CHARACTERIZATION OF 2-PRIMAL SNT -SEMIGROUPS

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Abstract: In this section we prove if S be a SNT -semigroup with DCC on the right ideals. $P(S)$ has IFP then $S_{\Gamma}(P)$ has the IFP for each $P \in mSpec(S)$ and also we prove if S be a SNT -semigroup and satisfies DCC right ideals. S is a 2 primal Γ -semigroup then S_{Γ_P} is a completely semiprime ideal.

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1. Introduction

In 1964, N. Nobusawa (see [4]) introduced the notion of a Γ -ring, more general than a ring. In 1966, W.E. Barnes (see [2]) weakened slightly the conditions in the definition of Γ -ring in the sense of Nobusawa. Many fundamental results in ring theory have been extended to Γ -rings by different authors obtaining various authors generalization analogous to corresponding parts in ring theory. In 1981, M.K. Sen (see [5]) and later in 1986. Sen and Saha in [6,7,8], introduced

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the concept of the Γ -semigroup as a generalization of semigroup and ternary semigroup. Many classical notions of semigroup have extended to Γ -semigroup, see [1,2].

In this paper we introduced and study characterization of 2 primal SNT -semigroups extended and generalizing the results obtained for semigroups.

Definition 2.1. A Γ -semigroup S is said to be 2-primal Γ -semigroup if and only if $\mathbf{P}(S) = \mathbf{N}(S)$, where $\mathbf{P}(S)$ denotes the intersection of all prime ideals of the Γ -semigroup S . i.e. the prime radical of S and $\mathbf{N}(S)$ denotes the set of all nilpotent elements of S .

Definition 2.2. A Γ -semigroup S is said to be right symmetric if for any $x, y, z \in S$, $x\Gamma y\Gamma z = 0$ implies $x\Gamma z\Gamma y = 0$. An ideal A is said to be right symmetric if for any $x, y, z \in S$, $x\Gamma y\Gamma z \subseteq A$ implies $x\Gamma z\Gamma y \subseteq A$. Similarly we can define left symmetric.

Definition 2.3. A Γ -semigroup S is said to be left symmetric if for any $x, y, z \in S$, $x\Gamma y\Gamma z = 0$ implies $y\Gamma x\Gamma z = 0$. An ideal A is said to be left symmetric if for any $x, y, z \in S$, $x\Gamma y\Gamma z \subseteq A$ implies $y\Gamma x\Gamma z \subseteq A$.

Lemma 2.4. Any Γ -semigroup with out strongly nilpotent elements is right symmetric.

Proof. Let S be a Γ -semigroup having no strongly nilpotent elements. Let $x, y, z \in S$ such that $x\Gamma y\Gamma z = 0$. Then

$$z\Gamma(x\Gamma y\Gamma z)\Gamma x\Gamma y = 0,$$

i.e $(z\Gamma x\Gamma y)\Gamma(z\Gamma x\Gamma y) = 0$. Since S has no strongly nilpotent elements, then

$$z\Gamma x\Gamma y = 0.$$

Hence

$$x\Gamma y\Gamma x\Gamma(z\Gamma x\Gamma y)\Gamma x\Gamma z = 0,$$

and so $x\Gamma y\Gamma x\Gamma z = 0$.

Thus $y\Gamma x\Gamma z\Gamma y\Gamma(x\Gamma y\Gamma x\Gamma z)\Gamma y\Gamma x = 0$. Therefore

$$y\Gamma x\Gamma z\Gamma y\Gamma x = 0,$$

and so that $x\Gamma z\Gamma(y\Gamma x\Gamma z\Gamma y\Gamma x)\Gamma z\Gamma y = 0$, i.e. $((x\Gamma z\Gamma y)\Gamma)^2(x\Gamma z\Gamma y) = 0$ and consequently $x\Gamma z\Gamma y = 0$.

Hence S is right symmetric.

Definition 2.5. An one sided ideal A of S has the intersection of factors property (or simply IFP) if for any $a, b \in S$, $a\Gamma b \subseteq A \Rightarrow a\Gamma S\Gamma b \subseteq A$.

Definition 2.6. Let S be a Γ -semigroup and P be a prime ideal of S then we define

$$S_\Gamma(P) = \{x \in S : x\Gamma S\Gamma y \subseteq \mathbf{P}(S) \text{ for some } y \in S \setminus P\},$$

$$S_{\Gamma_P} = \{x \in S : x\Gamma y \subseteq \mathbf{P}(S) \text{ for some } y \in S \setminus P\},$$

$$\overline{S}_{\Gamma_P} = \{x \in S : (x\Gamma)^n x \subseteq S_{\Gamma_P} \text{ for some positive integer } n\}.$$

Lemma 2.7. Let S be a Γ -semigroup then $S_\Gamma(P) \subseteq S_{\Gamma_P} \subseteq \overline{S}_{\Gamma_P}$ for every prime ideal P of S .

Proof. Let $x \in S_\Gamma(P)$. Then there exist a $y \in S \setminus P$ such that $x\Gamma S\Gamma y \subseteq \mathbf{P}(S)$. In this case, there exists $m \in S$ and $\alpha \in \Gamma$ such that $m\alpha y \notin P$. Suppose on the contrary, $S\Gamma y \subseteq P$. Then $y\Gamma S\Gamma y \subseteq P$. Since P is prime, $y \in P$, a contradiction. Thus, we get $x\Gamma m\alpha y \subseteq x\Gamma M\Gamma y \subseteq \mathbf{P}(S)$ and $m\alpha y \notin P$ for some $m \in S$ and $\alpha \in \Gamma$ and consequently $x \in S_{\Gamma_P}$. The proof of $S_{\Gamma_P} \subseteq \overline{S}_{\Gamma_P}$ is obvious.

Definition 2.8. A Γ -semigroup S is said to be SNT -semigroup. If $\mathbf{N}(S) = \mathbf{N}_\Gamma(S)$, where $\mathbf{N}_\Gamma(S)$ is the set of all strongly nilpotent element of S .

Notation 2.9. $\text{Spec}(S)$ denotes the set of all prime ideals of S and $m\text{Spec}(S)$ denote the set of all minimal prime ideals of S .

Theorem 2.10. Let S be a SNT -semigroup with DCC on right ideals. $\mathbf{P}(S)$ has IFP then $S_\Gamma(P)$ has the IFP for each $P \in m\text{Spec}(S)$.

Proof. Let $a, b \in S$ such that $a\Gamma b \subseteq S_\Gamma(P)$ for each $P \in m\text{Spec}(M)$. Then $a\Gamma b\Gamma S\Gamma y \subseteq \mathbf{P}(S)$ for some $y \in S \setminus P$. Since $\mathbf{P}(S)$ has IFP, $(a\Gamma S\Gamma b)\Gamma S\Gamma b \subseteq \mathbf{P}(S)$. Hence $(a\Gamma S\Gamma b)\Gamma S\Gamma b \subseteq S_\Gamma(P)$ and consequently $S_\Gamma(P)$ has the IFP.

Theorem 2.11. Let S be a SNT -semigroup with DCC on right ideals. $S_\Gamma(P)$ has the IFP for each $P \in m\text{Spec}(S)$ then S is a 2 primal Γ semigroup.

Proof. Let $x \in \mathbf{N}(S)$. Since S is a SNT -semigroup, there exists a positive integer n such that $(x\Gamma)^n x = 0$. We claim $x \in \mathbf{P}(S)$. If $x \notin \mathbf{P}(S)$, then there

exists $P \in \text{Spec}(S)$ such that $x \notin P$. Since P is prime and $x \notin P$, there exist $m_1 \in S$ and $\alpha_1, \alpha_2 \in \Gamma$ such that $x\alpha_1 m_1 \alpha_2 x \notin P$. Again use the primeness of P , there exists $m_2 \in S$, $\alpha_3, \alpha_4 \in \Gamma$ such that $x\alpha_1 m_1 \alpha_2 x \alpha_3 m_2 \alpha_4 x \notin P$. Proceeding like this, we get $m_1, m_2, \dots, m_n \in S$ and $\alpha_1, \alpha_2, \dots, \alpha_{2n} \in \Gamma$ such that $x\alpha_1 m_1 \alpha_2 x \alpha_3 \dots m_n \alpha_{2n} x \notin P$. Since $(x\Gamma)^n x = 0$ and $S_\Gamma(P)$ is a two sided ideal of S , $(x\Gamma)^n x \subseteq S_\Gamma(P)$. Again since $S_\Gamma(P)$ has *IFP*, $x\Gamma m_1 \Gamma m_2 \Gamma \dots \Gamma m_n \Gamma x \subseteq S_\Gamma(P)$.

Hence $x\alpha_1 m_1 \alpha_2 x \alpha_3 \dots m_n \alpha_{2n} x \in P$, a contradiction and consequently $x \in \mathbf{P}(S)$. Thus $\mathbf{N}(S) \subseteq \mathbf{P}(S)$. But always $\mathbf{P}(S) \subseteq \mathbf{N}(S)$ and so S is a 2 primal Γ -semigroup.

Proposition 2.12. *Let S be an SNT -semigroup with *DCC* on right ideals. S is a 2 primal Γ -semigroup then $S_\Gamma(P) = \overline{S}_{\Gamma_P}$ for each $P \in m\text{Spec}(S)$.*

Proof. Let $x \in \overline{S}_{\Gamma_P}$ for each $P \in m\text{Spec}(S)$. Then $(x\Gamma)^n x \subseteq S_{\Gamma_P}$ for some positive integer n . Hence there exists $y \in S \setminus P$ such that $(x\Gamma)^n x \Gamma y \subseteq \mathbf{P}(S)$. Since $\mathbf{P}(S)$ has *IFP*, $((x\Gamma y)\Gamma)^n x \Gamma y \subseteq \mathbf{P}(S)$. Since $\mathbf{P}(S)$ is completely semi prime, $x\Gamma y \subseteq \mathbf{P}(S)$. Again using *IFP* of $\mathbf{P}(S)$, we have $x\Gamma S \Gamma y \subseteq \mathbf{P}(S)$. Thus $x \in S_\Gamma(P)$ and so $\overline{S}_\Gamma(P) \subseteq S_\Gamma(P)$. By lemma 2.7, $S_\Gamma(P) \subseteq \overline{S}_\Gamma(P)$ and consequently $S_\Gamma(P) = \overline{S}_\Gamma(P)$.

Proposition 2.13. *Let S be an SNT -semigroup with *DCC* on right ideals $S_\Gamma(P) = \overline{S}_{\Gamma_P}$ for each $P \in m\text{Spec}(S)$ then $S_\Gamma(P) = S_{\Gamma_P} \Gamma \mathbf{P}$ for each $P \in m\text{Spec}(S)$.*

Proof. Suppose that $S_\Gamma(P) = \overline{S}_{\Gamma_P}$ for each $P \in m\text{Spec}(S)$. By lemma 2.7, $S_\Gamma(P) \subseteq S_{\Gamma_P} \subseteq \overline{S}_{\Gamma_P}$. Hence $S_\Gamma(P) = S_{\Gamma_P}$.

Theorem 2.14. *Let S be a SNT -semigroup. S is a 2 primal Γ -semigroup then $\mathbf{P}(S)$ is a completely semi prime ideal of S .*

Proof. Let $a \in S$ such that $a\Gamma a \subseteq \mathbf{P}(S)$. since S is a 2 primal and SNT -semigroup, there is a positive integer n such that $((a\Gamma a)\Gamma)^n (a\Gamma a) = 0$, i.e. $(a\Gamma)^{2n+1} a = 0$. So $a \in \mathbf{N}(S)$. Hence $a \in \mathbf{P}(S)$. Thus $\mathbf{P}(S)$ is a completely semi prime ideal of S .

Lemma 2.15. *Let S be a SNT -semigroup. $\mathbf{P}(S)$ is a completely semi prime ideal of S . Then $\mathbf{P}(S)$ is a left and right symmetric ideal of S .*

Proof. Let $a, b, c \in S$ such that $a\Gamma b\Gamma c \subseteq \mathbf{P}(S)$. Then $(c\Gamma a\Gamma b)\Gamma(c\Gamma a\Gamma b) \subseteq \mathbf{P}(S)$. Using the similar argument used in theorem 2.11, we can easily prove

that $a\Gamma c\Gamma b \subseteq \mathbf{P}(S)$. More over, $a\Gamma c\Gamma b \subseteq \mathbf{P}(S)$ implies $(b\Gamma a\Gamma c)\Gamma(b\Gamma a\Gamma c) \subseteq \mathbf{P}(S)$ and so $b\Gamma a\Gamma c \subseteq \mathbf{P}(S)$. There fore $\mathbf{P}(S)$ is a left and right symmetric ideal of S

Proposition 2.16. *Let S be an SNT -semigroup. $x\Gamma y \subseteq \mathbf{P}(S)$ implies $y\Gamma S\Gamma x \subseteq \mathbf{P}(S)$ for $x, y \in S$. Then S is a 2 primal Γ -semigroup.*

Proof. Let $x \in \mathbf{N}(S)$. Since S is an SNT -semigroup, $(x\Gamma)^n x = 0$ for some n . Hence $(x\Gamma)^n x = (x\Gamma)(x\Gamma)^{n-1}x \subseteq \mathbf{P}(S)$. By hypothesis $(x\Gamma)^{n-1}x\Gamma S\Gamma x \subseteq \mathbf{P}(S)$. Also

$$(x\Gamma)^{n-1}x\Gamma S\Gamma x = ((x\Gamma)^{n-2}x)\Gamma(x\Gamma S\Gamma x) \subseteq \mathbf{P}(S)$$

implies

$$x\Gamma S\Gamma x\Gamma S\Gamma (x\Gamma)^{n-2}x \subseteq \mathbf{P}(S).$$

Continuing this process, we get $x\Gamma S\Gamma x\Gamma \dots x\Gamma S\Gamma x \subseteq \mathbf{P}(S)$. Since $\mathbf{P}(S)$ is a semi prime ideal, $x \in \mathbf{P}(S)$. Thus $\mathbf{N}(S) \subseteq \mathbf{P}(S)$ and consequently S is a 2 primal Γ semigroup.

Theorem 2.17. *Let S be a SNT -semigroup and satisfies DCC right ideals. S is a 2 primal Γ -semigroup then S_{Γ_P} is a completely semi prime ideal.*

Proof. By proposition 2.13, $S_{\Gamma_P} = S_{\Gamma}(P)$ for each $P \in mSpec(s)$. First our aim is to prove that $S_{\Gamma_P} = \overline{S}_{\Gamma_P} = S_{\Gamma}(P)$ for each $P \in mSpec(S)$. If $a \in \overline{S}_{\Gamma_P}$, then $(a\Gamma)^n a\Gamma y \subseteq \mathbf{P}(S)$ for some positive integer n and some $y \in S \setminus P$. By theorem 2.10, $\mathbf{P}(S)$ has IFP, $((a\Gamma y)\Gamma)^n(a\Gamma y) \subseteq \mathbf{P}(S)$. Thus $a\Gamma y \subseteq \mathbf{P}(S)$ and so $a \in S_{\Gamma_P} = S_{\Gamma}(P)$. But by lemma 2.7, $S_{\Gamma_P} \subseteq \overline{S}_{\Gamma_P} \subseteq S_{\Gamma}(P)$. Hence $S_{\Gamma}(P) = S_{\Gamma_P} = \overline{S}_{\Gamma_P}$. Now we shall prove that $S_{\Gamma}(P)$ is a completely semi prime ideal of S . Let $a\Gamma a \subseteq S_{\Gamma}(P)$. Then there exists a positive integer n such that $((a\Gamma a)\Gamma)^n(a\Gamma a) \subseteq S_{\Gamma_P}$, because $S_{\Gamma_P} = \overline{S}_{\Gamma_P} = S_{\Gamma}(P)$. Therefore $(a\Gamma)^{2n+1}a \subseteq S_{\Gamma_P}$. So that $a \in S_{\Gamma}(P)$. Thus $S_{\Gamma}(P)$ is a completely prime ideal of S .

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