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ON AN EXTENSION OF SHARP DOMAINS

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Abstract: As an extension of the class of sharp domains introduced by Ahmad et al., we introduce and study a class of integral domains D characterized by the property that whenever X, Y_1, Y_2 are nonzero ideals of D with $X \supseteq Y_1Y_2$, there exist nonzero ideals Z_1 and Z_2 such that $X_w = (Z_1Z_2)_w$, $(Z_1)_w \supseteq Y_1$ and $(Z_2)_w \supseteq Y_2$. We call D with this property a w-sharp domain. We show that every fraction ring of a w-sharp domain is w-sharp, that a w-Dedekind domain is w-sharp and that every nonzero finitely generated ideal of a w-sharp domain is w-invertible.

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1. Introduction

In [9], Cohn introduced a Schreier domain. The study of Schreier domains was continued in [8, 19, 20]. Later many extensions of the Schreier domains were introduced and studied in [1, 2, 5, 10, 11, 13]. In [2], Ahmad et al. called a domain D a sharp domain if whenever X, Y_1, Y_2 are nonzero ideals of D with $X \supseteq Y_1Y_2$, there exist nonzero ideals Z_1 and Z_2 such that $X = Z_1Z_2$, $Z_1 \supseteq Y_1$ and $Z_2 \supseteq Y_2$. They showed that a sharp domain is a completely integrally closed generalized GCD domain, that a sharp domain is a Prüfer domain of dimension ≤ 1 and that a countable sharp domain is a Dedekind domain.

In this paper, we study the following extension of a sharp domain. We call a domain D a w-sharp domain if whenever X, Y_1, Y_2 are nonzero ideals of D with $X \supseteq Y_1Y_2$, there exist nonzero ideals Z_1 and Z_2 such that $X_w = (Z_1Z_2)_w$, $(Z_1)_w \supseteq Y_1$ and $(Z_2)_w \supseteq Y_2$.

We show that every fraction ring of a w-sharp domain is w-sharp (Proposition 2.3). A w-sharp domain is a PwMD of w-dimension ≤ 1 (Proposition 2.4). A w-Dedekind domain is a w-sharp domain (Proposition 2.5). The v-closure of every nonzero ideal of a w-sharp domain is w-invertible (Proposition 2.6). A w-sharp Mori domain is w-Dedekind (Corollary 2.7). Every nonzero finitely generated ideal of a w-sharp domain is w-invertible (Proposition 2.10). A countable w-sharp domain is a w-Dedekind domain (Corollary 2.13).

This new concept of w-sharp domain depends upon the notion of star operation, w-operation. A reader in need of a quick review on this topic may consult Section 32 and 34 of [15]. For the reader's convenience we give a working introduction here for the notions involved. Let D be a domain with quotient field K and let $F(D)$ denote the set of nonzero fractional ideals of D. A function $A \mapsto A^* : F(D) \to F(D)$ is called a *star operation* on D if $*$ satisfies the following three conditions for all $0 \neq a \in K$ and for all $I, J \in F(D)$: (1) $D^* = D$ and $(aI)^* = aI^*$, (2) $I \subseteq I^*$ and if $I \subseteq J$, then $I^* \subseteq J^*$, (3) $(I^*)^* = I^*$. An ideal $I \in F(D)$ is called a $*$-ideal if $I^* = I$. For all $I, J \in F(D)$, we have $(IJ)^* = (I^*J)^* = (I^*J^*)^*$. These equations define the so-called $*$-*multiplication.* If $\{I_\alpha\}$ is a subset of $F(D)$ such that $\cap I_\alpha \neq 0$, then $\cap I_\alpha^*$ is a $*$-ideal. Also, if $\{I_\alpha\}$ is a subset of $F(D)$ such that $\sum I_\alpha$ is a fractional ideal, then $(\sum I_\alpha)^* = (\sum I_\alpha^*)^*$. A star operation $*$ is said to be a *stable* star operation if $(I \cap J)^* = I^* \cap J^*$ for all $I, J \in F(D)$. The function $*_f : F(D) \to F(D)$ given by $I^{*_f} = \cup J^*$, where J ranges over all nonzero finitely generated sub-ideals of I, is also a star operation; $*$ is said to be a star operation of *finite character* if $* = *_f$. Clearly $(*_f)_f = *_f$. Let $Max_*(D)$ denote the set of maximal $*$-ideals, that is, ideals maximal among proper integral $*$-ideals of D. Every maximal $*$-ideal is a prime ideal. If $*$ is of finite character, then every proper $*$-ideal is contained in some maximal $*$-ideal, and $*$ is stable if and only if $I^* = \cap_{P \in Max_*(D)} ID_P$ for all $I \in F(D)$, cf. [3, Corollary 4.2]. A $*$-ideal I is of *finite type* if $I = (a_1, ..., a_n)^*$ for some $a_1, ..., a_n \in I$. An ideal $I \in F(D)$ is said to be $*$-*invertible* if $(II^{-1})^* = D$, where $I^{-1} = (D : I) = \{x \in K \mid xI \subseteq D\}$. If $*$ is of finite character, then I is $*$-invertible if and only if II^{-1} is not contained in any maximal $*$-ideal of D; in this case $I^* = (a_1, ..., a_n)^*$ for some $a_1, ..., a_n \in I$. Let $*_1, *_2$ be star operations on D. We write $*_1 \leq *_2$, if $I^{*_1} \subseteq I^{*_2}$ for all $I \in F(D)$. In this case we get $(I^{*_1})^{*_2} = I^{*_2} = (I^{*_2})^{*_1}$ and every $*_1$-invertible ideal is $*_2$-invertible. Some well-known star operations are: the *d-operation* (given by $I \mapsto I$), the *v-operation* (given by $I \mapsto I_v = (I^{-1})^{-1}$) and the *t-operation* (defined by $t = v_f$). The *w-operation* is the star operation given by $I \mapsto I_w = \{x \in K \mid Jx \subseteq I$ for some finitely generated ideal J of D with $J^{-1} = D\}$. The w-operation is a stable star operation of finite character. For every $I \in F(D)$, we have $I \subseteq I_w \subseteq I_t \subseteq I_v$; so a v-ideal is a t-ideal and a t-ideal is a w-ideal. It is known that $Max_w(D) = Max_t(D)$, cf. [4, Corollary 2.17] and $I_w = \cap_{P \in Max_t(D)} ID_P$ by [4, Corollary 2.13]. So, a nonzero fractional ideal is w-invertible if and only if it is t-invertible. Recall from [7] that D is called a $*$-*Dedekind domain* if every nonzero ideal of D is $*$-invertible.

Throughout this paper, all rings are commutative and unitary. Our standard references for any undefined notation or terminology are [15] and [18].

2. Main Results

Definition 2.1. We call a domain D a w-sharp domain if whenever X, Y_1, Y_2 are nonzero ideals of D with $X \supseteq Y_1Y_2$, there exist nonzero ideals Z_1 and Z_2 such that $X_w = (Z_1Z_2)_w$, $(Z_1)_w \supseteq Y_1$ and $(Z_2)_w \supseteq Y_2$.

Proposition 2.2. *Let D be a domain and $T \subseteq D$ a multiplicative set such that $X_w \subseteq (XD_T)_w$ for each non zero ideal X of D. If D is w-sharp, then the fraction ring D_T is w-sharp.*

Proof. Given that $X_w \subseteq (XD_T)_w$ which is equivalent to $(X_wD_T)_w = (XD_T)_w$. If X, Y_1, Y_2 are nonzero ideals of D with $(XD_T) \supseteq (Y_1Y_2D_T)$, then $Z = XD_T \cap D \supseteq Y_1Y_2$. Since D is w-sharp, there exist nonzero ideals Z_1 and Z_2 of D such that $Z_w = (Z_1Z_2)_w$, $(Z_1)_w \supseteq Y_1$ and $(Z_2)_w \supseteq Y_2$. As $(YD_T)_w = (Y_wD_T)_w$ for each nonzero ideal Y, we have $(XD_T)_w = (Z_wD_T)_w = ((Z_1Z_2)_wD_T)_w = (Z_1Z_2D_T)_w$, $(Z_1D_T)_w = ((Z_1)_wD_T)_w \supseteq Y_1D_T$ and $(Z_2D_T)_w \supseteq Y_2D_T$. □

Proposition 2.3. *Every fraction ring of a w-sharp domain is w-sharp.*

Proof. By Proposition 2.2, it is enough to show that $X_w \subseteq (XD_T)_w$ for each nonzero ideal X of D. If $r \in X_w$, then $rY \subseteq X$ for some nonzero finitely generated ideal Y of D such that $Y_v = D$. Hence $(YD_T)_v = D_T$ by [17, Lemma 3.4] and $rYD_T \subseteq XD_T$ implies $r \in (XD_T)_w$. □

Proposition 2.4. *If D is a w-sharp domain, then D_Q is a valuation domain with value group a complete subgroup of the reals for each $Q \in Max_w(D)$. In particular, D is a $PwMD$ of w-dimension ≤ 1.*

Proof. Let Q be a maximal w-ideal. If X is a nonzero ideal of D, then $X_wD_Q = XD_Q$ by [3, Corollary 4.2]. By Proposition 2.2, applied for $T = D - Q$, we get that D_Q is a sharp domain. Now apply [2, Theorem 11]. The "in particular" assertion is clear. □

Proposition 2.5. *If D is a w-Dedekind domain, then D is w-sharp.*

Proof. Let X, Y_1, Y_2 be nonzero ideals of D such that $X \supseteq Y_1Y_2$. Take $Z_1 = X + Y_1$ and $Z_2 = X(Z_1)^{-1}$. Note that $Z_1 \subseteq D$ and $Y_1 \subseteq Z_1$. Since $(Z_1(Z_1)^{-1})_w = D$, $X_w = (Z_1Z_2)_w$. From $Y_2Z_1 = Y_2(Y_1 + X) \subseteq X$, we get $Y_2 \subseteq (Y_2Z_1(Z_1)^{-1})_w \subseteq (X(Z_1)^{-1})_w = (Z_2)_w$. □

Proposition 2.6. *If D is a w-sharp domain, then X_v is w-invertible for each nonzero ideal X of D.*

Proof. If X is a nonzero ideal of D and $r \in X - \{0\}$, then $X(rX^{-1}) \subseteq rD$. Since D is w-sharp, there exist nonzero ideals Z_1 and Z_2 of D such that $(Z_1)_w \supseteq X$, $(Z_2)_w \supseteq rX^{-1}$ and $rD = (Z_1Z_2)_w$. Hence Z_1 is w-invertible and we get $Z_1^{-1} = (r^{-1}Z_2)_w \supseteq (rr^{-1}X^{-1})_w = X^{-1}$, so $(Z_1)_v \subseteq X_v$. The reverse inclusion follows from $(Z_1)_w \supseteq X$. Thus $X_v = (Z_1)_v$ is w-invertible, because $(Z_1)_w = (Z_1)_v$ since Z_1 is w-invertible. □

Corollary 2.7. *If D is a w-sharp and a Mori domain, then D is w-Dedekind.*

Proof. By Proposition 2.4, D is a $PwMD$, so $w = t$ by [14, Proposition 3.15]. As D is a Mori domain, $w = t = v$. By Proposition 2.6, D is w-Dedekind. □

Lemma 2.8. *Let D be a domain and $X, Z_1, Z_2 \in F(D)$.*
(1) *If $(X + Z_1)_w = D$, then $(X \cap Z_1)_w = (XZ_1)_w$.*
(2) *If X is w-invertible, then $(X(Z_1 \cap Z_2))_w = (XZ_1 \cap XZ_2)_w$.*

Proof. (1) Obviously, $(XZ_1)_w \subseteq (X \cap Z_1)_w$. Conversly, since $(X + Z_1)_w = D$, we have $(X \cap Z_1)_w = ((X \cap Z_1)(X + Z_1))_w \subseteq (XZ_1)_w$, thus $(X \cap Z_1)_w = (XZ_1)_w$.
(2) Obviously, $(X(Z_1 \cap Z_2))_w \subseteq (XZ_1 \cap XZ_2)_w$. Conversly, because X is w-invertible, we have $(XZ_1 \cap XZ_2)_w = (XX^{-1}(XZ_1 \cap XZ_2))_w \subseteq (X(X^{-1}XZ_1 \cap X^{-1}XZ_2))_w \subseteq (X(Z_1 \cap Z_2))_w$. □

Proposition 2.9. *Let D be a w-sharp domain. If X, Y are nonzero ideals of D such that $(X + Y)_v = D$, then $(X_v + Y_v)_w = D$.*

Proof. Let K be the quotient field of D. Changing X by X_v and Y by Y_v, we may assume that X, Y are w-invertible v-ideals, cf. Proposition 2.6. Since $(X + Y)^2 \subseteq X^2 + Y$ and D is w-sharp, there exist two nonzero ideals Y_1, Y_2 such that $(X^2 + Y)_w = (Y_1Y_2)_w$ and $X + Y \subseteq (Y_1)_w \cap (Y_2)_w$. We claim that $(X^2 + Y)_w : X = (X + Y)_w$. To prove our claim, we perform the following step-by-step computation. First, $(X^2 + Y)_w : X = ((X^2 + Y)_w : kX) \cap D = ((X^2 + Y)X^{-1})_w \cap D = (X + YX^{-1})_w \cap D$ because X is w-invertible. As w is stable star operation, we get $(X + YX^{-1} \cap D) = ((X + YX^{-1}) \cap D)_w = (X + (YX^{-1} \cap D))_w$ by modular distributivity. Since X is w-invertible, we get $(X+(YX^{-1}\cap D))_w = (X+X^{-1}(Y\cap X))_w$, cf. Lemma 2.8. Using the fact that X is w-invertible (hence v-invertible) and Lemma 2.8, we derive that $(X+X^{-1}(Y\cap X))_w \subseteq (X + X^{-1}(XY)_v)_w \subseteq (X + (XX^{-1}Y)_v)_w = (X + (Y)_v)_w = (X + Y)_w$. Putting all these facts together, we get $(X^2+Y)_w : X \subseteq (X+Y)_w$ and the other inclusion is clear. So the claim is proved. From $(X^2 + Y)_w = (Y_1Y_2)_w$, we get $(Y_1)_w \subseteq (X^2+Y)_w : (Y_2)_w \subseteq (X^2+Y)_w : X = (X+Y)_w$, so $(Y_1)_w = (X+Y)_w$. Similarly we get $(Y_2)_w = (X+Y)_w$, hence $(X^2+Y)_w = ((X+Y)^2)_w$. It follows that $(Y)_w \subseteq (X^2 + Y)_w = ((X + Y)^2)_w \subseteq ((Y)^2 + X)_w$. So $Y_w = Y_w \cap ((Y)^2 + X)_w = (Y \cap ((Y)^2 + X))_w = ((Y)^2 + (Y \cap X))_w$ where we used the fact that w is stable star operation and the modular distributivity. By the Lemma 2.8, we have $(X\cap Y) \subseteq (XY)_v$, so we get $(Y)_w = ((Y)^2+(Y\cap X))_w \subseteq ((Y)^2+(XY)_v)_w$. Since Y is w-invertible, we have $D = (Y(Y)^{-1})_w \subseteq (((Y)^2 + (XY)_v)(Y^{-1}))_w \subseteq (Y + X_v)_w = (Y + X)_w$. Thus $(X + Y)_w = D$. □

Proposition 2.10. *Let D be a w-sharp domain. Then every finitely generated nonzero ideal of D is w-invertible.*

Proof. Let $r, s \in D - \{0\}$. By Proposition 2.6, the ideal $X = (rD + sD)_v$ is w-invertible. (hence v-invertible), So $(rX^{-1}+sX^{-1})_v = D$. By Proposition 2.9, we get $(rX^{-1} + sX^{-1})_w = D$. Hence $X = ((rX^{-1} + sX^{-1})X)_w = (rD + sD)_w$

because X is w-invertible. Thus every two-generated nonzero ideal of D is w-invertible. Now the proof of [15, Proposition 22.2] can be easily adapted to show that every finitely generated nonzero ideal of D is w-invertible. □

Let D be a domain with quotient field K. According to [6], a family $\mathcal{F}$ of nonzero prime ideals of D is called independent of finite character family (IFC family), If (a) $D = \cap_{P\in\mathcal{F}} D_P$ (b) every nonzero $d \in D$ belongs to only finitely many members of $\mathcal{F}$ and (c) every nonzero prime ideal of D is contained in at most one member of $\mathcal{F}$.

Proposition 2.11. *Let D be a domain. Assume that*
(1) *every $r \in D - \{0\}$ is contained in only finitely many maximal w-ideals,* and (2) *for every $Q \in Max_w D$, D_Q is a valuation domain with value group a complete subgroup of the reals.*
Then D is a w-sharp domain.

Proof. By [16], $D = \cap_Q D_Q$ where Q runs in the set of maximal w-ideals. As each D_Q is a valuation domain with value group a complete subgroup of the reals, every Q has height one. It follows that $Max_w(D)$ is an IFC family. We show that D is w-sharp. Let X, Y_1, Y_2 be nonzero ideals of D such that $X \supseteq Y_1Y_2$. Let $L_1, ...L_n$ be the maximal w-ideals of D containing Y_1Y_2. Since D_{L_i} is sharp, there exist A_i, B_i ideals of D_{L_i} such that $XD_{L_i} = A_iB_i, A_i \supseteq Y_1D_{L_i}$ and $B_i \supseteq Y_2D_{L_i}$ for all i between 1 and n. Set $A'_i = A_i \cap D$, $B'_i = B_i \cap D$, $i = 1, 2.....n$, $A = A'_1...A'_n$ and $B = B'_1....B'_n$. By [6, Lemma 2.3], L_i is the only element of $Max_w(D)$ containing A'_i (resp. B'_i), thus it can be checked that $XD_L = (AB)D_L$, $AD_L \supseteq Y_1D_L$ and $BD_L \supseteq Y_2D_L$ for each $L \in Max_w(D)$. So, we have $X_w = (AB)_w$, $A_w \supseteq Y_1$ and $B_w \supseteq Y_2$. Consequently, D is w-sharp. □

Proposition 2.12. *Let D be a countable domain such that D is a PwMD and X_v is w-invertible for each nonzero ideal X of D. Then every nonzero element of D is contained in only finitely many maximal w-ideals.*

Proof. Let us deny. By [13, Corollary 5], there exists a nonzero element r and an infinite family $(X_k)_{k\geq 1}$ of w-invertible proper ideals containing r which are mutually w-comaximal (i.e., $(X_p + X_q)_w = D$ for every $p \neq q$). For each nonempty set Ω of natural numbers, consider the v-ideal $X_\Omega = \cap_{k\in\Omega} X_k$ (note that $r \in X_\Omega$). By hypothesis, X_Ω is w-invertible. We claim that $X_\Omega \neq X_{\Omega'}$ whenever Ω and Ω' are distinct nonempty sets of natural numbers. Deny. Then there exists a nonempty set of natural numbers Δ and some $l \notin \Delta$ such that $X_l \supseteq X_\Delta$. Consider the ideal $A = (X_l^{-1} X_\Delta)_w \supseteq X_\Delta$. If $q \in \Delta$, then $X_q \supseteq X_\Delta = (X_lA)_w$, so $X_q \supseteq A$, because $(X_q + X_l)_w = D$. It follows that

$X_\Delta \supseteq A$, so $X_\Delta = A = (X_l^{-1}X_\Delta)_w$. Since X_Δ is w-invertible, we get $X_l = D$, a contradiction. Thus the claim is proved. But then it follows that $\{X_\Omega \mid \emptyset \neq \Omega \subseteq \mathbb{N}\}$ is an uncountable set of w-invertible ideals. This leads to a contradiction, because D being countable, it has countably many w-ideals of finite type. □

Corollary 2.13. *Let D be a countable domain such that D is w-sharp. Then D is a w-Dedekind domain.*

Proof. We may assume that D is not a field. By Proposition 2.4, D is a PwMD. Now Propositions 2.6 and 2.12 show that every nonzero element of D is contained in only finitely many maximal w-ideals. Let Q be a maximal w-ideal of D. By Proposition 2.4, D_Q is a countable valuation domain with value group $\mathbb{Z}$ or $\mathbb{R}$, so D_Q is a DVR. Thus D is a w-Dedekind domain, cf. [7, Theorem 4.11]. □

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