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THE EDGE VERSION OF RANDIĆ, ZAGREB, ATOM BOND CONNECTIVITY AND GEOMETRIC-ARITHMETIC INDICES OF $HAC_5C_6C_7[P,Q]$ NANOTUBE

Yingying Gao[1], Wasim Sajjad[2],
Abdul Qudair Baig[3], Mohammad Reza Farahani[4 §]

[1]College of Pharmacy and Biological Engineering
Chengdu University
Chengdu, 610106, P.R. CHINA
[2]Department of Mathematics
University of Sargodha
Mandi Bahauddin Campus
Mandi Bahauddin, PAKISTAN
[3]Department of Mathematics
COMSATS Institute of Information Technology
Attock Campus, PAKISTAN
[4]Department of Applied Mathematics
Iran University of Science and Technology (IUST)
Narmak, Tehran, 16844, IRAN

Abstract: Let G be a simple molecular graph with vertex set $V(G)$ and edge set $E(G)$ respectively. The degree $deg(v)$ of the vertex $v \in V(G)$ is the number of vertices adjacent with vertex v. A graph can be recognized by a numeric number, a polynomial, a sequence of numbers or a matrix. A topological index is a numeric quantity associated with a graph which characterize the topology of graph and is invariant under graph automorphism. Topological indices play important role in mathematical chemistry especially in the quantitative structure-property relationship (QSPR) and quantitative structure-activity relationship (QSAR) studies.

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§Correspondence author

In this paper we consider the line graph of $HAC_5C_6C_7[p, q]$ nanotube for $p \geq 1$ and $q = 2$ and we compute the edge version of Randić, Zagreb, Atom Bond Connectivity its fourth version and Geometric-arithmetic index with its fifth version.

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1. Introduction and Preliminary Results

Graph theory has provided chemist with a variety of useful tools, such as topological indices. Molecules and molecular compounds are often modeled by molecular graph. A molecular graph is a representation of the structural formula of a chemical compound in terms of graph theory, whose vertices correspond to the atoms of the compound and edges correspond to chemical bonds. *Cheminformatics* is new subject which is a combination of chemistry, mathematics and information science. It studies Quantitative structure-activity (QSAR) and structure-property (QSPR) relationships that are used to predict the biological activities and properties of chemical compounds. In the QSAR /QSPR study, physico-chemical properties and topological indices such as Wiener index, Szeged index, Randić index, Zagreb indices and ABC index are used to predict bioactivity of the chemical compounds. A graph can be recognized by a numeric number, a polynomial, a sequence of numbers or a matrix. A topological index is a numeric quantity associated with a graph which characterize the topology of graph and is invariant under graph automorphism. There are some major classes of topological indices such as distance based topological indices, degree based topological indices and counting related polynomials and indices of graphs. Among these classes degree based topological indices are of great importance and play a vital role in chemical graph theory and particularly in chemistry. In more precise way, a topological index $Top(G)$ of a graph, is a number with the property that for every graph H isomorphic to G, $Top(H) = Top(G)$. The concept of topological indices came from Wiener [39] while he was working on boiling point of paraffin, named this index as *path number*. Later on, the path number was renamed as *Wiener index*.

Let G be a graph. Then the Wiener index of G is defined as

$$W(G) = \frac{1}{2} \sum_{(u,v)} d(u, v) \tag{1}$$

where (u, v) is any ordered pair of vertices in G and $d(u, v)$ is the distance between vertex u and vertex v in a graph G.

Now we discuss the edge version of some important degree based topological indices of a graph G.

1.1. Edge Version of Randić Index

The very first and oldest degree based topological index is *Randić index* [36] denoted by $R_{-\frac{1}{2}}(G)$ and introduced by Milan Randić in 1975 and defined as

$$R_{-\frac{1}{2}}(G) = \sum_{ef \in E(L(G))} \frac{1}{\sqrt{deg(e)deg(f)}} \tag{2}$$

Sometimes it is reffered as Connectivity index which is used to relate variety of Physical, Chemical and Pharmacological properties of organic molecules and it became one of the most popular molecular structure descriptor. After the first paper of Milan Randić the mathematical properties and generalization of Randić index were extensively studied. In 1998 *Bollobas* and *Erdos* [3] generalized this index by replacing square root by power of any real number which is called *General Randić index*. The edge version of General Randić index is defined as

$$_{e}R_{\alpha}(G) = \sum_{ef \in E(L(G))} (deg_{L(G)}(e).deg_{L(G)}(f))^{\alpha} \quad \text{for} \quad \alpha = 1, \frac{1}{2}, -1, -\frac{1}{2} \tag{3}$$

Where $deg_{L(G)}(e)$ is the degree of the vertex e and $deg_{L(G)}(f)$ is the degree of the vertex f of line graph of G.
The general Randić index for $\alpha = 1$ is the second Zagreb index for any graph G.

1.2. Edge Version of Zagreb Index

The degree based molecular descriptor called Zagreb index was introduced more than thirty years ago by *Gutman* and *Trinajstic* [22]. The edge version of zagreb index is defined as

$$_{e}M_1(G) = \sum_{ef \in E(L(G))} (deg_{L(G)}(e) + deg_{L(G)}(f)) \tag{4}$$

Where $deg_{L(G)}(e)$ is the degree of the vertex e and $deg_{L(G)}(f)$ is the degree of the vertex f of line graph of G.

1.3. Edge Version of Atom Bond Connectivity Index

The Atom Bond Connectivity index was introduced by Estrada et al [6]. The edge version of atom bond connectivity index is defined as

$$ {}_{e}ABC(G) = \sum_{ef \in E(L(G))} \sqrt{\frac{deg_{L(G)}(e) + deg_{L(G)}(f) - 2}{deg_{L(G)}(e).deg_{L(G)}(f)}} \tag{5} $$

Where $deg_{L(G)}(e)$ is the degree of the vertex e and $deg_{L(G)}(f)$ is the degree of the vertex f of line graph of G. The edge version of fourth version of atom bond connectivity index is defined as

$$ {}_{e}ABC_4(G) = \sum_{ef \in E(L(G))} \sqrt{\frac{s_{L(G)}(e) + s_{L(G)}(f) - 2}{s_{L(G)}(e)s_{L(G)}(f)}} \tag{6} $$

where $s_{L(G)}(e)$ is the sum of the degree of the vertices adjacent with e and $s_{L(G)}(f)$ is the sum of the degree of the vertices adjacent with f in the line graph of G.

1.4. Edge Version of Geometric Arithmetic Index

The Geometric Arithmetic index was invented by Vukiĉeviĉ and Furtula [38] which gives the difference between geometric and arithmetic means.The edge version of Geometric Arithmetic index is defined as

$$ {}_{e}GA(G) = \sum_{ef \in E(L(G))} \frac{2\sqrt{deg_{L(G)}(e)deg_{L(G)}(f)}}{deg_{L(G)}(e) + deg_{L(G)}(f)} \tag{7} $$

Where $deg_{L(G)}(e)$ is the degree of the vertex e and $deg_{L(G)}(f)$ is the degree of the vertex f of line graph of G.
Also the edge version of fifth geometric arithmetic index is defined as

$$ {}_{e}GA_5(G) = \sum_{ef \in E(L(G))} \frac{2\sqrt{s_{L(G)}(e)s_{L(G)}(f)}}{s_{L(G)}(e) + s_{L(G)}(f)} \tag{8} $$

where $s_{L(G)}(e)$ is the sum of the degree of the vertices adjacent with e and $s_{L(G)}(f)$ is the sum of the degree of the vertices adjacent with f in the line graph of G.

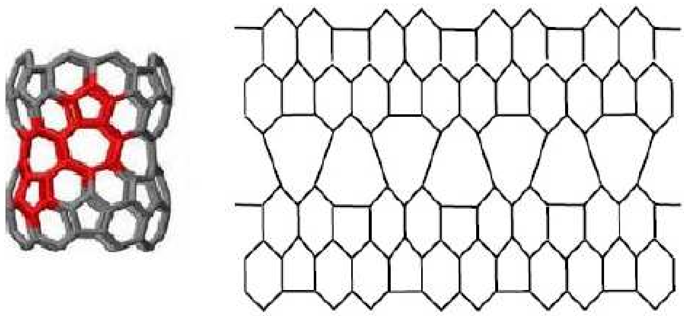

Figure 1: $HAC_5C_6C_7[p,q]$ Nanotube

2. Carbon Nanotubes

Carbon nanotubes form an interesting class of carbon nonmaterial. There are three types of nanotubes namely, armchair, chiral and zigzag structures nanotubes. These carbon nanotubes shows remarkable mechanical properties. Experimental studies have shown that they belong to the stiffest and elastic known materials. Diudea wa s the first chemist who consider the problem of topological indices of nano-structures.

In this paper we continue this program and compute the edge version of some degree based topological indices of $HAC_5C_6C_7[p,q]$ nanotube.

3. Main Results

$HAC_5C_6C_7[p,q]$ shown in Fig.1 is constructed by alternating C_5, C_6 and C_7 carbon cycles. It is tube shaped material but we consider it in the form of sheet shown in Fig.1. The two dimensional lattice of $HAC_5C_6C_7[p,q]$ consists of p rows and q periods. Here p denotes the number of pentagons in one row and q is the number of periods in whole lattice. A period consist of three rows (See references [11]-[19],[31],[32]).

We now compute the edge version of Randić, Zagreb, Atom bond connectivity with its fourth version and Geometric Arithmetic index with its fifth version of $HAC_5C_6C_7[p,q]$ nanotubes with p columns and q rows. The line graph of $HAC_5C_6C_7[p,q]$ nanotube has $88p-12$ edges with degree vertices 2, 3 and 4. The first edge partition has 2 edges with $d_{L(G)}(e) = d_{L(G)}(f) = 2$, the second edge partition has 12 edges with $d_{L(G)}(e) = 2$ and $d_{L(G)}(f) = 3$, the third edge partition has $6p+1$ edges with $d_{L(G)}(e) = d_{L(G)}(f) = 3$, the fourth edge parti-

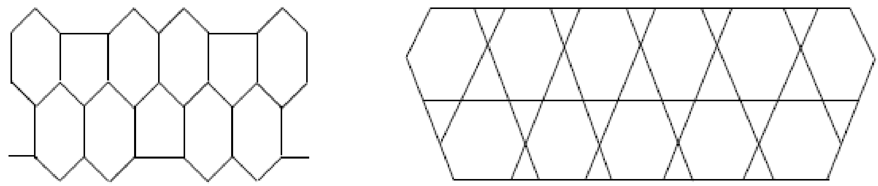

Figure 2: $HAC_5C_6C_7$ and $L(HAC_5C_6C_7)$

tion has $12p+10$ edges with $d_{L(G)}(e)=3$ and $d_{L(G)}(f)=4$ and the fifth edge partition has $70p-37$ edges with $d_{L(G)}(e)=d_{L(G)}(f)=4$. In this paper for $HAC_5C_6C_7[p,q]$ we consider $p\geq 1$ and $q=2$.

Theorem 3.1. *For every $p\geq 1$ and $q=2$, consider the graph of $G\cong HAC_5C_6C_7[p,q]$ nanotube. Then the ${}_eR_\alpha(G)$ is equal to*

$$ {}_eR_\alpha(HAC_5C_6C_7[p,q]) = \begin{cases} 1318p-383, & \alpha=1; \\ (298+24\sqrt{3})p+12\sqrt{6}+20\sqrt{3}-141, & \alpha=\frac{1}{2}; \\ \frac{145}{24}p+\frac{163}{144}, & \alpha=-1; \\ (\frac{39}{2}+2\sqrt{3})p+2\sqrt{6}+\frac{5}{\sqrt{3}}-\frac{95}{12}), & \alpha=-\frac{1}{2}. \end{cases} $$

Proof. Let G be the graph of $HAC_5C_6C_7[p,q]$ nanotube. Since from (3) we have

$$ {}_eR_\alpha(G) = \sum_{ef\in E(L(G))} (deg_{L(G)}(e).deg_{L(G)}(f))^\alpha \quad \text{for} \quad \alpha = 1, \frac{1}{2}, -1, -\frac{1}{2} $$

For $\alpha=1$
Now we apply the formula of ${}_eR_\alpha(G)$ for $\alpha=1$.

$$ {}_eR_1(G) = \sum_{ef\in E(L(G))} (deg_{L(G)}(e).deg_{L(G)}(f)) $$

By using edge partition from Table.1, we get

${}_eR_1(G) = 2(2\times2)+12(2\times3)+(6p+1)(3\times3)+(12p+10)(3\times4)+(70p-37)(4\times4)$
After an easy simplification, we obtain

$$ \Longrightarrow_e R_1(HAC_5C_6C_7[p,q]) = 1318p-383 $$

For $\alpha = \frac{1}{2}$
We apply the formula of ${}_eR_\alpha(G)$ for $\alpha = \frac{1}{2}$.

$$ {}_eR_{\frac{1}{2}}(G) = \sum_{ef \in E(L(G))} \sqrt{(deg_{L(G)}(e).deg_{L(G)}(f))} $$

By using edge partition from Table.1, we get

${}_eR_{\frac{1}{2}}(G) = 2\sqrt{2 \times 2} + 12\sqrt{2 \times 3} + (6p+1)\sqrt{3 \times 3} + (12p+10)\sqrt{3 \times 4} + (70p - 37)\sqrt{4 \times 4}$
After an easy simplification, we obtain

$$ \Longrightarrow_e R_{\frac{1}{2}}(HAC_5C_6C_7[p,q]) = (298 + 24\sqrt{3})p + 12\sqrt{6} + 20\sqrt{3} - 141 $$

For $\alpha = -1$
We apply the formula of ${}_eR_\alpha(G)$ for $\alpha = -1$.

$$ {}_eR_{-1}(G) = \sum_{ef \in E(L(G))} (deg_{L(G)}(e).deg_{L(G)}(f))^{-1} $$

By using edge partition from Table.1, we get

${}_eR_{-1}(G) = \frac{2}{2\times2} + \frac{12}{2\times3} + \frac{6p+1}{3\times3} + \frac{12p+10}{3\times4} + \frac{70p-37}{4\times4}$
After an easy simplification, we obtain

$$ \Longrightarrow_e R_{-1}(HAC_5C_6C_7[p,q]) = \frac{145}{24}p + \frac{163}{144} $$

For $\alpha = -\frac{1}{2}$
We apply the formula of ${}_eR_\alpha(G)$ for $\alpha = -\frac{1}{2}$.

$$ {}_eR_{-\frac{1}{2}}(G) = \sum_{ef \in E(L(G))} (deg_{L(G)}(e).deg_{L(G)}(f))^{-\frac{1}{2}} $$

By using edge partition from Table.1, we get

${}_eR_{-\frac{1}{2}}(G) = \frac{2}{\sqrt{2\times2}} + \frac{12}{\sqrt{2\times3}} + \frac{6p+1}{\sqrt{3\times3}} + \frac{12p+10}{\sqrt{3\times4}} + \frac{70p-37}{\sqrt{4\times4}}$
After an easy simplification, we obtain

$$ \Longrightarrow_e R_{-1}(HAC_5C_6C_7[p,q]) = (\frac{39}{2} + 2\sqrt{3})p + 2\sqrt{6} + \frac{5}{\sqrt{3}} - \frac{95}{12} $$

□

(d_u, d_v) where $uv \in E(G)$	Number of edges
$(2, 2)$	2
$(2, 3)$	12
$(3, 3)$	$(6p + 1)$
$(3, 4)$	$(12p + 10)$
$(4, 4)$	$(70p - 37)$

Table 1: Edge partition of $L(HAC_5C_6C_7[p, q])$ based on degrees of end vertices of each edge.

□

Theorem 3.2. *For every $p \geq 1$ and $q = 2$, consider the graph of $G \cong HAC_5C_6C_7[p, q]$ nanotube. Then its edge version of first Zagreb index is*

$$_eM_1(HAC_5C_6C_7[p, q]) = 680p - 152$$

Proof. Let G be the graph of $HAC_5C_6C_7[p, q]$ nanotube. Since from (4) we have

$$_eM_1(G) = \sum_{ef \in E(L(G))} (deg_{L(G)}(e) + deg_{L(G)}(f))$$

By using edge partition from Table.1, we get

$_eM_1(L(G)) = 2(2 + 2) + 12(2 + 3) + (6p + 1)(3 + 3) + (12p + 10)(3 + 4) + (70p - 37)(4 + 4)$
After doing some calculation, we get

$$\Longrightarrow_e M_1(HAC_5C_6C_7[p, q]) = 680p - 152$$

□

□

Theorem 3.3. *For every $p \geq 1$ and $q = 2$, consider the graph of $G \cong HAC_5C_6C_7[p, q]$ nanotube. Then its edge version of Atom bond connectivity index is*

$$_eABC(HAC_5C_6C_7[p, q]) = (4 + 2\sqrt{15} + \frac{35\sqrt{6}}{2})p + \sqrt{2} + 6\sqrt{2} + \frac{2}{3} + \frac{5\sqrt{5}}{\sqrt{3}} - \frac{37\sqrt{6}}{4}$$

Proof. Let G be the graph of $HAC_5C_6C_7[p,q]$ nanotube. Since from (5) we have

$$_eABC(G) = \sum_{ef\in E(L(G))} \sqrt{\frac{deg_{L(G)}(e) + deg_{L(G)}(f) - 2}{deg_{L(G)}(e).deg_{L(G)}(f)}}$$

By using edge partition from Table.1, we get

$$_eABC(G) = 2\sqrt{\tfrac{2+2-2}{2\times 2}} + 12\sqrt{\tfrac{2+3-2}{2\times 3}} + (6p+1)\sqrt{\tfrac{3+3-2}{3\times 3}} + (12p+10)\sqrt{\tfrac{3+4-2}{3\times 4}} + (70p-37)\sqrt{\tfrac{4+4-2}{4\times 4}}$$

After an easy simplification, we obtain

$$\Longrightarrow_e ABC(HAC_5C_6C_7[p,q]) = (4+2\sqrt{15}+\frac{35\sqrt{6}}{2})p+\sqrt{2}+6\sqrt{2}+\frac{2}{3}+\frac{5\sqrt{5}}{\sqrt{3}}-\frac{37\sqrt{6}}{4}$$

$\square$

Theorem 3.4. *For every $p \geq 1$ and $q = 2$, consider the graph of $G \cong HAC_5C_6C_7[p,q]$ nanotube. Then its fourth version of the atom-bond connectivity index is*

$$_eABC_4(HAC_5C_6C_7[p,q]) = (4\sqrt{\tfrac{19}{110}} + 4\sqrt{\tfrac{11}{70}} + 2\tfrac{\sqrt{18}}{10} + 4\sqrt{\tfrac{23}{154}} + 4\sqrt{\tfrac{8}{55}} + \tfrac{12}{\sqrt{70}} + \sqrt{2} + 8\sqrt{\tfrac{29}{240}} + 54\sqrt{\tfrac{15}{128}})p + \eta$$

where

$$\eta = \tfrac{4\sqrt{2}}{5} + \tfrac{4}{\sqrt{15}} + \tfrac{\sqrt{26}}{5} + 2\sqrt{2} + \sqrt{\tfrac{14}{15}} + \tfrac{8}{9} + 4\sqrt{\tfrac{17}{90}} + \tfrac{8}{\sqrt{6}} + 2\sqrt{\tfrac{19}{110}} + 6\sqrt{\tfrac{11}{70}} + \tfrac{\sqrt{18}}{10} + 4\sqrt{\tfrac{23}{150}} + 2\sqrt{\tfrac{23}{154}} + 2\sqrt{\tfrac{8}{55}} + \tfrac{24}{\sqrt{70}} + 2\sqrt{2} + 10\sqrt{\tfrac{29}{240}} + 7\sqrt{\tfrac{15}{128}}$$

Proof. Let G be the graph of $HAC_5C_6C_7[p,q]$ nanotube. Since from (6) we have

$$_eABC_4(G) = \sum_{ef\in E(L(G))} \sqrt{\frac{s_{L(G)}(e) + s_{L(G)}(f) - 2}{s_{L(G)}(e)s_{L(G)}(f)}}$$

By using edge partition from Table.2, we get

$$_eABC_4(G) = 2\sqrt{\tfrac{5+5-2}{5\times 5}} + 2\sqrt{\tfrac{5+9-2}{5\times 9}} + 2\sqrt{\tfrac{5+10-2}{5\times 10}} + 6\sqrt{\tfrac{6+9-2}{6\times 9}} + 2\sqrt{\tfrac{6+10-2}{6\times 10}} + 2\sqrt{\tfrac{9+9-2}{9\times 9}} + 4\sqrt{\tfrac{9+10-2}{9\times 10}} + 8\sqrt{\tfrac{9+14-2}{9\times 14}} + (2p+1)\sqrt{\tfrac{10+10-2}{10\times 10}} + (4p+2)\sqrt{\tfrac{10+11-2}{10\times 11}} + (4p+6)\sqrt{\tfrac{10+14-2}{10\times 14}} + 4\sqrt{\tfrac{10+15-2}{10\times 15}} + (4p+2)\sqrt{\tfrac{11+14-2}{11\times 14}} + (4p+2)\sqrt{\tfrac{11+15-2}{11\times 15}} + (4p+8)\sqrt{\tfrac{14+15-2}{14\times 15}} + (4p+8)\sqrt{\tfrac{14+16-2}{14\times 16}} + (8p+10)\sqrt{\tfrac{15+16-2}{15\times 16}} + (54p+7)\sqrt{\tfrac{16+16-2}{16\times 16}}$$

After an easy simplification, we obtain

(S_u, S_v), where $uv \in E(G)$	Number of edges	(S_u, S_v), where $uv \in E(G)$	Number of edges
$(5,5)$	2	$(10,11)$	$4p+2$
$(5,9)$	2	$(10,14)$	$4p+6$
$(5,10)$	2	$(10,15)$	4
$(6,9)$	6	$(11,14)$	$4p+2$
$(6,10)$	2	$(11,15)$	$4p+2$
$(9,9)$	2	$(14,15)$	$4p+8$
$(9,10)$	4	$(14,16)$	$4p+8$
$(9,14)$	8	$(15,16)$	$8p+10$
$(10,10)$	$(2p+1)$	$(16,16)$	$54p+7$

Table 2: Edge partition of $L(HAC_5C_6C_7[p,q])$ based on degrees sum of end vertices of each edge.

$_eABC_4(HAC_5C_6C_7[p,q]) = (4\sqrt{\frac{19}{110}} + 4\sqrt{\frac{11}{70}} + 2\frac{\sqrt{18}}{10} + 4\sqrt{\frac{23}{154}} + 4\sqrt{\frac{8}{55}} + \frac{12}{\sqrt{70}} + \sqrt{2} + 8\sqrt{\frac{29}{240}} + 54\sqrt{\frac{15}{128}})p + \eta$ □

Theorem 3.5. *For every $p \geq 1$ and $q = 2$, consider the graph of $G \cong HAC_5C_6C_7[p,q]$ nanotube. Then its edge version of Geometric-Arithmetic index is*

$$_eGA(HAC_5C_6C_7[p,q]) = (6 + \frac{48\sqrt{3}}{7} + 70)p + \frac{24\sqrt{6}}{5} + \frac{40\sqrt{3}}{7} - 34$$

Proof. Let G be the graph of $HAC_5C_6C_7[p,q]$ nanotube. Since from (7) we have

$$_eGA(G) = \sum_{ef \in E(L(G))} \frac{2\sqrt{deg_{L(G)}(e)deg_{L(G)}(f)}}{deg_{L(G)}(e) + deg_{L(G)}(f)}$$

By using edge partition from Table.1, we get

$_eGA(G) = 4\frac{\sqrt{2\times2}}{2+2} + 24\frac{\sqrt{2\times3}}{2+3} + 2(6p+1)\frac{\sqrt{3\times3}}{3+3} + 2(12p+10)\frac{\sqrt{3\times4}}{3+4} + 2(70p-37)\frac{\sqrt{4\times4}}{4+4}$
After an easy simplification, we obtain

$$\Longrightarrow_e GA(HAC_5C_6C_7[p,q]) = (6 + \frac{48\sqrt{3}}{7} + 70)p + \frac{24\sqrt{6}}{5} + \frac{40\sqrt{3}}{7} - 34$$

□

Theorem 3.6. *For every $p \geq 1$ and $q = 2$, consider the graph of $G \cong HAC_5C_6C_7[p,q]$ nanotube. Then its fifth version of Geometric-Arithmetic index is*

$_eGA_5(HAC_5C_6C_7[p,q]) = (\frac{8\sqrt{110}}{21} + \frac{\sqrt{140}}{3} + \frac{8\sqrt{154}}{25} + \frac{4\sqrt{165}}{13} + \frac{8\sqrt{210}}{29} + \frac{4\sqrt{224}}{15} + \frac{16\sqrt{240}}{31} + 56)p + \eta$

where

$\eta = \frac{6\sqrt{5}}{7} + \frac{4\sqrt{2}}{3} + \frac{12\sqrt{6}}{5} + \frac{\sqrt{15}}{2} + \frac{24\sqrt{10}}{19} + \frac{48\sqrt{14}}{23} + \frac{4\sqrt{110}}{21} + \frac{\sqrt{140}}{2} + \frac{8\sqrt{150}}{25} + \frac{4\sqrt{154}}{25} + \frac{2\sqrt{165}}{13} + \frac{16\sqrt{210}}{29} + \frac{8\sqrt{224}}{15} + \frac{20\sqrt{240}}{31} + 12$

Proof. Let G be the graph of $HAC_5C_6C_7[p,q]$ nanotube. Since from (8) we have

$$_eGA_5(G) = \sum_{ef \in L(E(G))} \frac{2\sqrt{s_{L(G)}(e)s_{L(G)}(f)}}{s_{L(G)}(e) + s_{L(G)}(f)}$$

By using edge partition from Table.2, we get

$_eGA_5(G) = 4\frac{\sqrt{5\times5}}{5+5} + 4\frac{\sqrt{5\times9}}{5+9} + 4\frac{\sqrt{5\times10}}{5+10} + 12\frac{\sqrt{6\times9}}{6+9} + 4\frac{\sqrt{6\times10}}{6+10} + 4\frac{\sqrt{9\times9}}{9+9} + 8\frac{\sqrt{9\times10}}{9+10} + 16\frac{\sqrt{9\times14}}{9+14} + 2(2p+1)\frac{\sqrt{10\times10}}{10+10} + 2(4p+2)\frac{\sqrt{10\times11}}{10+11} + 2(4p+6)\frac{\sqrt{10\times14}}{10+14} + 8\frac{\sqrt{10\times15}}{10+15} + 2(4p+2)\frac{\sqrt{11\times14}}{11+14} + 2(4p+2)\frac{\sqrt{11\times15}}{11+15} + 2(4p+8)\frac{\sqrt{14\times15}}{14+15} + 2(4p+8)\frac{\sqrt{14\times16}}{14+16} + 2(8p+10)\frac{\sqrt{15\times16}}{15+16} + 2(54p+7)\frac{\sqrt{16\times16}}{16+16}$

After an easy simplification, we obtain

$\Longrightarrow_e GA_5(HAC_5C_6C_7[p,q]) = (\frac{8\sqrt{110}}{21} + \frac{\sqrt{140}}{3} + \frac{8\sqrt{154}}{25} + \frac{4\sqrt{165}}{13} + \frac{8\sqrt{210}}{29} + \frac{4\sqrt{224}}{15} + \frac{16\sqrt{240}}{31} + 56)p + \eta$ $\square$

4. Conclusion

In this paper, we have discussed edge version of Randić, Zagreb, Atom Bond Connectivity its fourth version and Geometric-arithmetic index with its fifth version. We have considered the line graph of $HAC_5C_6C_7[p,q]$ nanotube and we have computed the edge version of Randić, Zagreb, Atom Bond Connectivity its fourth version and Geometric-index with its fifth version.

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