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ON w-α-OPEN SETS AND α-WEAKLY gw-CLOSED SETS IN w-SPACES

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Abstract: The purpose of this article is to introduce the notions of w-α-open sets and α-weakly gw-closed sets in w-spaces, and to study properties of them. In particular, the relationships among weakly gw-closed sets, s-weakly gw-closed sets and α-weakly gw-closed sets are investigated.

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1. Introduction

In [16], Siwiec introduced the notions of weak neighborhoods and weak base in a topological space. We introduced the weak neighborhood systems defined by using the notion of weak neighborhoods in [7]. The weak neighborhood system induces a weak neighborhood space which is independent of neighborhood spaces [2] and general topological spaces [1]. The notions of weak structure and w-space were investigated in [8]. In fact, the set of all g-closed subsets [3] in a topological space is a kind of weak structure.

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We introduced the notion of gw-closed set in [9] and some its basic properties. In [13], we introduced and studied the notion of weakly gw-closed sets for the sake of extending the notion of gw-closed sets in w-spaces.

The purpose of this article is to introduce the notions of w-α-open sets and α-weakly gw-closed sets in w-spaces, and to study properties of them. First, we we introduce the notion of w-α-open sets in weak spaces, and investigate its properties. Second, in order to extend the notion of gw-closed sets in w-spaces, by using the notion of w-α-open sets, we introduce the notion of α-weakly gw-closed sets in weak spaces, and investigate its properties. In particular, the relationships among weakly wg-closed sets, w-α-closed sets [13] and s-weakly g-closed sets [14] and α-weakly g-closed sets are investigated.

2. Preliminaries

Let S be a subset of a topological space X. The closure (resp., interior) of S will be denoted by $cl(S)$ (resp., $int(S)$). A subset S of X is called a *pre-open* [6] (resp., α-open [15], *semi-open* [4]) set if $S \subset int(cl(S))$ (resp., $S \subset int(cl(int(S))), S \subset cl(int(S))$). The complement of a pre-open (resp., α-open, *semi-open*) set is called a *pre-closed* (resp., α-closed, *semi-closed*) set. The family of all pre-open (resp., α-open, semi-open) sets in X will be denoted by $PO(X)$ (resp., $\alpha(X), SO(X)$). We know the family $\alpha(X)$ is a topology finer than the given topology on X.

A subset A of a topological space (X, τ) is said to be: g-closed [3] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X. The family of all g-open sets in X will be denoted by $GO(X)$.

Let X be a nonempty set. A subfamily w_X of the power set $P(X)$ is called a *weak structure* [8] on X if it satisfies the following:

(1) $\emptyset \in w_X$ and $X \in w_X$.

(2) For $U_1, U_2 \in w_X$, $U_1 \cap U_2 \in w_X$.

Then the pair (X, w_X) is called a *w-space* on X. Then $V \in w_X$ is called a *w-open* set and the complement of a w-open set is a *w-closed* set.

Then the family τ, $\alpha(X)$ and $GO(X)$ are all weak structures on X. But $PO(X)$ and $SO(X)$ are not weak structures on X. A subfamily m_X of the power set $P(X)$ of a nonempty set X is called a *minimal structure* on X [5] if $\emptyset \in w_X$ and $X \in w_X$. Thus clearly every weak structure is a minimal structure.

Let (X, w_X) be a w-space. For a subset A of X, the *w-closure* of A and the *w-interior* [8] of A are defined as follows:

(1) $wC(A) = \cap\{F : A \subseteq F, X - F \in w_X\}$.

(2) $wI(A) = \cup\{U : U \subseteq A, U \in w_X\}$.

Theorem 2.1 ([8]). *Let (X, w_X) be a w-space and $A \subseteq X$.*

(1) $x \in wI(A)$ if and only if there exists an element $U \in W(x)$ such that $U \subseteq A$.

(2) $x \in wC(A)$ if and only if $A \cap V \neq \emptyset$ for all $V \in W(x)$.

(3) If $A \subseteq B$, then $wI(A) \subseteq wI(B)$; $wC(A) \subseteq wC(B)$.

(4) $wC(X - A) = X - wI(A)$; $wI(X - A) = X - wC(A)$.

(5) If A is w-closed (resp., w-open), then $wC(A) = A$ (resp., $wI(A) = A$).

Let (X, w_X) be a w-space and $A \subseteq X$. Then A is called

(1) *a generalized w-closed set* (simply, gw-closed set) [9] if $wC(A) \subseteq U$, whenever $A \subseteq U$ and U is w-open.

(2) *a weakly generalized w-closed set* (simply, weakly gw-closed set) [13] if $wC(wI(A)) \subseteq U$ whenever $A \subseteq U$ and U is w-open;

(3) *s-weakly generalized w-closed* (simply, s-weakly gw-closed) [14]
if $wI(wC(A)) \subseteq U$ whenever $A \subseteq U$ and U is w-open.

In [14], we have showed the implications below:

$$
\begin{array}{ccccc}
 & & & & s\text{-weakly } gw\text{-closed} \\
 & & & \nearrow & \\
w\text{-closed} & \rightarrow & gw\text{-closed} & & \not\uparrow \;\; \not\downarrow \\
 & & & \searrow & \\
 & & & & \text{weakly } gw\text{-closed}
\end{array}
$$

3. w-α-Open Sets

In this section, we introduce the notion of w-α-open sets and study some basic properties:

Definition 3.1. Let (X, w_X) be a w-space and $A \subseteq X$. A subset A of X is called a *w-α-open set* if $A \subseteq wIwCwI(A)$. The complement of a w-α-open set is called a *w-α-closed set*. The family of all w-α-open sets in X will be denoted by $W_\alpha(X)$.

From Definition of 3.1, obviously the following statements are obtained:

Lemma 3.2. *Let (X, w_X) be a w-space. Then*

(1) Every w-open set is w-α-open.

(2) A is an w-α-closed set if and only if $wC(wI(wC(A))) \subseteq A$.

Remark 3.3. Recall that: Let (X, w_X) be a w-space and $A \subseteq X$. Then A is called

(1) *w-semi-open* (resp., *w-semi-closed*) [11] if $A \subseteq wCwI(A)$ (resp., $wIwC(A) \subseteq A$).

(2) a *w-preopen set* (resp., *w-preclosed set* [10] if $P \subseteq wIwC(P)$ (resp., $wCwI(P) \subseteq P$.

(3) a *w-regular open set* (resp., *w-regular closed set* [12] if $A = wIwC(A)$ (resp., $A = wCwI(A)$.

w-semi-open

$\nearrow$

w-regular open $\rightarrow$ w-open$\rightarrow$ w-α-open

$\searrow$

w-preopen

Theorem 3.4. *Let (X, w_X) be a w-space. Then the intersection of any two w-α-open sets is w-α-open.*

Proof. Let A and B be w-α-open sets. Then $A \subseteq wIwCwI(A)$ and $B \subseteq wIwCwI(B)$. and $A \cap B \subseteq wIwCwI(A) \cap wIwCwI(B) = wI(wCwI(A) \cap wCwI(B))$. So, for the proof, it is sufficient to show that we $wI(wCwI(A) \cap wCwI(B)) \subseteq wIwCwI(A \cap B)$. For each $x \in wI(wCwI(A) \cap wCwI(B))$, there exists a w-open set U_x containing x such that $x \in U_x \subseteq wCwI(A) \cap wCwI(B)$. Then since $U_x \subseteq wCwI(A) \cap wCwI(B)$, for each $z \in U_x$ and for each w-open set W_z containing z, since $U_x \cap W_z$ is a w-open set containing z, $(U_x \cap W_z) \cap wI(A) \neq \emptyset$. Then there is some w-open set $V_A \subseteq A$ such that $(U_x \cap W_z) \cap V_A \neq \emptyset$. Since $(U_x \cap W_z \cap V_A) \subseteq U_x$, we have that $(U_x \cap W_z \cap V_A) \subseteq wCwI(B)$, and so $(U_x \cap W_z \cap V_A) \cap wI(B) \neq \emptyset$. It implies that $\emptyset \neq U_x \cap W_z \cap wI(A) \cap wI(B) = (U_x \cap W_z) \cap wI(A \cap B) \subseteq W_z \cap wI(A \cap B)$. So, $z \in wCwI(A \cap B)$, and it implies that $U_x \subseteq wCwI(A \cap B)$. Finally, $x \in wIwCwI(A \cap B)$. □

Theorem 3.5. *Let (X, w_X) be a w-space. Then any union of w-α-open sets is w-α-open.*

Proof. Let A_i be a w-α-open set for $i \in J$. From Definition 3.1 and Theorem 2.1, it follows $A_i \subseteq wIwCwI(A_i) \subseteq wIwCwI(\cup A_i)$. This implies $\cup A_i \subseteq wIwCwI(\cup A_i)$. Hence $\cup A_i$ is a w-α-open set. □

Definition 3.6. Let (X, w_X) be a w-space. For a subset A of X, the *w-α-closure* of A and the *w-α-interior* of A, denoted by $w\alpha C(A)$ and $w\alpha I(A)$, respectively, are defined as the following:

$w\alpha C(A) = \cap\{F \mid A \subseteq F,\ F \text{ is } w\text{-}\alpha\text{-closed in } X\}$;

$w\alpha I(A) = \cup\{U \mid U \subseteq A,\ U \text{ is } w\text{-}\alpha\text{-open in } X\}$.

Theorem 3.7. *Let (X, w_X) be a w-space and $A \subseteq X$. Then*
(1) $w\alpha I(A) \subseteq A$.
(2) If $A \subseteq B$, then $w\alpha I(A) \subseteq w\alpha I(B)$.
(3) A is w-α-open iff $w\alpha I(A) = A$.
(4) $w\alpha I(w\alpha I(A)) = w\alpha I(A)$.
(5) $w\alpha C(X - A) = X - w\alpha I(A)$ and $w\alpha I(X - A) = X - w\alpha C(A)$.

Proof. (1), (2) Obvious.
(3) It follows from Theorem 3.4.
(4) It follows from (3).
(5) For $A \subseteq X$,

$$\begin{aligned} X - w\alpha I(A) &= X - \cup\{U \mid U \subseteq A,\ U \text{ is } w\text{-}\alpha\text{-open }\} \\ &= \cap\{X - U \mid U \subseteq A,\ U \text{ is } w\text{-}\alpha\text{-open }\} \\ &= \cap\{X - U \mid X - A \subseteq X - U,\ U \text{ is } w\text{-}\alpha\text{-open }\} \\ &= w\alpha C(X - A). \end{aligned}$$

Similarly, we have $w\alpha I(X - A) = X - w\alpha C(A)$. □

Theorem 3.8. *Let (X, w_X) be a w-space and $A, B, F \subseteq X$. Then*
(1) $A \subseteq w\alpha C(A)$.
(2) If $A \subseteq B$, then $w\alpha C(A) \subseteq w\alpha C(B)$.
(3) F is w-α-closed iff $w\alpha C(F) = F$.
(4) $w\alpha C(w\alpha C(A)) = w\alpha C(A)$.

Proof. It is similar to the proof of Theorem 3.7. □

Theorem 3.9. *Let (X, w_X) be a w-space and $A \subseteq X$. Then*

(1) $x \in w\alpha C(A)$ if and only if $A \cap V \neq \emptyset$ for every w-α-open set V containing x.

(2) $x \in w\alpha I(A)$ if and only if there exists a w-α-open set U such that $U \subseteq A$.

Proof. Obvious. □

Lemma 3.10 ([8]). *Let (X, w_τ) be a w-space and $A, B \subseteq X$. Then the following things hold:*

(1) $wI(A) \cap wI(B) = wI(A \cap B)$.

(2) $wC(A) \cup wC(B) = wC(A \cup B)$.

Theorem 3.11. *Let (X, w_X) be a w-space. Then for $A \subseteq X$,*

(1) $w\alpha C(A) = A \cup wCwIwC(A)$;

(2) $w\alpha I(A) = A \cap wIwCwI(A)$.

Proof. (1) First, we show that $A \cup wCwIwC(A)$ is w-α-closed. From Lemma 3.10, $wCwIwC(A \cup wCwIwC(A)) = wCwI(wC(A) \cup wCwCwIwC(A)) = wCwIwC(A) \subseteq A \cup wCwIwC(A)$. So, $A \cup wCwIwC(A)$ is w-α-closed.

Let F be any w-α-closed set such that $A \subseteq F$. Then $wCwIwC(A) \subseteq wCwIwC(F) \subseteq F$. Since $A \cup wCwIwC(A) \subseteq F$ and $A \cup wCwIwC(A)$ is w-α-closed, $w\alpha C(A) = A \cup wCwIwC(A)$.

(2) It is similar to the proof of (1). □

4. α-Weakly Generalized w-Closed Sets

In this section, we introduce the notion of α-weakly gw-closed set and study its properties. In particular, we study the relations among weakly gw-closed sets, s-weakly gw-closed sets and α-weakly gw-closed sets.

Definition 4.1. Let (X, w_X) be a w-space and $A \subseteq X$. Then A is said to be *α-weakly generalized w-closed* (simply, α-weakly gw-closed) if $wCwIwC(A) \subseteq U$ whenever $A \subseteq U$ and U is w-open.

Theorem 4.2. *(1) Every w-α-closed set is α-weakly gw-closed.*

(2) Every gw-closed set is α-weakly g-closed.

(3) Every α-weakly g-closed set is both weakly gw-closed and s-weakly gw-closed.

Proof. (1) Let A be a w-α-closed set and U be a w-open set containing A. Since $wCwIwC(A) \subseteq A$, obviously it satisfies $wCwIwC(A) \subseteq U$. So, A is α-weakly gw-closed.

(2)(3) For any nonempty set A, since $wCwIwC(A) \subseteq wC(A)$, $wIwC(A) \subseteq wCwIwC(A)$ and $wCwI(A) \subseteq wCwIwC(A)$, the statements are obtained. □

Remark 4.3. In general, the converses of the theorem above are not true as shown in the examples below:

Example 4.4. (1) Let $X = \{a, b, c, d\}$ and $w = \{\emptyset, \{a\}, \{c\}, \{c, d\}, X\}$ be a weak structure in X. For a set $A = \{a, b, c\}$, obviously, A is α-weakly gw-closed. But, since $wC(A) = X$, it is not w-α-closed.

For a w-open set $A = \{d\}$, note that $wC(A) = \{b, d\}$ and $wCwIwC(A) = wCwI(\{b, d\}) = wC(\emptyset) = \emptyset$. So A is α-weakly gw-closed but not gw-closed.

(2) Let $X = \{a, b, c\}$ and $w = \{\emptyset, \{a\}, \{b\}, X\}$ be a weak structure in X.

For a w-open set $A = \{b\}$, note that $wC(A) = \{b, c\}$, $wIwC(A) = A$ and $wCwIwC(A) = wCwI(wC(A)) = \{b, c\}$. So A is s-weakly gw-closed but not α-weakly gw-closed.

(3) Let $X = \{a, b, c, d\}$ and $w = \{\emptyset, \{a\}, \{c, d\}, X\}$ be a weak structure in X. For a w-open set $A = \{c\}$, note that $wI(A) = \emptyset$ and $wCwIwC(A) = wCwI(\{b, c, d\}) = wC(\{c, d\}) = \{b, c, d\}$. So A is weakly gw-closed but not α-weakly gw-closed.

From the above theorems and examples, the following relations are obtained:

s-weakly gw-closed
$\nearrow$
w-closed $\rightarrow$ gw-closed $\rightarrow$ α-weakly gw-closed $\quad\quad \not\uparrow \ \not\downarrow$
$\searrow$
weakly gw-closed

Lemma 4.5. *Let (X, w_X) be a w-space. Then the following things hold:*
(1) For w-α-open sets A and B,
$$wIwCwI(A) \cap wIwCwI(B) = wIwCwI(A \cap B).$$
(2) For w-α-closed sets A and B,
$$wCwIwC(A) \cup wCwIwC(B) = wCwIwC(A \cup B).$$

Proof. (1) Let A and B be w-α-open sets. Then, from the proof of Theorem 3.4, we know that $wIwCwI(A) \cap wIwCwI(B) = wI(wCwI(A) \cap wCwI(B)) \subseteq wIwCwI(A \cap B)$. Now, from Theorem 2.1 and Lemma 3.10, it follows that $wIwCwI(A \cap B) = wIwC(wI(A) \cap wI(B)) \subseteq wI(wCwI(A) \cap wCwI(B)) = wIwCwI(A) \cap wIwCwI(B)$. Consequently, $wIwCwI(A) \cap wIwCwI(B) = wIwCwI(A \cap B)$.

(2) By (4) of Theorem 2.1 and the statement (1), it is obtained. $\square$

Theorem 4.6. *Let (X, w_X) be a w-space. Then the union of two α-weakly gw-closed sets is α-weakly gw-closed.*

Proof. Let A and B be two α-weakly gw-closed sets. Let G be any w-open set such that $A \cup B \subseteq G$. Then $A \subseteq G$ and $B \subseteq G$. Since A and B are α-weakly gw-closed sets, $wCwIwC(A) \subseteq G$ and $wIwCwC(B) \subseteq G$. By Lemma 4.5, $wCwIwC(A \cup B) = wCwIwC(A) \cup wCwIwC(B) \subseteq G$. So, $A \cup B$ is α-weakly gw-closed. □

Remark 4.7. In general, the intersection of two α-weakly gw-closed sets is not α-weakly gw-closed as shown in the next example:

Example 4.8. For $X = \{a, b, c, d\}$, let $w = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, d\}, X\}$ be a w-structure in X. For $A = \{a, b, c\}$ and $B = \{a, c, d\}$, obviously, A and B are α-weakly gw-closed. But for $A \cap B = \{a, c\}$, since $\{a, c\}$ is w-open and $wCwIwC(\{a, c\}) = wCwI(\{a, c, d\}) = wC(\{a, c, d\}) = \{a, c, d\}$, $A \cap B = \{a, c\}$ is not α-weakly gw-closed.

Theorem 4.9. *Let (X, w_X) be a w-space and $A \subseteq X$. Then A is α-weakly wg-closed if and only if $w\alpha C(A) \subseteq U$ whenever $A \subseteq U$ and U is w-open.*

Proof. Let A be an α-weakly gw-closed subset of X and let U be any w-open set such that $A \subseteq U$. Then since $wCwIwC(A) \subseteq U$, $A \cup wCwIwC(A) \subseteq U$. So, by Theorem 3.11, $w\alpha C(A) \subseteq U$.

For $A \subseteq X$, suppose that $w\alpha C(A) \subseteq U$ whenever $A \subseteq U$ and U is w-open. Let U be any w-open set with $A \subseteq U$. Then from hypothesis and Theorem 3.11, $wCwIwC(A)) \subseteq A \cup wCwIwC(A) = w\alpha C(A) \subseteq U$. Hence, A is α-weakly gw-closed. □

Theorem 4.10. *Let (X, w_X) be a w-space. Then if A is an α-weakly gw-closed set, then $wCwIwC(A) - A$ contains no any non-empty w-closed set.*

Proof. For an α-weakly gw-closed set A, F be a w-closed subset such that $F \subseteq wCwIwC(A) - A$. Then $A \subseteq X - F$ and $X - F$ is w-open. Since A is α-weakly gw-closed, $wCwIwC(A) \subseteq X - F$. It implies that $F \subseteq X - wCwI(wC(A))$ and $F \subseteq wCwIwC(A) - A$. So, $F = \emptyset$. □

Corollary 4.11. *Let (X, w_X) be a w-space. Then if A is an α-weakly gw-closed set, then $w\alpha C(A) - A$ contains no any non-empty w-closed set.*

Proof. Since $wCwI(wC(A)) - A = (A \cup wCwI(wC(A))) - A = w\alpha C(A) - A$, by Theorem 4.10, the statement is satisfied. □

Now, we introduce the notion of α-weakly gw-open sets and study its basic properties.

Definition 4.12. Let (X, w_X) be a w-space and $A \subseteq X$. Then A is called *an α-weakly generalized open set* (simply, α-weakly gw-open set) if $X - A$ is α-weakly gw-closed.

Theorem 4.13. *Let (X, w_X) be a w-space. Then the intersection of two α-weakly gw-open sets is α-weakly gw-open.*

Proof. It is similar to the proof of Theorem 4.6. □

Remark 4.14. In general, the union of two α-weakly gw-closed sets is not α-weakly gw-closed (cf. Remark 4.7).

From Theorem 4.13 and Remark 4.14, the following theorem is obtained:

Theorem 4.15. *Let (X, w_X) be a w-space. Then the family of all α-weakly gw-closed sets is a weak structure in X.*

Theorem 4.16. *Let (X, w_X) be a w-space and $A \subseteq X$. Then A is α-weakly gw-open if and only if $F \subseteq wIwCwI(A)$ whenever $F \subseteq A$ and F is w-closed.*

Proof. It is obvious from Definition 4.1. □

Theorem 4.17. *Let (X, w_X) be a w-space and $A \subseteq X$. Then A is α-weakly gw-open if and only if $F \subseteq w\alpha I(A)$ whenever $F \subseteq A$ and F is w-closed.*

Proof. Let A be an α-weakly gw-open subset of X and F be a w-closed set such that $F \subseteq A$. Then $F \subseteq wIwCwI(A)$. Since $F \subseteq A \cap wIwCwI(A)$, by Theorem 3.11, $F \subseteq w\alpha I(A)$.

Suppose that the statement is satisfied. For $A \subseteq X$, let F be w-closed and $F \subseteq A$. Then by hypothesis and Theorem 3.11, $F \subseteq w\alpha I(A) = A \cap wIwCwI(A)$, and so $F \subseteq wIwCwI(A)$. Hence, A is α-weakly gw-open. □

Theorem 4.18. *Let (X, w_X) be a w-space and $A \subseteq X$. Then if A is α-weakly gw-open, then $U = X$, whenever $wIwCwI(A) \cup (X - A) \subseteq U$ and U is w-open.*

Proof. Let U be any w-open set and $wIwCwI(A) \cup (X - A) \subseteq U$. Then $X - U \subseteq (X - wIwCwI(A)) \cap A = wCwIwC(X - A) \cap A = wCwIwC(X - A) - (X - A)$. Since $X - A$ is α-weakly gw-closed, by Theorem 4.10, the w-closed set $X - U$ must be empty. Hence, $U = X$. □

Corollary 4.19. *Let (X, w_X) be a w-space and $A \subseteq X$. Then if A is α-weakly gw-open, then $U = X$, whenever $w\alpha I(A) \cup (X - A) \subseteq U$ and U is w-open.*

Proof. Since

$$w\alpha I(A) \cup (X - A) = (A \cap wIwCwI(A)) \cup (X - A) = wIwCwI(A) \cup (X - A),$$

by the above theorem, it is obtained. □

Theorem 4.20. *Let (X, w_X) be a w-space. Then if A is an α-weakly gw-closed set, then $wCwIwC(A) - A$ is α-weakly gw-open.*

Proof. If A is an α-weakly gw-closed set, then by Theorem 4.10, $\emptyset$ is the only one w-closed subset of $wCwIwC(A) - A$. So, $\emptyset \subseteq wIwCwI(wCwIwC(A) - A)$. Hence, $wCwIwC(A) - A$ is α-weakly gw-open. □

Corollary 4.21. *Let (X, w_X) be a w-space. Then if A is an α-weakly gw-closed set, then $w\alpha C(A) - A$ is α-weakly gw-open.*

Proof. From $w\alpha C(A) - A = (A \cup wCwIwC(A)) - A = wCwIwC(A) - A$, it is obtained. □

Theorem 4.22. *Let (X, w_X) be a w-space. Then if A is an α-weakly gw-open set, then $wIwCwI(A) \cup (X - A)$ is α-weakly gw-closed.*

Proof. If A is an α-weakly gw-open set, then by Theorem 4.18, X is the only one w-open set containing $wIwCwI(A) \cup (X - A)$. So, obviously, $wIwCwI(A) \cup (X - A)$ is α-weakly gw-closed. □

Corollary 4.23. *Let (X, w_X) be a w-space. Then if A is an α-weakly gw-open set, then $w\alpha I(A) \cup (X - A)$ is α-weakly gw-closed.*

Proof. It follows from $w\alpha I(A) \cup (X - A) = (A \cap wIwCwI(A)) \cup (X - A) = wIwCwI(A) \cup (X - A)$ and Theorem 4.22. □

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