10}^{(8)}(t) \, \text{Ⓢ} \, V_0(t) + (Q_{11}^{(84)}(t) + Q_{11}^{(34)}(t)) \, \text{Ⓢ} \, V_1(t))$$
$$V_2(t) = Q_{20}^{(7)}(t) \, \text{Ⓢ} \, V_0(t) + (Q_{22}^{(76)}(t) + Q_{22}^{(56)}(t)) \, \text{Ⓢ} \, V_2(t)$$

Taking LaplaceStieltjes Transforms (L.S.T.) of these relations and solving them by applying crammer rule for $V_0^{**}(s)$, we obtain

$$V_0^{**}(s) = \frac{N_3(s)}{D_1(s)}$$

where

$$N_3(s) = (Q_{01}^{**}(s) + Q_{02}^{**}(s))(1 - (Q_{11}^{**(84)}(s) + Q_{11}^{**(34)}(s))$$
$$(1 - (Q_{22}^{**(76)}(s) + Q_{22}^{**(56)}(s))$$

and $D_1(s)$ is already specified.

$$V_0 = \lim_{s \to 0}(sV_0^*(s)) = \lim_{s \to 0}\left(\frac{s(N_3(s))}{D_1(s)}\right) = \frac{N_3(0)}{D_1'(0)} = \frac{N_3}{D_1}$$

where $N_3 = 1 - p_{10}^{(8)} p_{10}^{(}(8))$ and D_1 is already specified.

8. Cost-Benefit Analysis

The expected total profit incurred to the system in steady state is given by

$$P = C_0 A_0 - C_1 B_0 - C_2 V_0$$

where

$\qquad C_0$ = revenue per unit up time of the system

$\qquad C_1$ = cost per unit time for which expert repairman is busy

$\qquad\qquad$ in repairing in the field unit

$\qquad C_2$ = cost per visit of expert.

9. Graphical Study

Let us assume that the repair rate are exponentially distributed as under:

$$g_1(t) = \alpha_1 e^{-\alpha_1 t} \ \ \text{and} \ \ g_2(t) = \alpha_2 e^{-\alpha_2 t}$$

$$p_{01} = \lambda_1/(\lambda_2 + \lambda_1), \ p_{18} = \lambda_2/(\lambda_2 + \lambda_1),$$

$$p_{13} = \lambda_1/(\lambda_2 + \alpha_1), \ p_{34} = 1, \ p_{41} = 1, \ p_{18} = \alpha_1/(\lambda_1 + \alpha_1),$$

$$p_{84} = \lambda_1/(\lambda_1 + \alpha_2), \ p_{80} = \alpha_2/(\lambda_1 + \alpha_2),$$

$$p_{10}^{(8)} = \alpha_1/(\lambda_1 + \alpha_1) \cdot \alpha_2/(\lambda_1 + \alpha_2),$$

$$p_{11}^{(34)} = \lambda_1/(\lambda_1 + \alpha_1), \ p_{11}^{(84)} = \frac{\alpha_1}{(\lambda_1 + \alpha_1)} \cdot \frac{\lambda_1}{(\lambda_1 + \alpha_2)},$$

$$\mu_0 = 1/(\lambda_2 + \lambda_1), \ m_{13} = \frac{\alpha_1}{\lambda_1 + \alpha_1} \cdot 1/(\lambda_1 + \alpha_1),$$

$$m_{18} = \frac{\lambda_1}{\lambda_1 + \alpha_1} \cdot 1/(\lambda_1 + \alpha_1)$$

$$m_{34} = 1/\alpha_1, \ m_{41} = 1/\alpha_2, \ m_{80} = \frac{\alpha_2}{\lambda_1 + \alpha_2} \cdot 1/(\lambda_1 + \alpha_2),$$

$$m_{84} = \frac{\lambda_1}{\lambda_1 + \alpha_2} \cdot 1/(\lambda_1 + \alpha_2)$$

$$M_0 = \frac{1}{\lambda_2 + \lambda_1}, \ M_1 = \frac{1}{\alpha_1 + \lambda_1}, \ W_1 = \frac{1}{\alpha_1}$$

10. Graphical Interpretation

By giving some numerical values to the parameters involved, various graphs has been plotted using particular case and the following interpretations have been drawn.

Figure 2 shows the behaviour operative unit for different values of repair rate (α_1). From the graph, we can see of MTSF with respect to failure rate (λ_1) of the that the MTSF decreases as λ_1 increases, but has higher values for higher values of α_1.

Figure 3 shows the behaviour of availability (A_0) with respect to failure rate (λ_2) of the operative unit for different values of repair rate (α_2). From the graph, we can see that the A_0 decreases as λ_2 increases, but has higher values for higher values of α_2.

Figure 4 shows the behaviour of profit (P) with respect to failure rate (λ_2) of the operative unit for different values of repair rate (α_2). From the graph, we
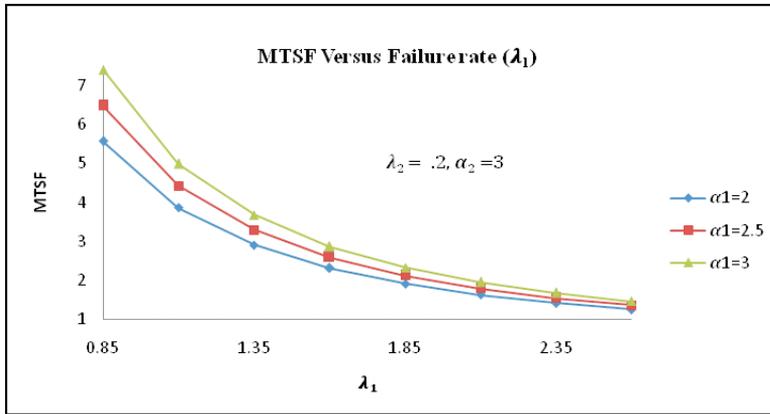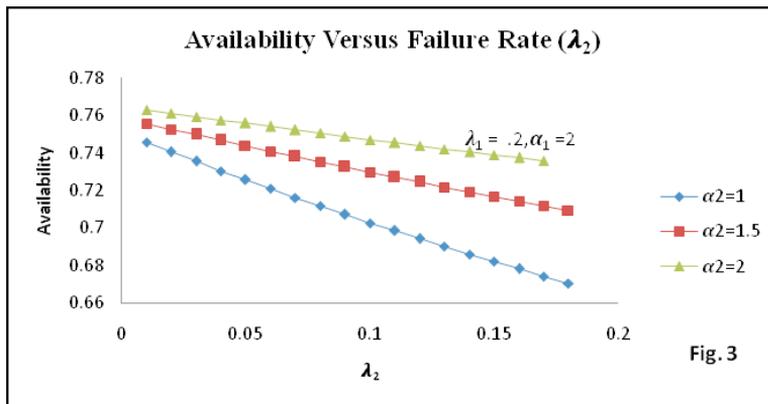
Figure 2

Figure 3

can see that the P decreases as λ_2 increases, but has higher values for higher values of α_2.

Following conclusions can be drawn from Figure 3.5:

(i) If $\alpha = 7$, then profit (P) is $>$ or $=$ or < 0 according as failure rate (λ_2) is $<$ or $=$ or > 1.67. Therefore, the quality of the unit should be such that it has failure rate (λ_2) less than 1.67, otherwise the system will give negative profit.

(ii) If $\alpha_2 = 9$, then profit (P) is $>$ or $=$ or < 0 according as failure rate (λ_2) is $<$ or $=$ or > 1.802. Therefore, the quality of the unit should be such that it has failure rate (λ_2) less than 1.802, otherwise the system will give

Figure 4
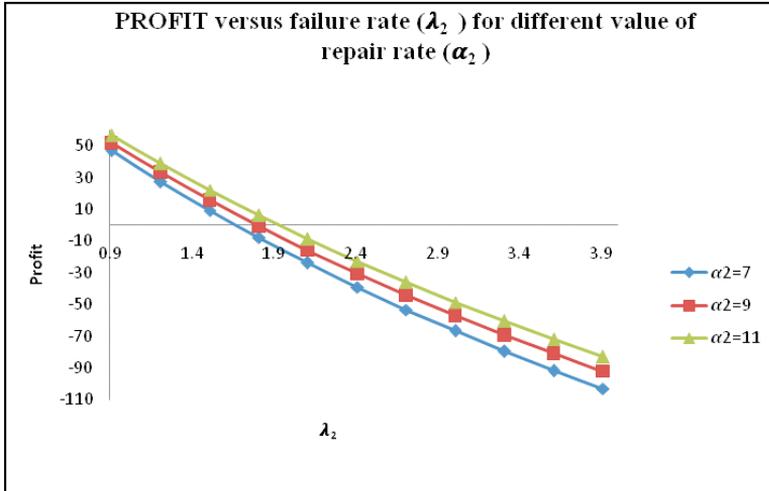
negative profit.

(iii) If $\alpha_2 = 11$, then profit (P) is $>$ or $=$ or < 0 according as failure rate (λ_2) is $<$ or $=$ or > 1.935. Therefore, the quality of the unit should be such that it has failure rate (λ_2) less than 1.935, otherwise the system will give negative profit.
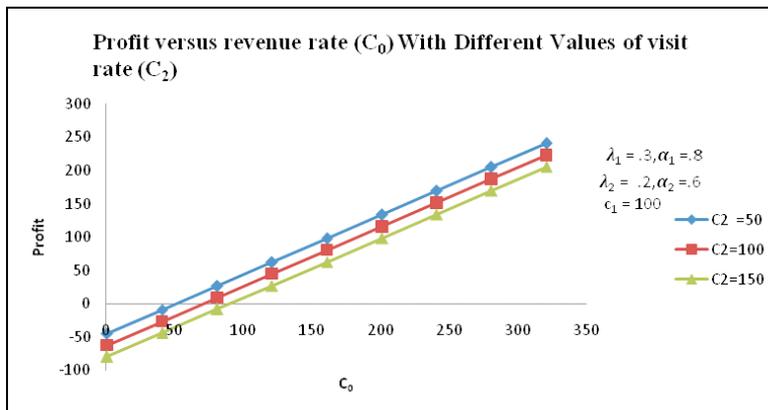

Figure 5

Figure 5 shows the behavior of profit (P) with respect to cost per unit visit up revenue (C_0) for different values of cost per visit (C_2) of the repairman.It

reaveals that profit (P) increase with increase in the values of C_0, but it gets lowered for higher of C_2.

Following conclusions can also be drawn from the graph.

(i) For $C_2 = 50$, the profit (P) is $>$ or $=$ or < 0 according as C_0 is $>$ or $=$ or < 50.92. Thus, the price of profit should be fixed in such a way so that the revenue is atleast 50.92.

(ii) For $C_2 = 100$, the profit (P_2) is $>$ or $=$ or < 0 according as C_0 is $>$ or $=$ or < 70.62. Thus, the price of profit should be fixed in such a way so that the revenue is atleast 70.62.

(iii) For $C_2 = 150$, the profit (P_2) is $>$ or $=$ or < 0 according as C_0 is $>$ or $=$ or < 90.32. Thus, the price of profit should be fixed in such a way so that the revenue is atleast 90.32.

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