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SECOND LAW ANALYSIS FOR A POROUS CHANNEL FLOW WITH ASYMMETRIC SLIP AND CONVECTIVE BOUNDARY CONDITIONS

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Abstract: The present work addresses the entropy generation in the flow of an incompressible viscous fluid through porous parallel walls subjected to asymmetric slip and convective heating conditions. The differential equations for momentum and energy are formulated, made dimensionless, solved and used to compute the entropy generation rate in the flow channel. The effect of various physical parameters on the velocity, temperature and entropy generation profiles are presented graphically and discussed.

AMS Subject Classification: 76D05, 35Q30

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1. Introduction

Irreversibility analysis in thermal set up is a fundamental approach that drives many flow and heat transfer processes. Using the second law of thermodynamics approach, studies conducted on the entropy minimization in the flow of hot moving fluid between parallel plates with heat transferred from one point to the other are much in recent times. These studies sterns on the earlier works Bejan [1, 2] developed the method for minimizing entropy generation rate in irreversible processes. Lucia [3, 4] presented irreversibility analysis for open systems. Moreover, Jha and Ajibade [5] presented a thermodynamics analysis that explains the role of suction and injection on entropy generation rate in a leaky wall. Tasnim et al. [6] examined the inherent irreversibility in a porous channel flow based on hydrodynamics principles. Hookman and Ejlali [7] implemented the second law analysis to explain the heat dynamics in a vertical tube.

On a micro-channel scale, entropy generation analysis have been performed by taking the velocity gradient and the slip length at porous walls into consideration. For instance, Eegunjobi and Makinde [8] discussed the influence of Navier slip condition on entropy generation rate in vertical channel. Yildrim and Sezer [9] analysed the partial slip effect on peristaltic channel flow. Das et al. [10] presented a novel analysis MHD non-Newtonian nanofluid with convective heat transfer using the second law procedure. Entropy analysis on unsteady porous channel flow was presented by Chinyoka and Makinde [11] with thermal boundary conditions. Das and Jana [12] gave a detailed and elaborate study on couple stress slip flow using the classical thermodynamics laws. Adesanya and Makinde [13, 14, 15] documented heat irreversibility analysis for slip and convective flows in parallel walls.

Motivated by several important results on slip flows and thermal boundary conditions [16, 17, 18, 19, 20, 21, 22], the specific objective of this article is to examine the entropy generation in a horizontal channel with asymmetrical slip and convective heating conditions at the porous walls. In the next section, the equations governing the fluid flow are formulated, made dimensionless, solved and used to compute expressions for the entropy generation rate and the Bejan heat ratio. Section four dealt with the discussion of results while the paper is concluded in the last section.

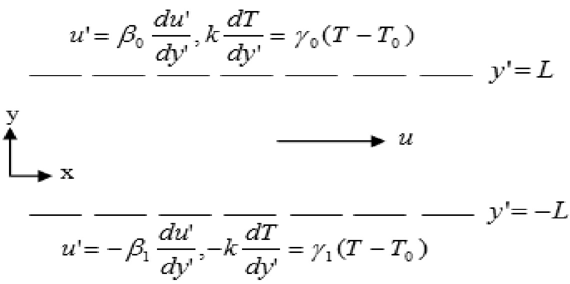

Figure 1: Geometry of the problem

2. Mathematical Analysis

Consider the steady flow of an incompressible, viscous fluid through parallel porous walls of distance $2L$ apart. The channel is taken to be infinitely long such that the flow is thermodynamically and hydrodynamically developed. Due to wall porosity, slip effects are considered at the walls, and the walls are assumed to exchange heat with the ambient as shown in Figure 1 below.

Noting the flow assumption, therefore, the temperature and velocity fields are functions of y alone. Based on these assumptions, the basic equations governing the slip fluid flow can be written as:

$$
\begin{aligned}
-\frac{dP}{dx} + \rho v_0 \frac{du'}{dy'} + \mu \frac{d^2u'}{dy'^2} &= 0, \\
k\frac{d^2T}{dy'2} + v_0\rho C_p \frac{dT}{dy'} + \mu\left(\frac{du'}{dy'}\right)^2 &= 0,
\end{aligned}
\tag{1}
$$

subject to the asymmetrical slip and convective boundary conditions

$$
\begin{aligned}
u'(-L) = \beta_0 \frac{du'(-L)}{dy'}, k\frac{dT(-L)}{dy'} &= \gamma_0\left(T(-L) - T_0\right) \\
u'(L) = -\beta_1 \frac{du'(L)}{dy'}, -k\frac{dT(L)}{dy'} &= \gamma_1\left(T(L) - T_0\right)
\end{aligned}
\tag{2}
$$

where $\beta_{0,1}$ are the Navier slip coefficients at the walls, μ is the fluid viscosity, u' is the axial velocity, P is the fluid pressure, ρ is the fluid density, v_0 measures the channel porosity due to suction and injection. C_p measures the specific heat capacity of the fluid, T and k are the dimensional fluid temperature and thermal conductivity of the material respectively, T_0 and T_1 are referenced fluid temperatures and $\gamma_{0,1}$ measures the Newtonian cooling rate at the walls. Introducing the following dimensionless variables

$$y = \frac{y'}{L}, u = \frac{u'}{U}, G = \frac{-L^2}{\mu U}\frac{dP}{dx}, \alpha_1 = \frac{\beta_1}{\text{Ł}}, s = \frac{v_0 L}{\nu},$$
$$\theta = \frac{T - T_0}{T_1 - T_0}, Pe = \frac{v_0 \rho C_p L}{k}, \alpha_0 = \frac{\beta_0}{\text{Ł}}, Br = \frac{\mu U^2}{k(T_1 - T_0)} \quad (3)$$

we obtain the following nonlinear ordinary differential equations with appropriate boundary conditions

$$G + s\frac{du}{dy} + \frac{d^2u}{dy^2} = 0, \quad (4)$$

$$\frac{d^2\theta}{dy^2} + Pe\frac{d\theta}{dy} + Br\left(\frac{du}{dy}\right)^2 = 0, \quad (5)$$

subject to the boundary conditions

$$u(-1) = \alpha_0\frac{du(-1)}{dy}, \frac{d\theta(-1)}{dy} = Bi_0\theta(-1)$$
$$u(1) = -\alpha_1\frac{du(1)}{dy}, \frac{d\theta(1)}{dy} = -Bi_1\theta(1) \quad (6)$$

where u is the dimensionless fluid velocity, θ is the dimensionless fluid temperature, $\alpha_{0,1}$ are the dimensionless slip parameters at the walls, s is the fluid suction/injection parameter due to channel porosity, Pe is the Peclet number, Br is the Brinkman number while $Bi_{0,1}$ are the Biot's numbers. The solutions of equations Eqns (4-6) are obtained by using D-solve algorithm in Mathematica, the graphical solutions are presented below as figures 2-5.

3. Entropy Generation Analysis

In dimensional form, the equation for the total entropy generation per unit volume in the flow channel is given by

$$Q^m = \frac{k}{T_0^2}\left(\frac{dT}{dy'}\right)^2 + \frac{\mu}{T_0}\left(\frac{du'}{dy'}\right)^2 \quad (7)$$

where the first term is the irreversibility due to heat transfer while the second term represents the irreversibility due to fluid friction. In dimensionless form, we have

$$N_s = \frac{Q^m T_0^2 L^2}{k(T_1 - T_0)^2} = \left(\frac{d\theta}{dy}\right)^2 + \frac{Br}{\Omega}\left(\frac{du}{dy}\right)^2 \quad (8)$$

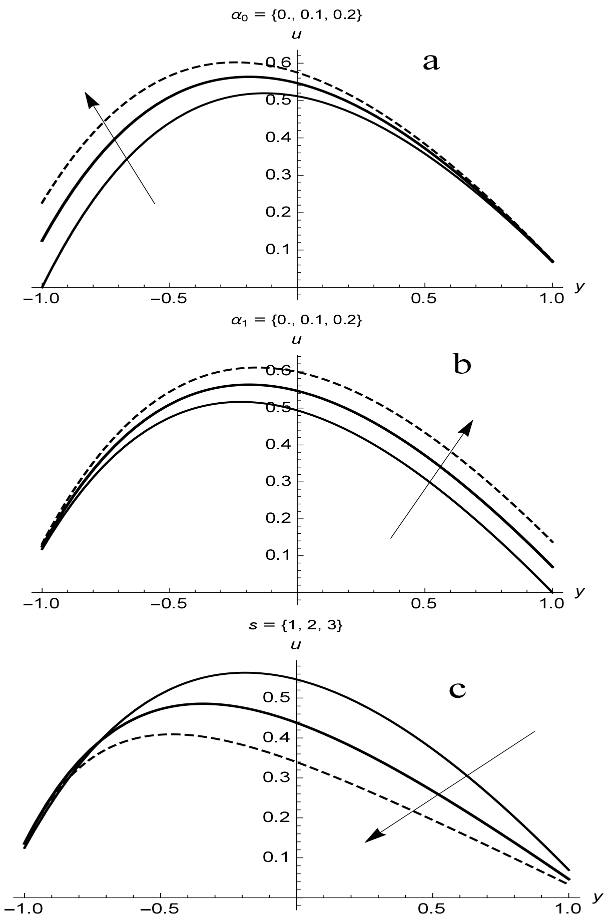

Figure 2: Effect of different fluid parameters on the fluid velocity: (a) varying lower slip parameter α_0, with $s = 1$, $\alpha_1 = 0.1$; (b) varying upper slip parameter α_1, with $s = 1$, $\alpha_0 = 0.1$; (c) varying suction parameter s, with $\alpha_0 = \alpha_1 = 0.1$.

where N_s, $\Omega = \frac{T_1 - T_0}{T_0}$ represents the dimensionless entropy generation and temperature difference parameters respectively. The heat irreversibility distribution ratio (Φ) is the ratio of entropy generation due to fluid friction and entropy generation due to heat transfer. Hence, we denote

$$N_1 = \left(\frac{d\theta}{dy}\right)^2, N_2 = \frac{Br}{\Omega}\left(\frac{du}{dy}\right)^2 \tag{9}$$

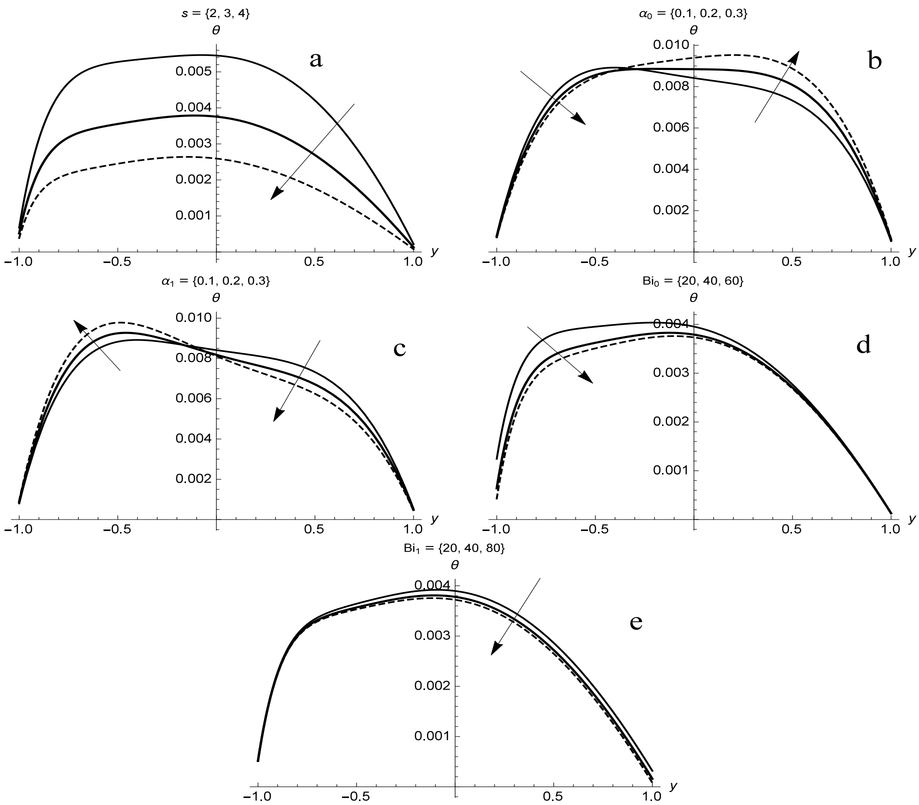

Figure 3: Effect of different fluid parameters on the temperature distribution of the fluid: (a) varying suction parameter s, with $\alpha_0 = \alpha_1 = 0.1$, $Bi_0 = Bi_1 = 50$, $Br = 0.1$, $Pe = 1$; (b) varing lower slip parameter α_0, with $\alpha_1 = 0.1$, $Bi_0 = Bi_1 = 50$, $Br = 0.1$, $Pe = 1$, $s = 0.1$; (c) varying upper slip parameter α_1, with $\alpha_0 = 0.1$, $Bi_0 = Bi_1 = 50$, $Br = 0.1$, $Pe = 1$, $s = 1$; (d) varying lower Biot number Bi_0, with $\alpha_0 = \alpha_1 = 0.1$, $Bi_1 = 50$, $Br = 0.1$, $Pe = 1$, $s = 3$; (e) varying upper Biot number Bi_1, with $\alpha_0 = \alpha_1 = 0.1$, $Bi_0 = 50$, $Br = 0.1$, $Pe = 1$, $s = 3$.

such that $N_s = N_1 + N_2$, where N_1 and N_2 represents the irreversibility due to heat transfer and fluid friction respectively.

Then the Bejan number for heat irreversibility ratio can be written as

$$Be = \frac{N_1}{N_1 + N_2} \tag{10}$$

From Eqn (10), it is evident that Be is bounded i.e. $0 \leq Be \leq 1$. This implies that when $Be = 0$, the fluid friction irreversibility dominates over irreversibility due to heat transfer and $Be = 1$ shows that the irreversibility due to heat transfer dominates over the irreversibility due to fluid friction. The special case when $Be = 0.5$ implies equal contributions to the entropy generation rate.

4. Discussion of Results

The present section displays the graphical solutions of the velocity and temperature profiles together with entropy generation and ratio for the heat irreversibility. Figure 2a shows the effect of the lower slip parameter on the fluid flow. It is observed that increase in the lower slip parameter enhances the fluid flow velocity at the lower wall. In other words, the average transport rate of the flow system is improved by incresing the slip parameter at the lower solid-fluid interface. Moreover, figure 2b exhibits the effect of the upper slip parameter on the fluid flow. The response of the flow rate to increasing upper slip parameter is comparable to the observation in figure 2a. As shown, the increase in the upper slip parameter enhances the velocity of the flow at the upper wall. In both cases, there is decrease in the adhesiveness of the fluid particles to the solid walls. Figure 2c represents the influence of the asymmetric suction/injection parameter on the fluid flow. As observed, an increase in suction/injection parameter leads to drift in the flow pattern of the fluid towards the suction wall.

Figure 3 shows the effect of changes in some fluid parameters on the fluid temperature distribution within the flow channel. As seen in figure 3a, an increase in the suction parameter decreases the fluid temperature distribution within the flow channel while fluid injection enhances the temperature distribution. Here, fluid suction reduces the translational kinetic energy of the fluid. In figure 3b, a dual temperature profile is observed in the flow system as the lower slip parameter increases. While a decrease in temperature of the fluid particles characterises the fluid region closer to the lower wall, the fluid experiences increased temperature in the neighbourhood of the upper wall. However, a reverse phenomenon is exhibited as the upper slip parameter increases as shown in figure 3c. Figure 3d represents the effect of lower Biot number on the temperature distribution within the flow channel. As observed, an increase in the lower Biot number decreases the temperature of the fluid particles closer to the lower wall while an increase in the upper Biot number decreases the fluid temperature near the upper wall as shown in figure 3e.

Interestingly, it is observed that as both upper and lower Biot number increases, the energy level of the thermal set up decreases as shown in figures

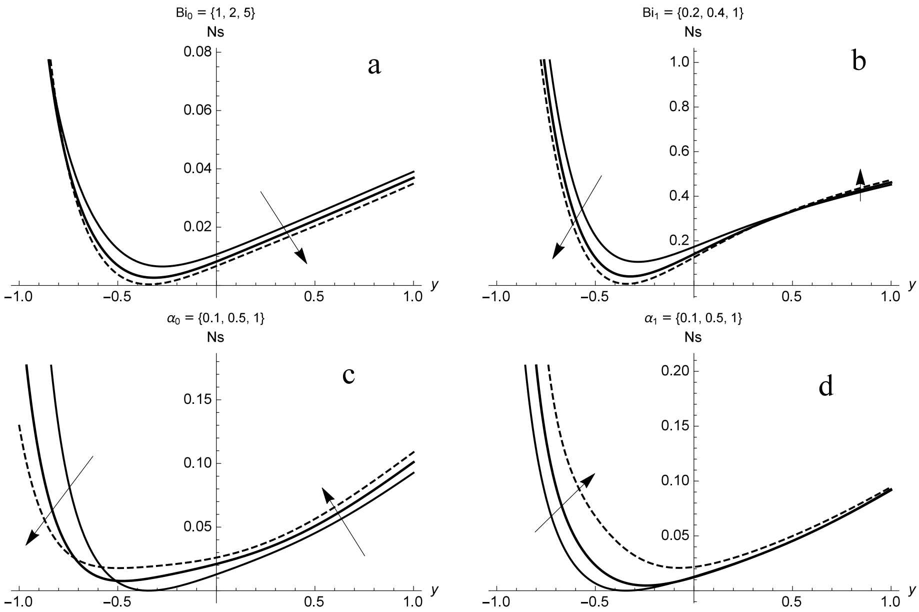

Figure 4: Effect of different fluid parameters on the entropy generation rate: (a) varying lower Biot number Bi_0, with $\alpha_0 = \alpha_1 = 0.1$, $Bi_1 = 10$, $Br = 1$, $Pe = 1$, $s = 2$, $\Omega = 10$; (b) varing upper Biot number Bi_1, with $\alpha_0 = \alpha_1 = 0.1$, $Bi_0 = 10$, $Br = 2$, $Pe = 1$, $s = 2$, $\Omega = 1$; (c) varying lower slip parameter α_0, with $\alpha_1 = 0.1$, $Bi_0 = Bi_1 = 10$, $Br = 2$, $Pe = 1$, $s = 2$, $\Omega = 10$; (d) varying upper slip parameter α_1, with $\alpha_0 = 0.1$, $Bi_0 = Bi_1 = 10$, $Br = 2$, $Pe = 1$, $s = 2$, $\Omega = 10$.

4a-b. One also observed that entropy generation rate is on the decrease as the lower slip parameter increases in the region closer to the lower wall, while the entropy generation rate increases elsewhere as shown in figure 4c. It is clear from figure 4d that an increase in the upper slip parameter encourages the growth of entropy. Figure 5 depicts the irreversibility ratio in the fluid, where the Bejan number, Be, is plotted against the width of the channel, y. It is observed from figures 5a-d that as slip parameters and the Biot numbers increases, the irreversibility due to heat transfer dominates the fluid motion over the irreversibility due to fluid friction.

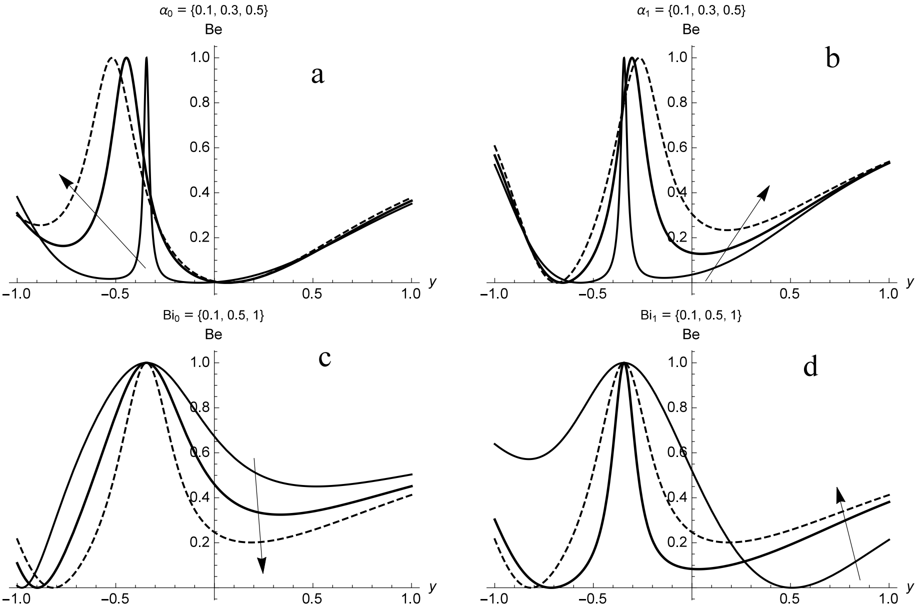

Figure 5: Effect of different fluid parameters on the irreversibility ratio: (a) varying lower slip parameter α_0, with $\alpha_1 = 0.1$, $Bi_0 = Bi_1 = 50$, $Br = 1$, $Pe = 1$, $s = 2$, $\Omega = 10$; (b) varing upper slip parameter α_1, with $\alpha_0 = 0.1$, $Bi_0 = Bi_1 = 10$, $Br = 2$, $Pe = 1$, $s = 2$, $\Omega = 10$; (c) varying lower Biot number Bi_0, with $\alpha_0 = \alpha_1 = 0.1$, $Bi_1 = 1$, $Br = 1$, $Pe = 1$, $s = 2$, $\Omega = 10$; (d) varying upper Biot number Bi_1, with $\alpha_0 = \alpha_1 = 0.1$, $Bi_0 = 1$, $Br = 1$, $Pe = 1$, $s = 2$, $\Omega = 10$.

5. Conclusion

The steady flow of Newtonian fluid flow through a porous channel with slip and thermal boundary condition is considered. The equations governing the fluid flow are formulated, non-dimensionalized and solved. From the results, it is found that slip and convective conditions do not only have significant effect on the flow and temperature fields, it also affects the entropy generation rate and the irreversibility ratio within the flow channel. In particular, increasing the slip parameters increases the average flow rate and decreases the effective temperature of the system, thereby scaling down the rate of entropy generation in the fluid system. Moreover, increasing the suction parameter decreases the rate of change of momentum in the system.

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