SOME REMARKS ON SOFT SEPARATION AXIOMS

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Abstract: The paper deals with various forms of soft separation axioms in soft topological spaces. The main goal is to point out any soft topological space is homeomorphic to a topological space with topology on Cartesian product, consequently many soft topological notions and results can be derived from general topology.

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1. Introduction

Despite the reservations to exaggerated expectations in soft theory research, there are still many non-systemic results in the case of soft topological spaces. In the next, we will try to clarify some inconsistencies and explain the issue from general topology point of view. The purpose of the article is to point out possible ways of research of soft topological spaces in such a way that research corresponds to common procedures known from general topology.

We recall the basic concept related to soft sets, soft topological spaces and soft separation axioms. For details see references.

Let X, A be nonempty sets. A pair (F, A) is called a soft set over X with respect to a set of parameters A , where $F : A \rightarrow 2^X$ is a set valued mapping from A to the power set 2^X of X . The soft union (soft intersection) of two

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soft sets $(F, A), (G, A)$ is defined as a soft set (H, A) where $H(e) = F(e) \cup G(e)$ ($H(e) = F(e) \cap G(e)$) for all $e \in A$ and the soft complement of (F, A) is a soft set (F^c, A) defined by a set valued mapping $F^c(e) = X \setminus F(e)$ for all $e \in A$. A soft set (F, A) is said to be a null (an absolute) soft set, if $F(e) = \emptyset$ ($F(e) = X$) for any $e \in A$. If a collection τ of soft sets contains the null soft set, the absolute soft set and τ is closed under arbitrary union and finite intersection of soft sets, then τ is called a soft topology and (X, τ, A) is called a soft topological space.

2. Topological View on Soft Separation Axioms

A graph of a set valued mapping $F : A \rightarrow 2^X$ is the set $Gr(F) := \{[e, x] \in A \times X : x \in F(e)\}$ and it is a subset of $A \times X$. Since a set valued mapping can be understood as a subset of the Cartesian product $A \times X$, a soft set over X with respect to a set of parameters A can be introduced as a subset of $A \times X$. Consequently, the basic soft operations (the soft union, the soft intersection, the soft complement) can be defined by the corresponding set operations on the graphs of set valued mappings.

As it has already been pointed out in [7], [8] a soft topological space is nothing else as a topology on $A \times X$. More precisely, if (X, τ, A) is a soft topological space, then $(A \times X, \tau_{A \times X})$ is a topological space, where $\tau_{A \times X} = \{Gr(F) : (F, A) \in \tau\}$. It is clear that for any $e \in A$, (X, τ_e) is a topological space where $\tau_e = \{F(e) : (F, A) \in \tau\}$. On the other hand, if $(A \times X, \sigma)$ is a topological space, then (X, τ, A) is a soft topological space, where $\tau = \{(F_S, A) : S \in \sigma\}$ and $F_S : A \rightarrow 2^X$, $F_S(e) = \{x \in X : [e, x] \in S\}$ for all $e \in A$.

For the purpose of this article we introduce the next terminological definition.

Definition 2.1. Let A, X be nonempty sets. A set $\{e\} \times Y$ ($A \times \{x\}$) is called a vertical set (a horizontal line), $Y \subset X$, $x \in X$, $e \in A$. If $x, y \in X$ are distinct points, X_1, X_2 are disjoint subsets of X and $e \in A$, then the points $[e, x], [e, y]$ are called vertical distinct points, the sets $\{e\} \times X_1$, $\{e\} \times X_2$ are called vertical disjoint sets and the sets $A \times \{x\}$, $A \times \{y\}$ are called horizontal distinct lines.

Recall some notions from general topology. Two distinct points are topologically distinguishable, if at least one of them has a neighborhood that is not a neighborhood of the other. Two points are separated (separated by neighborhoods) if each of them has a neighborhood that is not a neighborhood of the other (if they have disjoint neighborhoods). More generally, two subsets are separated by neighborhoods if they have disjoint neighborhoods.

Next two notions are not commonly used, but for our purpose we introduce them. Two sets are distinguishable (weakly separated), if at least one of them (each of them) has a neighborhood that is not a neighborhood of the other.

The separation axioms are a series of definitions in topology that allow the classification of various topological spaces. The following axioms are typically defined.

Definition 2.2. A topological space is said to be

- (1) T_0 (T_1 , T_2) if any two distinct points are topologically distinguishable (separated, separated by neighborhoods),
- (2) regular if, given any point x and closed set F such that x does not belong to F , they are separated by neighborhoods.

As we will see from the next results all soft topological notions are topological ones and many relationships among soft separation axioms seem to be very trivial. In fact, it is just about rewording soft concepts into topological ones based on correspondence between a set valued mapping and its graph and the proofs are left to the reader. Furthermore, many soft separation axioms separate only specific soft subsets (even not disjoint soft subsets) what is not typical in general topology.

In the next theorems we use notations introduced in [1].

Theorem 2.1. A soft topological space (X, τ, A) is

- (1) soft- T_0^G ([5]) if and only if it is soft- T_0^T ([11]) if and only if for any $e \in A$, (X, t_e) is T_0 ,
- (2) soft- T_0^H ([6]) (soft- T_0^S ([10])) if and only if any two disjoint subsets of X (two horizontal distinct lines) are distinguishable in (X, t_e) for any $e \in A$ (in $(A \times X, \tau_{A \times X})$).

Theorem 2.2. A soft topological space (X, τ, A) is

- (1) soft- T_1^H ([6]) if and only if it is soft- T_1^T ([11]) if and only if for any $e \in A$, (X, t_e) is T_1 ,
- (2) soft- T_1^S ([10]) if and only if any two horizontal distinct lines are weakly separated in $(A \times X, \tau_{A \times X})$.

Theorem 2.3. A soft topological space (X, τ, A) is

- (1) soft- T_2^G ([5]) if and only if for any $e \in A$, (X, t_e) is T_2 ,

- (2) $\text{soft-}T_2^H$ ([6]) ($\text{soft-}T_2^S$ ([10]), $\text{soft-}T_2^T$ ([11])) if and only if any two vertical disjoint sets (horizontal distinct lines, vertical distinct points) are separated by neighborhoods in $(A \times X, \tau_{A \times X})$.

Theorem 2.4. A soft topological space (X, τ, A) is

- (1) soft-regular^T ([11]) ($\text{soft-}T_3^G$ ([5])) if and only if $(A \times X, \tau_{A \times X})$ (for any $e \in A$, (X, t_e)) is regular,
- (2) soft-regular^H ([6]) (soft-regular^S ([10])) if and only if, given any closed set S in $(A \times X, \tau_{A \times X})$ and vertical set (horizontal line) which is not contained in S , they are separated by neighborhoods in $(A \times X, \tau_{A \times X})$.

From the theorems above we can see the soft separation axioms can be characterized in the corresponding topological space $(A \times X, \tau_{A \times X})$ by classical separation axioms mostly applied on specific subsets of $A \times X$ or in (X, t_e) .

It is even natural to ask about the usefulness of introduced soft terms that may seem very questionable unless it is supported by applications. Since soft topology is nothing else but a topology on Cartesian product, does it make sense to talk about a new theory? Furthermore some soft separation axioms separate two subsets which are not disjoint they seem to be very strange. It would be better to define the soft separation axioms in accordance with general topology. Last but not least, the formal and stylistic aspects of soft theory are too complicated.

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