International Journal of Pure and Applied Mathematics
Volume 115 No. 3 2017, 561-575
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v115i3.10

HEAT ABSORPTION BY HEAT-TRANSFER AGENT IN A FLAT PLATE SOLAR COLLECTOR

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Abstract: The authors consider heat absorption by heat-transfer agent in a flat plate solar collector as well as construction of such solar collectors. At given effective channel

Received: May 29, 2017
Revised: July 2, 2017
Published: July 27, 2017

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diameter, thermal efficiency of the plant grows as ϕ (the ratio of the channel diameter to the distance between channel axes) increases. The upper limit of ϕ is restricted by the given temperature of heated water output. It reaches its peak at $t_k = t_{out}^w$ because $q'_{nk} >> q_{cn}$ (t_k is water temperature at the upper part of solar collector, q'_{nk} is the output heat flow and q_{cn} is the input heat flow). All other factors being the same, the thermal efficiency of the plant is reduced but slightly as channel diameter grows from 14 mm to 30 mm. The effect becomes stronger as channel diameter is decreased below 14 mm. As water temperature is decreased at the plant output, the thermal efficiency is increased and shifts toward growing ϕ values. Under the normal operating conditions of a flat plate solar collector, the maximum practically achievable value of thermal efficiency at $t = 60^\circ C$ is 0.28, and it grows to 0.4 as the temperature of output water is reduced by $10^\circ C$.

AMS Subject Classification: 74F05, 58J35, 81T28
Key Words: solar radiation, thermal efficiency, flat plate solar collector, angle of solar collector

1. Introduction

The EC Fund of housing involves 25% houses with solar collectors on their roofs (that supply house heating and hot water). This is the most complicated sector of housing [1] whose power efficiency is priority for energy modernization [2] and [3]. The additional technical elements and promising proposals for market updating are given by development of new solar heat supply systems [4], [5] and [6].

To provide hot water for residential use as well as for house heating and cooling, solar collectors are placed at the vertical surfaces (30% the total number of collectors) [6]. Several commercial applications of solar collectors for repairs as well as some available products for solar thermal power plants are known [7]. The idea of applying metal absorbers in solar thermal power plants for solar heating of houses was developed and studied in the European project [8]. A heater prototype and a new approach were developed in the European project to optimize arrangement of absorber with an aluminum solar collector [9].

Knowing the radiative and convective heat transfer coefficients of collector, one can correctly simulate the heat loss factor [10].

2. Methods of Calculations

If the quantum of water heated per 1 hour by $1m^2$ of glassed surface of solar collector is $1kg$, heat capacity of water C is $4.19 \cdot 10^3 kJ/kg^{\circ}C$ and temperature of water flowing through tube input (output) is t_{in}^w (t_{out}^w), then the quantity of heat absorbed by water is

$$q_{abs} = ReC(t_{in}^w - t_{out}^w) \tag{1}$$

(Re is the Reynolds number). On the other hand, this quantity of heat is transferred from the surface of solar collector tubes by radiation and convection. The total heat flow consists of the following components:

1) Heat radiated on the tubes by the inner glass surface:

$$q_{st} = A\left[\left(\frac{T_{c1}}{100}\right)^4 - \left(\frac{T_c}{100}\right)^4\right] \tag{2}$$

2) Heat radiated on the channels by the free part of the heat receiver:

$$Q_{sk} = A\left[\left(\frac{T_k}{100}\right)^4 - \left(\frac{T_c}{100}\right)^4\right] \tag{3}$$

3) Heat of the direct solar radiation absorbed by the surface of heat-removing channels:

$$q_{nk} = E \perp a_{st}\varepsilon_k \tag{4}$$

4) Heat transmitted by convection from air in the solar collector to the surface of heat-removing channels:

$$q_{bk} = a_c\pi\phi(T_c - T_k) \tag{5}$$

Here a_c is calculated by the following expression:

$$a_c = 1.10 \cdot \left(\frac{T_c - T_k}{d_{ek}}\right)^{\frac{1}{4}} \tag{6}$$

Substitution of a_c from Eq. (6) in Eq. (5) gives:

$$q_{bk} = \frac{1.1\pi\phi}{d^{\frac{1}{4}}}(T_c - T_k)^{\frac{1}{4}}$$

or

$$q_{bk} = A_1 \cdot 316 \cdot \left(\frac{T_c}{100} - \frac{T_k}{100}\right)^{\frac{1}{4}}, \quad (6a)$$

where

$$A_1 = \frac{1.1\pi\phi}{d_{ek}^{\frac{1}{4}}}.$$

3. Results and Discussion

Table 1. The A_1 values at different ϕ and d (C_{eq} is the equivalent specific heat).

C_{eq}/ϕ	00	0.08	0.13	0.18	0.23	0.28	0.33	0.38	0.43
8	0.00	0.164	0.267	0.369	0.473	0.575	0.678	0.783	0.885
13	0.00	0.146	0.236	0.328	0.418	0.510	0.599	0.692	0.781
18	0.00	0.138	0.224	0.301	0.396	0.468	0.552	0.635	0.720
23	0.00	0.125	0.204	0.282	0.360	0.438	0.518	0.596	0.677
23	0.00	0.120	0.195	0.271	0.345	0.421	0.516	0.570	0.645
33	0.00	0.115	0.187	0.260	0.332	0.405	0.475	0.548	0.621
38	0.00	0.112	0.180	0.250	0.319	0.390	0.459	0.529	0.600
43	0.00	0.107	0.175	0.243	0.310	0.378	0.445	0.512	0.580

It is possible to make considerably easier use of Eq. (6a) by presenting $316 \cdot \left(\frac{T_c}{100} - \frac{T_k}{100}\right)^{1/4}$ as a polynomial with integer exponents. For example, an analysis shows that, in the case of operating temperature difference from $100^{\circ}C$ to $110^{\circ}C$ (in which a hot box solar collector operates), this factor can be written down (with sufficient accuracy) as polynomial of degree one:

$$316 \cdot \left(\frac{T_c}{100} - \frac{T_k}{100}\right)^{1/4} = 316 \cdot \left(\frac{T_c}{100} - \frac{T_k}{100}\right) + D$$

Then Eq. (6) will look as follows:

$$q_{bk}' = A_1[316\left(\frac{T_c}{100} - \frac{T_k}{100}\right) + D] \quad (7)$$

where D is the tube diameter; the numerical values of specific heat C (331 and 24.8) are true at $T_k - T_c \geq 5$ because

$$q_{tot} = q_{st} + q_{st} + q_{bk} + q_{bk}^{\prime} \tag{8}$$

After substituting the values of summands q_{st}, q_{bk} and $q_{bk}^{\prime}$ to the right-hand side of Eq. (8) we obtain

$$q_{tot} = A\big[(\frac{T_k}{100})^4 + (\frac{T_{c1}}{100})^4 - 2(\frac{T_c}{100})^4\big] + \\ +A_1 316\big[(\frac{T_k}{100} - \frac{T_c}{100}))\big]^{5/4} + q_{nk} \tag{9}$$

or

$$q_{tot} = A\left[\left(\frac{T_k}{100}\right)^4 + \left(\frac{T_{c1}}{100}\right)^4 - 2\left(\frac{T_c}{100}\right)^4\right] + \\ + A_1 C\left[\left(\frac{T_k}{100} - \frac{T_c}{100}\right)\right] + A_1 D + q_{nk}. \tag{10}$$

Besides Eqs. (9) or (10) that determine q_{tot}, an auxiliary equation has to be set up, for example, the heat transfer equation for water transfer from the outer surfaces of channels (tubes) to the liquid. If the average temperature of channel (tube) surfaces is $\bar{T}_k$ and that of the water in the channels of solar collector is $\bar{t}_b$, then the heat transfer equation for parallel channels is

$$q_{tot} = \frac{\pi m(\bar{T}_k - \bar{T}_b)}{\frac{2.3}{2A_c} lg\frac{d}{d_0} + \frac{1}{a_b d_0}} = \frac{\pi \alpha(\bar{T}_k - \bar{T}_b)}{\frac{2.3}{2\lambda_k} lg\frac{d}{d_0} + \frac{1}{a_b d_0}} \tag{11}$$

If it is impossible to neglect heat loss from the outer surfaces, then the total heat loss will be twice as much, and consequently a useful quantity of heat absorbed by water will be less. In this case, the total heat loss may be practically taken into account by the coefficient x1:

$$x_1 = 1 - \frac{k_b}{k_c} \cdot \frac{F_b}{F_c} \tag{12}$$

In other words, at joint solving the above equations, Eq. (11) should be replaced by

$$q_{on} = (1 + \frac{k_b}{k_c} \cdot \frac{F_b}{F_c} q_n \tag{13}$$

where F_c (F_b) is the solar collector surfaces without (with) glass.

Equations (7) and (13) have eight unknowns, namely, q_{nk}, q_n, q_{tot}, T_k, $\bar{T}_k$, T_{c_1}, T_{c_2} and Re. After such modification, Eq. (13) is

$$q_n = \alpha_2(T_{c_2} - T_0) + 4.9Ec\left[\left(\frac{T_{c_2}}{100}\right)^4 - \left(\frac{T_0}{100}\right)\right]^4. \quad (14)$$

When considering the set up equations, we see that they are quartic. To conveniently calculate analytical solution, let us try to reduce these equations to those of the first degree. Indeed, at normal solar collector operation T_k, T_c, T_{c_1} and T_{c_2} vary practically from $40^\circ C$ to $100^\circ C$, while the air temperature varies from $10^\circ C$ to $50^\circ C$. In such temperature ranges, the dependence $\left(\frac{T}{100}\right)^4$ can be expressed with sufficient accuracy as straight-line equation.

The following equation is true for the first four temperatures:

$$\left(\frac{T}{100}\right)^4 = M\left(\frac{T}{100}\right) + N = 168\left(\frac{T}{100}\right) - 438, \quad (15)$$

for air

$$\left(\frac{T_0}{100}\right)^4 = R\left(\frac{T_0}{100}\right) + S = 103\left(\frac{T_0}{100}\right) - 227, \quad (16)$$

One can see that the corresponding replacement of T_k, T_c, T_{c_1} and T_{c_1} by the expressions compiled in the form of Eq. (15) and of T_0 by the expressions compiled in the form of Eq. (16) reduces the above equations to those of the first degree that are solved in the ordinary way.

For approximate calculations that provide acceptable results, the computing method for solar collector can be simplified by supposing that

$$T_{c_1} = C_1T_k + (1 - C_1)T_0 \quad (17)$$

$$T_{c_2} = C_2T_k + (1 - C_2)T_0 \quad (18)$$

The above equations can be obtained from the expressions for heat loss in the steady-state:

$$q_nk_c(T_k - T_0) = (\alpha_1 - \alpha_{l_1})(T_k - T_{c_1}) = \frac{\lambda_c}{\delta_c}(T_{c_1} - T_{c_2}) = (\alpha_2 - \alpha_{\lambda_2})(T_{c_2} - T_0), \quad (19)$$

where α_{l_1} is the factor of heat transfer from the surfaces of channels (tubes) and screen to the inner surface of glass; α_{l_2} is the heat-transfer factor taking into account radiation from the outer surface of glass to environment.

To prove that T_{c_1} and T_{c_2} can be presented as in Eqs. (17) and (18), let us divide Eq. (19) by $(\alpha_1 + \alpha_{l_1}$. Then

$$k_0 = (T_k - T_0) = (T_k - T_{c_1}) = \frac{\alpha_3}{\alpha_1 + \alpha_l}(T_{c_1} - T_{c_2}) = \\ = \frac{\alpha_2 + \alpha_{l_2}}{\alpha_1 + \alpha_{l_2}}(T_{c_2} - T_0), \quad (20)$$

where $k_0 = \frac{k_c}{\alpha_1+\alpha_{l_1}}$ and $\alpha_3 = \frac{\lambda_c}{\delta_c}$. From Eq. (20) we find:

$$T_{c_1} = C_1T_k + (1 - C_1)T_0, \; T_{c_2} = C_2T_k + (1 - C_2)T_0,$$

where $C_1 = 1 - k_0$ and $C_2 = k_0\frac{\alpha_1+\alpha_{l_1}}{\alpha_2+\alpha_{l_2}}$.

T_{c_1} and T_{c_2} may be presented using Eqs. (19) and (20); so further application of the above equations becomes possible if a method for determining numerical values of C_1 and C_2. They can be found by inserting the values of T_{c_1} and T_{c_2}:

$$(\alpha_1 + \alpha_{l_1})(1 - C_1) = \alpha_3(C_1 - C_2)(\alpha_2 + \alpha_{l_1})C_2. \quad (21)$$

Then, substituting to Eq. (20) instead of Eq. (21) its value α_{λ_1} from Eq. (19), we obtain:

$$[2(1 - C_1)^{1/4} \cdot (T_k - T_0)^{1/4} + \alpha_{l_1}](1 - C_1) = \varepsilon_3(C_1 - C_2) = C_2(\alpha_1 + \alpha_{l_2}). \quad (22)$$

Because in the obtained set of equations with two unknowns $C_1 < 0.5$, $(1 - C_1)$ can be written as

$$(1 - C_1)^{1/4} = 1 - \frac{C_1}{4} = 1 - 0.25C_1. \quad (23)$$

Then, denoting $T_k - T_0)^{1/4}$ by x in Eq. (22) and taking into account the value of $(1C1)$ from Eq. (23), we can rewrite Eq. (23) as two equations with unknowns $C1$ and $C2$:

$$[2(1 - 0.25C_1)x + \alpha_{l_1}](1 - C_1) = \alpha_3(C_1 - C_2)$$

and

$$\alpha_3(C_1 - C_2) = (\alpha_2 + \alpha_{l_1})C_2.$$

A joint solving this set of equations gives:

$$C_1 = \frac{2.5x + \alpha_{l_1} + U - \sqrt{(2.5x + \alpha_{l_1} + U)^2 - 2x(2x + \alpha_{l_1})}}{x} \quad (24)$$

and

$$C_2 = \frac{\alpha_3}{\alpha_2 + \alpha_{l_2} + \alpha_3} \cdot C_1 = Z_{c_1} \quad (25)$$

where

$$U = \frac{\alpha_3(\alpha_3 + \alpha_{l_2})}{\alpha_2 + \alpha_{l_2} + \alpha_3}. \quad (26)$$

During operation of a hot box solar collector with single glazing, the value of $T_k - T_{c_1}$ varies from $40^\circ C$ to $80^\circ C$; this corresponds to x variation from 2.5 to 3. An analysis of Eq. (26) shows that the above variation affects the $C1$ value but slightly, so it is possible to enter the mean value $x = 2.75$ into Eq. (26). Taking the mean value $\alpha_{l_1} = \alpha_{l_2} = 5.3W/m^2K$, one can rewrite Eq. (26) as

$$C_1 = 4.8 + 0.363 - \sqrt{(4.8 + 0.36U)^2 - 8.6}. \quad (27)$$

As the thickness of solar collector glass is varied from $2.5mm$ to $4mm$ at $\lambda_c = 0.64W/m^2K$, the numerical values of coefficients U and Z slightly depend on δ, but they depend to a greater extent on α_2 and wind speed V_b.

Table 2. Dependence of numerical values of coefficients on the wind speed.

V_b	1	2	3	4	5	
a_1	586	636	681	706	747	$a_1 = (100a_2 + 4.9\varepsilon_{cm}M)C_2$
a_2	288	338	383	408	449	$a_2 = a_1 + 4.9\varepsilon_{cm}(RM)$
a_3	968	968	968	968	968	$a_3 = 4.9\varepsilon_{cm}(SN)$

Similarly, the equation can be transformed by substituting the T_{c_1} values:

$$q_{tot} = (a_4^{\prime} + a_4^{\prime\prime})\frac{T_k}{100} + a_5\frac{T_0}{100} - a_6\frac{T_c}{100} + a_7 + q_{nk}, \quad (28)$$

where $a_4^{\prime} = AM(1 - C_1)a_6 = 2MA + A_1C$, $a_4^{\prime\prime} = A_1C$ $a_7 = A_1D$ and $a_5 = MA(1 - C_1)q_{bk} = E_1a_c\xi\varepsilon_k\phi$.

Shown in Table 3 are numerical values of the coefficients $a_4^{\prime}$, $a_4^{\prime\prime}$, a_5, a_6, and a_7 as functions of ϕ and d. It will be recalled that the initial relations were derived on the assumption that the area F_c of solar collector surface is 1 m^2 and the coefficients of light area of glass (η_ϕ) and direct solar radiation absorbed by the surfaces of channels (tubes) (η_ϕ) are equal to unity.

To move to thermal design of solar collectors with total surface area of 10 m^2 by calculating 1 m^2 of the total glass surface area, the right-hand side of expression for q_{ng} should be multiplied by η_c and expression for q_{nk} in Eq. (27) should be multiplied by $\eta_c\eta_\phi$. To take into account heat loss through the side face, the right-hand side of expression for q_n should be multiplied by x_1 from (a). Then

$$q_{ng}^{\prime} = \eta_c q_{ng} \tag{29}$$

$$q_{tot}^{\prime} = q_{tot} - q_{nk}(1 - \eta_c\eta_\phi) \tag{30}$$

$$q_{on} = x_1 q_n \tag{31}$$

The coefficient η_c in Eqs. (30) and (31) is the ratio between the light ($F_{c.c}$) and total (F_c) areas of glass covering of solar collector ($F_{c.c}$) i.e., the surface without accounting for sash shading):

$$\eta_c = \frac{F_{c.c}}{F_c} = 1 - \frac{F_{cm}}{F_c} \tag{32}$$

Here $F_{c.c}$ is the difference between the total glass area F_c and shadow area F_{cm}. The latter depends on the altitude of the sun over the horizon and structural dimension of the solar collector. F_{cm} can be tentatively calculated from the expression

$$F_{cm} = (a_0 + c)[S(e - ua_0)] + c(l - ua_0 + b_0) \tag{33}$$

where u is the number of cross strips of section $a_0 x(c + b_0)$, m; a_0 (c and b) are the $X-$ ($Y-$) dimensions of strips, m; S is the number of longitudinal (continuous) strips; l is the solar collector length, m.

The coefficient η_ϕ, that is the ratio of a sum of the illuminated screen area and area of projection of the illuminated surface of channels (tubes) F_{cb} to the total screen area F_{ok} is determined similarly. It is a function of the altitude of the sun over the horizon, height of solar collector H and structural dimension of the solar collector. Since $F_{ok} = F_c$,

Table 3. Dependences of numerical values of the coefficients a_4', a_4'', a_5, a_6 and a_7 on ϕ and d.

ϕ	0	0.08	0.13	0.18	0.23	0.28	0.33	0.38	0.43
$V_b(m/s)a_4' = AM(1-C_1)(a)$									
1	0	72.7	192	326	495	688	892	1112	1353
2	0	67.8	185	315	478	665	862	1074	1308
3	0	66.2	181	308	467	649	841	1049	1276
4	0	64.7	177	301	456	635	823	1026	1248
5	0	63.9	175	297	451	627	813	1013	1233
$Da_4'' = A_1C(b)$									
d_{eff}/ϕ	0	0.08	0.13	0.18	0.23	0.28	0.33	0.38	0.43
8	0	54	88	122	156	190	224	259	293
13	0	48	78	108	138	168	198	229	258
18	0	45	74	99	131	155	182	210	238
23	0	41	69	93	119	145	173	197	224
28	0	39	64	89	114	138	170	188	213
33	0	38	62	86	109	134	181	181	205
38	0	37	59	83	105	129	152	175	198
43	0	35	57	80	102	125	147	169	192
$V_b(m/s)a_5 = MA(1-C_1)(c)$									
ϕ	0	0.08	0.13	0.18	0.23	0.28	0.33	0.38	0.43
1	0	20.5	56	145	95	201	260	325	396
2	0	23	62	161	106	224	291	363	442
3	0	24	67	173	114	240	312	389	473
4	0	26	71	183	121	255	330	412	501
5	0	27	73	188	124	262	340	424	516
$d_{eff}mma_6 = 2MA + A_1C(d)$									
ϕ	0	0.08	0.13	0.18	0.23	0.28	0.33	0.38	0.43
8	0	145	302	545	796	1080	1379	1679	2043
13	0	139	296	531	778	1058	1353	1667	2008
18	0	136	293	522	771	1045	1337	1648	1988
23	0	132	289	516	760	1035	1326	1635	1974
28	0	130	287	512	754	1029	1326	1626	1963
33	0	128	286	509	749	1024	1312	1619	1955
38	0	127	285	505	745	1019	1307	1613	1948
43	0	126	283	503	742	1015	1302	1607	1942
ϕ	0	0.08	0.13	0.18	0.23	0.28	0.33	0.38	0.43
8	0	-4.06	-6.6	-9.1	-11.7	-14.2	-16.8	-19.4	-21.9
13	0	-3.6	-5.8	-8.1	-10.3	-12.6	-14.3	-17.1	-19.3
$d_{eff}a_7 = A_1C(e)$									

$$\eta_\phi = \frac{F_{cb}}{F_{ok}} = \frac{F_{cb}}{F_c} = 1 - \frac{F_{mk}}{F_c} \tag{34}$$

18	0	-3.4	-5.5	-7.4	-9.8	-11.6	-13.6	-15.7	-17.8
23	0	-3.1	-5.0	-6.9	-8.9	-10.8	-12.8	-14.7	-16.7
28	0	-2.9	-4.8	-6.7	-8.5	-10.4	-12.7	-14.1	-15.9
33	0	-2.8	-4.6	-6.4	-8.2	-10.0	-11.7	-13.6	-15.4
38	0	-2.7	-4.4	-6.2	-7.9	-9.6	-11.3	-13.1	-14.8
43	0	-2.6	-4.3	-6.0	-7.6	-9.3	-11.0	-12.6	-14.3

where F_{mk} is the difference between the total area (the open part of absorber) and F_{cb}. It can be tentatively calculated from the following expression:

$$F_{mk} = (H + C + f)[(S + 1)(b - ua_0) + b] + [a_0 + (c + b_0)]ub, \qquad (35)$$

where f is the distance from the vertical walls of solar collector to the inner edge of sash, m.

The factor in x_1 may be calculated as follows. Supposing that there is no heat loss from the lateral surface, q_n and temperature conditions of solar collector are determined from the following expressions:

$$q'_{ng} = \eta_c q_{ng} = q'_{tot} + q_n, \qquad (36)$$

$$q'_{tot} = q_{tot} - (1 - \eta_c \eta_\phi) q_{nk}. \qquad (37)$$

Then K_c is found, supposing that it depends on T_k but slightly over the operating temperature range. This means that K_c calculated without allowance for heat loss from the lateral surface slightly differs from the real K_c value (with allowance for heat loss from the lateral surface). The calculation is performed using the following expression:

$$K_c = \frac{q_n}{T_k - T_0}. \qquad (38)$$

The total thermal resistance of glass and lateral surface of solar collector is a sum thermal resistances of radiation, convection and heat conduction:

$$\frac{1}{KC} = R_{\lambda k} + R_c, \qquad (39)$$

$$\frac{1}{K_b} = R_{\lambda_1 k_1} + R_b. \qquad (40)$$

Here $R_{\lambda k} \approx R_{\lambda_1 k_1}$ is the total radiation and convection resistance: R_λ and R_b are the heat resistance of glass and lateral surface of solar collector, respectively. It is rather easy to get $\frac{K_b}{K_c}$ by supposing that $R_{\lambda k} \approx R_{\lambda_1 k_1}$:

$$\frac{K_b}{K_c} = \frac{1}{1 + (R_b \pm R_c) K_c}. \tag{41}$$

Having the $\frac{K_b}{K_c}$ values, we determine F_b and F_c taking into account that a solar collector is a rectangular parallelepiped $H \times b \times l$. Indeed,

$$F_b = bl + 2lH + 2bH \tag{42}$$

$$F_c = bc \tag{43}$$

Therefore,

$$\frac{F_b}{F_c} = 1 + 2H\left(\frac{1}{C} + \frac{1}{b}\right). \tag{44}$$

If heat loss through the solar collector bottom are low and can be neglected, then

$$\frac{F_b}{F_c} = 2H\left(\frac{1}{l} + \frac{1}{b}\right).$$

A working equation for determination of coefficient X_1 is obtained by substituting $\frac{K_b}{K_c}$ from Eq. (41) and $\frac{F_b}{F_c}$ from Eq. (44) into expression Eq. (45):

$$X_1 = 1 + \frac{2H\left(\frac{1}{l} + \frac{1}{b}\right)}{1 + \left(\sum \frac{\delta}{\lambda} - \frac{\delta_c}{\lambda_c}\right) K_c}. \tag{45}$$

So the working equations used to tentatively design a solar collector are as follows:

$$q_{ng}^{\Box} = q_{tot}^{\Box} + q_{on} = q_{ng}\eta_c, \tag{I}$$

$$q_{on} = xq_n = x\left(a_1 \frac{T_k}{100} - a_2 \frac{T_0}{100} - a_3\right), \tag{II}$$

$$q_{tot}^{\Box} = q_{tot} - (1 - \eta_c \eta_\phi) q_{nk} = (q_4^{\Box} + a^{\Box}) \frac{T_k}{100} + a_5 \frac{T_0}{100} - a_6 \frac{T_c}{100} + a_7 + q_{nk}^{\Box}, \tag{III}$$

where $q_{nk} = \eta_c \eta_\phi \eta_{nk} = E \perp a_{cm} \varepsilon_k \eta_c \eta_\phi \phi$.

The order of solving the above equations is as follows. At first, the equations (I), (II) and (III) are solved assuming that $X = 1$. This makes it possible to determine the coefficients K_c (and consequently X). After this resolving these equations enables one to calculate $q^{\square}_{tot}$, q_{tot} and thermal efficiency η of solar collector that are close to real ones. The results of calculations (with regard to heat loss) at $q^{\square}_{ng} = 397W$, $t^w_{in} = 17^\circ C$ and $V_b = 3m/s$ are presented as functions of ϕ, with $d_{eq} = 23mm$ and $t^w_{out} = 60^\circ C$, $70^\circ C$ and $80^\circ C$ in Tables 4 and 5.

Table 4. Dependence of energy data of flat plate solar collectors on ϕ at $V = 2m/s$, $V = 3m/s$ and $t_{hw} = 60^\circ C$.

ϕ	0	0.08	0.13	0.18	0.23	0.28	0.33	0.38	0.43
$d_{eq} = 23mm, t_{hw} = 60°C,\ V = 3m/s$									
t_k	0	105	83	77	77	71	66	61	-
η, %	0	18	37	42	52	52	65	68	-
Re	0	2.5	4.8	5.5	6.8	7.7	8.5	9.0	-
q_{tot}	0	103	207	238	291	333	364	385	-
$d_{eq} = 23mm,\ t_{hw} = 60°C,\ V = 2m/s$									
ϕ	1.1		0.55		0.36		0.27		0.22
t_k			55		64		68		
η, %			0.82		0.68		0.64		
Re			10.8		8.9		8.4		
q_{tot}			463		381		359		

4. Conclusion

One can see from Tables 4 and 5 that the following items are to be complied when designing a flat plate solar collector:

1) At a specified efficient diameter of channel, thermal efficiency of a plant grows as ϕ (the ratio of the channel diameter to the distance between channel axes) increases. The upper limit of ϕ is restricted by the given temperature of heated water output. It reaches its peak at $t_k = t^w_{out}$ because $q^{\square}_{nk} >> q_{cn}$ ($q^{\square}_{nk}$ is the output heat flow and q_{cn} is the input heat flow).

2) All other factors being the same, the thermal efficiency of the plant is reduced but slightly as channel diameter grows from 14 mm to 30 mm. The effect becomes stronger as channel diameter is decreased below 14 mm.

Table 5. Dependence of energy data of flat plate solar collectors on ϕ at $t_{hw} = 70^{\circ}C$ and $t_{hw} = 80^{\circ}C$.

$d_{eq} = 23mm,\ t_{hw} = 70^{\circ}C$					
ϕ	1.1	0.55	0.36	0.27	0.22
t_k	0	105	83	77	77
η, %	0	18	37	42	52
Re	0	2.5	4.8	5.5	6.8
q_{tot}	0	103	207	238	291
$d_{eq} = 23mm,\ t_{hw} = 80^{\circ}C$					
ϕ	1.1	0.55	0.36	0.27	0.22
t_k		64	67	79	
η,%		65	63	49	
Re		8.45	8.1	6.4	
q_{tot}		363	350	273	

3) As water temperature is decreased at the plant output, the thermal efficiency is increased and shifts toward growing ϕ values.

4) Under the normal operating conditions of a flat plate solar collector, the maximum practically achievable value of thermal efficiency at $t = 60^{\circ}C$ is 0.28, and it grows to 0.4 as the temperature of output water is reduced by $10^{\circ}C$.

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