ASSESSING THE POTENTIAL OF MEDICAL-MASK AND DRUG-TREATMENT IN CONTROLLING INFLUENZA DISEASE

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Abstract: A classical SIS model for influenza disease with two intervention strategies such as medical mask and drug treatment from deterministic framework was extended in this paper into a stochastic differential equation (SDE). The SDE model was constructed by introducing a random perturbation in successful contact rate parameter. The concern of this study is focused in to two aspects based on from mathematical and epidemiological point of view. From mathematical point of view, the basic reproduction number from deterministic framework (known as R_0) was analyzed and compared with the stochastic threshold parameter (we call it as ρ_0). As well as R_0 in deterministic model, the ρ_0 also linked into the extinction and persistence condition of endemic equilibrium. If $\rho_0 < 1$, then the SDE system will reach the extinction of influenza disease with probability one. Otherwise, if $\rho_0 > 1$ then influenza disease will persist. However, we found a situation where ρ_0 gives a contrast result compared to R_0 as a consequence of random perturbation in successful contact rate parameter. From epidemiological point of view, we found that medical mask and drug treatment intervention and successful contact rate influence the dynamic of influenza spread in our model, which can give us a direction about the best strategy to regulate influenza spreads. Some numerical simulations were generated to support the analytical results.

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1. Introduction

Influenza, also called as Flu, is a respiratory illness caused by influenza viruses. People infected by influenza virus show some symptoms like fever, headaches, sore throat or cough. This disease is easily spread among human, mainly from droplets made when infected people talk, sneeze or cough [4, 10]. The most important thing to prevent people from infection of influenza is by taking a vaccination each year. Also it is recommended to staying away from infected people or prevent yourself by wearing a medical mask (it is more recommended that infected person is the one who should wear medical mask [6]).

Mathematical modelling can lead the way in purpose of understanding the complexity of various disease and could become as the scientific back-up for controlling the disease before implementation in to the field. Since Kermack and Mckendric [7] introduced the SIR model to understand the dynamic of epidemic system, many mathematical models have been proposed since then with the intention to understand the qualitative behaviour for various disease like dengue [15, 16, 18], malaria [11, 12], West Nile virus [13, 14], Influenza [6, 19], swine flu [17], etc. Many mathematical model have been developed to understand the spread of influenza and explored strategies for control the spread [6, 19, 20, 21, 22]. In this article, we will reconstruct the influenza model in [6] as a Stochastic Differential Equation (SDE) model. The model in [6] was introduced as a two dimensional deterministic model with involving intervention of medical mask and drug treatment interventions, and given by :

$$\left. \begin{aligned} \frac{d\bar{S}}{dt} &= -\frac{(p\beta u_1 \bar{S} + (1-u_1)\beta \bar{S})\bar{I}}{N} + (\alpha + (\gamma_0 + \gamma_1 u_2))\bar{I} \\ \frac{d\bar{I}}{dt} &= \frac{(p\beta u_1 \bar{S} + (1-u_1)\beta \bar{S})\bar{I}}{N} - (\alpha + (\gamma_0 + \gamma_1 u_2))\bar{I} \end{aligned} \right\}. \quad (1)$$

with $\bar{S}$ and $\bar{I}$ represent the number of susceptible and infected person, $u_1, u_2 \in [0, 1]$ as medical mask and drug treatment intervention respectively, $\alpha > 0$ as natural death rate, $\gamma_0, \gamma_1 \geq 0$ as the natural and drug-induced recovery rates respectively, $\beta > 0$ as successful infection rate and $p \in [0, 1]$ as a reduction of β because of medical mask use. The β parameters in the deterministic model is a constant all the time, while in the real world, β is not constant all the time. This fact will be discussed in the model that we proposed in this article.

In the next section, the proposed method will be discussed and followed with some results in the third section. Some conclusions will be given in the last section.

2. The Proposed Method

In this section, the development of the model in 1 will be given as an SDE model in first sub-section and then followed with statistical analysis about the persistence and coexistence of equilibrium points in second sub-section.

2.1. The SDE Model

Given the deterministic model for influenza spread with medical mask (u_1) and treatment (u_2) in [6]. The model assumed constant parameters, whereas in reality, the nature of epidemic growth and spread is inherently random due to the unpredictability of person-to-person contacts [9]. Therefore, to understand the effect of stochastic factor in the influenza spread, we introduce stochastic noise into model parameter. Since the environmental stochasticity factor is related to the successful contact rate β , βdt is then changed into a random variable $\tilde{\beta} dt$, that is

$$\tilde{\beta} dt = \beta dt + \sigma dW(t), \quad (2)$$

where σ is a positive constant that denotes the noise intensity and $dW(t) = W(t + dt) - W(t)$ is the increment of Wiener process.

Replacing βdt with $\tilde{\beta} dt$, the deterministic model in [6] becomes the Itô SDE as follows

$$\begin{aligned} dS(t) = & \left(-\frac{\beta}{N} S(u_1[p-1] + 1)I + (\alpha + \gamma_0 + \gamma_1 u_2)I \right) dt \\ & - \sigma \frac{S}{N} I(u_1[p-1] + 1) dW(t), \end{aligned} \quad (3a)$$

$$\begin{aligned} dI(t) = & \left(\frac{\beta}{N} S(u_1[p-1] + 1)I - (\alpha + \gamma_0 + \gamma_1 u_2)I \right) dt \\ & + \sigma \frac{S}{N} I(u_1[p-1] + 1) dW(t). \end{aligned} \quad (3b)$$

Since the population is assumed to be constant ($S(t) + I(t) = N$), it is sufficient to study the characteristics of solution of SDE (3b) only.

2.2. The SDE Model Solution

The deterministic model solution and basic reproduction number (R_0) were already discussed in [6]. The basic reproduction number (R_0) of the deterministic model [6] as follows

$$R_0 = \frac{\beta(u_1[p-1] + 1)}{(\alpha + \gamma_0 + \gamma_1 u_2)}, \quad (4)$$

and $\lim_{t \rightarrow \infty} I(t) = 0$ if and only if $R_0 \leq 1$.

Let

$$\rho_0 = \frac{\beta(u_1[p-1] + 1)}{(\alpha + \gamma_0 + \gamma_1 u_2)} - \frac{\sigma^2(u_1[p-1] + 1)^2}{2(\alpha + \gamma_0 + \gamma_1 u_2)}, \quad (5)$$

as the stochastic threshold parameter of SDE (3b) and the value of ρ_0 can be negative. The following theorem discuss the characteristic of ρ_0 related to the extinction and persistence condition of influenza spread.

Theorem 1. *If*

$$\rho_0 = \frac{\beta(u_1[p-1] + 1)}{(\alpha + \gamma_0 + \gamma_1 u_2)} - \frac{\sigma^2(u_1[p-1] + 1)^2}{2(\alpha + \gamma_0 + \gamma_1 u_2)} < 1, \quad (6)$$

and

$$\sigma^2(u_1[p-1] + 1)^2 \leq \beta(u_1[p-1] + 1), \quad (7)$$

then for any initial value $I(0) = I_0 \in (0, N)$, the solution of SDE (3b) obeys

$$\begin{aligned} \limsup_{t \rightarrow \infty} \frac{1}{t} \log I(t) &\leq \beta(u_1[p-1] + 1) - (\alpha + \gamma_0 + \gamma_1 u_2) - \\ &\frac{1}{2}(\sigma(u_1[p-1] + 1))^2 < 0 \quad a.s. \end{aligned} \quad (8)$$

namely, $I(t)$ tends to zero exponentially almost surely, i.e. the disease dies out with probability one.

Proof. Write $dI(t)$ in (3b) as

$$\begin{aligned} dI(t) &= \left(\beta I(u_1[p-1] + 1) - \frac{\beta}{N} I^2(u_1[p-1] + 1) - (\alpha + \gamma_0 + \gamma_1 u_2) I \right) dt \\ &+ \sigma \frac{N - I}{N} I(u_1[p-1] + 1) dW(t). \end{aligned} \quad (9)$$

To prove implication (8), use Itô formula by letting $Y(t) = \log I(t)$, hence from (9) we get

$$\log I(t) = \log I_0 + \int_0^t f(I(s))ds + \int_0^t \frac{\sigma}{N}(N - I)(u_1[p - 1] + 1)dW(s), \quad (10)$$

where $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by

$$\begin{aligned} f(I) = & \beta(u_1[p - 1] + 1) - \frac{\beta}{N}I(u_1[p - 1] + 1) - (\alpha + \gamma_0 + \gamma_1 u_2) - \\ & \frac{1}{2} \left(\sigma \left(1 - \frac{I}{N} \right) (u_1[p - 1] + 1) \right)^2 \end{aligned}$$

Under condition (7), we have

$$\begin{aligned} \log I(t) \leq & \log I(0) + t(\beta(u_1[p - 1] + 1) - (\alpha + \gamma_0 + \gamma_1 u_2)) \\ & - \frac{1}{2}\sigma^2(u_1[p - 1] + 1)^2 + \int_0^t \sigma \frac{N - I}{N}(u_1[p - 1] + 1)dW(s). \end{aligned} \quad (11)$$

This implies

$$\begin{aligned} \limsup_{t \rightarrow \infty} \frac{1}{t} \log I(t) \leq & (\beta(u_1[p - 1] + 1) - (\alpha + \gamma_0 + \gamma_1 u_2)) \\ & - \frac{1}{2}\sigma^2(u_1[p - 1] + 1)^2 + \limsup_{t \rightarrow \infty} \frac{1}{t} \int_0^t \sigma \frac{N - I}{N}(u_1[p - 1] + 1)dW(s) \end{aligned} \quad (12)$$

Based on [8],

$$\lim_{t \rightarrow \infty} \frac{1}{t} W(t) = 0 \quad a.s. \quad (13)$$

Hence, equation (12) becomes

$$\begin{aligned} \limsup_{t \rightarrow \infty} \frac{1}{t} \log I(t) \leq & \beta(u_1[p - 1] + 1) - (\alpha + \gamma_0 + \gamma_1 u_2) - \\ & \frac{1}{2}\sigma^2(u_1[p - 1] + 1)^2 a.s. \end{aligned} \quad (14)$$

Under condition (6), it then follows from (14) that

$$\begin{aligned} \limsup_{t \rightarrow \infty} \frac{1}{t} \log I(t) \leq & \beta(u_1[p - 1] + 1) - (\alpha + \gamma_0 + \gamma_1 u_2) - \\ & \frac{1}{2}(\sigma(u_1[p - 1] + 1))^2 < 0 \quad a.s. \end{aligned} \quad (15)$$

□

Compared to deterministic reproduction number which reach the extinction of Influenza if and only if $R_0 \leq 1$, ρ_0 in Theorem 1 needs an additional term, namely $\sigma^2(u_1[p-1] + 1)^2 \leq \beta(u_1[p-1] + 1)$. However, by listing all possible conditions that make $\rho_0 \leq 1$, we can prove the extinction of disease which is analogous with R_0 of deterministic model. Such conditions are discussed below.

Theorem 2. *If*

$$\sigma^2(u_1[p-1] + 1)^2 > \max(\beta(u_1[p-1] + 1), \frac{\beta^2(u_1[p-1] + 1)^2}{2(\alpha + \gamma_0 + \gamma_1 u_2)}), \quad (16)$$

then the value of stochastic threshold parameter $\rho_0 \leq 1$ and for any initial value $I(0) = I_0 \in (0, N)$, the solution of SDE (3b) obeys

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \log I(t) \leq \frac{1}{2} \frac{\beta^2}{\sigma^2} - (\alpha + \gamma_0 + \gamma_1 u_2) < 0 \quad a.s. \quad (17)$$

namely, $I(t)$ tends to zero exponentially almost surely, i.e. the disease dies out with probability one.

Proof. Consider condition (16) into two separate cases, that is Case 1.

$$\sigma^2(u_1[p-1] + 1)^2 > \beta(u_1[p-1] + 1), \quad (18)$$

which implies

$$\beta(u_1[p-1] + 1) \geq \frac{\beta^2(u_1[p-1] + 1)^2}{2(\alpha + \gamma_0 + \gamma_1 u_2)}. \quad (19)$$

The equation (18) can be written as

$$\frac{\beta(u_1[p-1] + 1)}{2(\alpha + \gamma_0 + \gamma_1 u_2)} \leq 1. \quad (20)$$

By considering case 1 in (18), ρ_0 in equation (5) can be written as

$$\rho_0 \leq \frac{\beta(u_1[p-1] + 1)}{(\alpha + \gamma_0 + \gamma_1 u_2)} - \frac{\beta(u_1[p-1] + 1)}{2(\alpha + \gamma_0 + \gamma_1 u_2)} = \frac{\beta(u_1[p-1] + 1)}{2(\alpha + \gamma_0 + \gamma_1 u_2)}, \quad (21)$$

and by using equation (20), the last equation becomes

$$\rho_0 \leq \frac{\beta(u_1[p-1] + 1)}{2(\alpha + \gamma_0 + \gamma_1 u_2)} \leq 1.$$

Case 2.

$$\sigma^2(u_1[p-1] + 1)^2 > \frac{\beta^2(u_1[p-1] + 1)^2}{2(\alpha + \gamma_0 + \gamma_1 u_2)}, \quad (22)$$

which implies

$$\frac{\beta^2(u_1[p-1] + 1)^2}{2(\alpha + \gamma_0 + \gamma_1 u_2)} \geq \beta(u_1[p-1] + 1) \quad (23)$$

The proof is analogue, thus both cases show that $\rho_0 \leq 1$. Furthermore, summoning the function (11) and equation (18), it takes its maximum value $f(I_*)$ at the stationary point

$$I_* = N \left(1 - \frac{\beta}{\sigma^2(u_1[p-1] + 1)} \right) \in (0, N). \quad (24)$$

From equation (22), the maximum value of function (11) is

$$f(I_*) = \frac{1}{2} \frac{\beta^2}{\sigma^2} - (\alpha + \gamma_0 + \gamma_1 u_2) < 0. \quad (25)$$

Recall the log function (10) and use equation (25) together with equation (13) implies

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \log I(t) \leq \frac{1}{2} \frac{\beta^2}{\sigma^2} - (\alpha + \gamma_0 + \gamma_1 u_2) < 0 \quad a.s.$$

□

Now the deterministic and stochastic threshold parameter are analogous, that is the disease vanishes when the threshold parameter value is less or equal to one. The following theorem discusses the persistence of Influenza in population.

Theorem 3. *If $\rho_0 > 1$, then for any initial value $I(0) = I_0 \in (0, N)$, the solution of SDE (3b) obeys*

$$\liminf_{t \rightarrow \infty} I(t) \leq I_\xi \leq \limsup_{t \rightarrow \infty} I(t) \quad a.s. \quad (26)$$

where

$$I_\xi = \frac{N \left(\sqrt{\beta^2 - 2\sigma^2(\alpha + \gamma_0 + \gamma_1 u_2)} - (\beta - \sigma^2(u_1[p-1] + 1)) \right)}{\sigma^2(u_1[p-1] + 1)}. \quad (27)$$

Proof. Before proving further, we bring forth I_ξ in equation (27) first by considering the function $f(I)$. Note that $f(I)$ in equation (11) have positive and negative root and we only consider the positive root. Under condition that $\rho_0 > 1$, it is found that I_ξ is the unique positive root of $f(I)$ and $f(0) > 0$. In addition, due to the positive values of $\alpha, \gamma_0, \gamma_1$ and u_2 , $f(N) = -(\alpha + \gamma_0 + \gamma_1 u_2) < 0$. Furthermore, by comparing I_ξ to the stationary point I_* , it is easy to see that $I_* < I_\xi$. which makes $f(I_*)$ possibly the candidate of maximum value of $f(I)$ at

$$f(I_*) = \frac{\beta^2}{2\sigma^2} - (\alpha + \gamma_0 + \gamma_1 u_2) > 0. \quad (28)$$

It follows that if $f(I_*) \geq f(0)$,

$$f(I) > 0 \quad \text{increasing function on } I \in (0, I_*), \quad (29)$$

$$f(I) > 0 \quad \text{strictly decreasing function on } I \in (I_*, I_\xi), \quad (30)$$

and

$$f(I) < 0 \quad \text{strictly decreasing function on } I \in (I_\xi, N). \quad (31)$$

Otherwise if $f(0) \geq f(I_*)$,

$$f(I) > 0 \quad \text{decreasing function on } I \in (0, I_\xi), \quad (32)$$

and

$$f(I) < 0 \quad \text{strictly decreasing function on } I \in (I_\xi, N). \quad (33)$$

To show assertion (26), first we prove that

$$\liminf_{t \rightarrow \infty} I(t) \leq I_\xi \quad (34)$$

is true. Suppose that (34) is not true, that is $\liminf_{t \rightarrow \infty} I(t) > I_\xi$ a.s., then there is $\delta \in (0, 1)$ such that $\mathbb{P}(\Omega_1) > \delta$ where $\Omega_1 = \{\liminf_{t \rightarrow \infty} I(t) \geq I_\xi + 2\delta\}$. That is, for all $\omega \in \Omega_1$ there is $t = \tau > 0$ such that

$$I(t) \geq I_\xi + \delta \quad \text{as } t \geq \tau. \quad (35)$$

From (31), (33), and (35), we can say

$$f(I(t)) \leq f(I_\xi + \delta) < f(I_\xi) = 0 \quad \text{as } t \geq \tau. \quad (36)$$

Recalling (10), it then follows that

$$\log I(t) \leq \log I(0) + \int_0^\tau f(I(s))ds + f(I_\xi + \delta)(t - \tau) + \int_0^t \sigma \frac{(N - I)}{N} (u_1[p - 1] + 1) dW(t). \quad (37)$$

Using equation (13) and (36), equation (37) becomes

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \log I(t) \leq f(I_\xi + \delta) < 0 \quad a.s. \quad (38)$$

Based on [8], inequality (38) implies that

$$\lim_{t \rightarrow \infty} I(t) = 0. \quad (39)$$

This contradicts (35). Therefore, (34) must be true. Furthermore, to prove that $I_\xi \leq \limsup_{t \rightarrow \infty} I(t)$ *a.s.*, we go through similar manner. $\square$

Note that level of I_ξ is the endemic level for $I(t)$ in stochastic model. By definition of R_0 in (4), ρ_0 now can be written as follows.

$$\rho_0 = \frac{\beta(u_1[p-1] + 1)}{(\alpha + \gamma_0 + \gamma_1 u_2)} - \frac{\sigma^2(u_1[p-1] + 1)^2}{2(\alpha + \gamma_0 + \gamma_1 u_2)} = R_0 - \frac{\sigma^2(u_1[p-1] + 1)^2}{2(\alpha + \gamma_0 + \gamma_1 u_2)}. \quad (40)$$

If $R_0 > 1$, there exist certain value of σ such that $\rho_0 < 1$. It means that while deterministic model indicates persistence of a disease, the stochastic model may indicate the extinction of a disease, which also stated in [5].

3. Results

3.1. Sensitivity Analysis of ρ_0

The sensitivity analysis is presented to show the change of ρ_0 value when some parameters are changed, while the rest held constant. In this session, we observe how the change of parameter u_1 and u_2 affect the change of ρ_0 value in Figure 2 (for two cases, let call it as small ($\sigma = 0.05$) and large ($\sigma = 0.15$) stochastic environmental factor). The constant parameters and their description is given in Table 1 (except it is stated otherwise).

In Figure 2, each coloured line presents the magnitude of ρ_0 with respect to u_1 and u_2 values. Even though both figures show similar characteristic that is enlarging u_1 and u_2 will reduce ρ_0 , note that between Figure 2 (a) and (b), there is a significant shift in ρ_0 values. Suppose σ is small enough ($\sigma = 0.05$) in (a), if nobody gets drug treatment ($u_2 = 0$), then in order to become extinct, that is $\rho_0 \leq 1$, we need more than 30% infected individuals to wear medical mask. Whereas with bigger value of σ in (b), we only need at least 22% infected individuals to wear medical mask so that the disease dies out.

Next sensitivity analysis in Figure 3 is given to compare the value of u_1 and u_2 with respect to R_0 and ρ_0 . Figure 3 consists of three different region :

Parameters	Values	Description
N	100	We assume total population is 100 people
β	$0.1 \frac{\text{person}}{\text{day}}$	It needs 10 successful contacts with infected person to transmit influenza
α	$\frac{1}{71 \times 365 \text{ days}}$	Based on WHO life expectancy [23]
γ_0	$\frac{1}{14 \text{ days}}$	We assume the natural recovery rate for influenza disease is 14 days
γ_1	$\frac{1}{14 \text{ days}}$	We assume that drug treatment will increase recovery rate by a half from $\frac{1}{14}$ to $\frac{1}{7}$. Therefore, we choose $\gamma_1 = \frac{1}{14}$. Please see details in [6].
p	0.1	We assume that medical mask will reduce the risk of getting influenza by 90%.
u_1	$u_1 \in [0, 1]$	The rate of used medical mask each day
u_2	$u_2 \in [0, 1]$	The rate of drug treatment intervention each day

Table 1: Parameters description

1. Region 1 . In this region, both deterministic and stochastic model present the same result i.e disease will dies out. It is shown by the magnitude of R_0 and ρ_0 are equal or smaller than one.
2. Region 2 . In this region, a contradiction result between deterministic and stochastic model is obtained, where $R_0 > 1$ but in the other hand $\rho_0 < 1$. This result presents that the stochastic environment has an important role in determining the existence of disease.
3. Region 3 . In this region, both deterministic and stochastic model present the same result i.e disease will exist. It is shown by the magnitude of R_0 and ρ_0 are larger than one.

As a consequence of the environment perturbation, we can find $\rho_0 < 1$ but in the other hand $R_0 > 1$ with the same used parameter values. In other words, we can find some situations when deterministic and SDE model will gave a different result about the coexistence of influenza disease depend on how large is the stochastic environment factor.

Figure 2: Sensitivity analysis of ρ_0 with respect to u_1 and u_2 for (a) $\sigma = 0.05$ and (b) $\sigma = 0.15$

3.2. Numerical Simulations

In this subsection, some numerical simulations are shown to give an interpretation about analytical results discussed in previous section. There are some cases to discuss as follows.

Simulating the deterministic model (1), we discuss the prevention and reduction of an Influenza outbreak that is shown in Figure 5. We then separate the case where an outbreak just happen, that is the initial number of infected person is above the endemic level (see Figure 5 (a) in which we analyse the endemic reduction plan using medical mask and drug), and another case where an outbreak does not happen yet that is depicted in Figure 5 (b) which shows the endemic prevention using the same method.

We found that in the Figure 5 (a), medical mask can reduce the disease up to 86% of the infected population, while drug treatment can only bring down 74% of it. Figure 5 (b) releases similar result, that is, medical mask can reduce the disease up to 98% of the infected population, while drug treatment can only bring down 84%. From both figure above, we see that preventing and reducing the disease using medical mask is more effective than treatment with drugs.

Next, we simulate the stochastic model (3b). The first simulation is performed to show an initial situation when there are no interventions given to the population ($u_1 = u_2 = 0$). We assume that $R_0 > 1$ and $\rho_0 > 1$ in $t = 0$. For this purpose, parameters value are given in table 1. It is important to stress that the mortality and recovery parameter values (α , γ_0 and γ_1) remain the same, while the infection rate β and the intervention parameter values (u_1 and u_2)

Figure 3: Comparison of sensitivity between u_1 and u_2 for R_0 and ρ_0 .

are changed in order to explore the behaviour of the Influenza spread.

Figure 6 generally show that the trend of the stochastic solutions agree with the corresponding deterministic solutions. Note that the number of infected individuals starts small and continuously rise up to an endemic level. As a consequence, the number of susceptible individuals becomes less than its initial value as t increases. Observe that Figure 6 (right figure) suggests that as the infection rate is higher, sat a certain time t the number of infected individuals is larger than the susceptible.

Second, the case where $R_0 < 1$ and $\rho_0 < 1$. This case considers the intervention of medical mask and drug treatment to a half of population, that is $u_1 = u_2 = 0.5$. By using interventions, some significant changes from the first simulation are served in Figure 7. The Influenza disease dies out starting from certain time and the whole population become susceptible. To investigate the effect of varying the infection rate on our model, β was given different sets of values. We see that the disease disappears sooner with lower rate of infection. Lastly, the case where $R_0 > 1$ and $\rho_0 < 1$. Using parameter value $\beta = 0.143$, Figure 8 depict the contradictory situation between deterministic and stochastic solutions. Figure 8 (a), where $R_0 = 1.01$ and $\rho_0 = 0.99$, the deterministic graph shows that in the long term the infected individuals do not go away from the population at endemic equilibrium $I_\xi = 1.8$ while the stochastic solution

Figure 5: Endemic reduction and prevention

Figure 6: Stochastic and deterministic solutions for persistence condition with $\beta = 0.1$ (left) and $\beta = 0.15$ (right) .

show that the disease will eventually vanish. When compared to Figure 8 (b), which has a greater value of R_0 but a smaller ρ_0 ($R_0 = 1.24, \rho_0 = 0.94$), shows the disease will persist at a bigger endemic level $I_\xi = 19$ by deterministic solution while the stochastic solution demonstrates a much sooner extinction of disease than Figure 8 (a).

4. Conclusions

In this paper, we have constructed a stochastic model for Influenza spread with medical mask u_1 and treatment u_2 intervention. From analysis of stochastic threshold parameter (ρ_0), we find that enlarging u_1 and u_2 will reduce ρ_0 . The disease will extinct if $\rho_0 < 1$ and persist otherwise. The disease will extinct sooner if the value of ρ_0 smaller. We also find that while deterministic model indicates persistence of a disease, the stochastic model may indicate the

Figure 7: Stochastic and deterministic solution with (a) $\beta = 0.1$ (b) $\beta = 0.15$

Figure 8: Stochastic and deterministic solution with (a) $\sigma = 0.1, u_1 = u_2 = 0.35$ (b) $\sigma = 0.3, u_1 = u_2 = 0.25$

extinction of a disease.

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