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COMMON FIXED POINT RESULTS FOR RATIONAL TYPE CONTRACTION IN COMPLEX VALUED b-METRIC SPACES

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Abstract: In this paper, we present some results of fixed point theory in a recently introduced generalization of the metric space, that is, complex valued b-metric space where the metric assumes values in the set of complex number. The presented theorems extend and generalize the well known results in the literature.

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Key Words: common fixed point, complete complex valued b-metric space, Cauchy sequence

1. Introduction

The concept of a complex valued metric space which is a generalization of the classical metric space was recently introduced by Azam et. al.[1]. Later, a number of articles in this field have been dedicated to the improvement and generalization of [1] in several ways (See [8], [9], [10], [11] and [12]).

In 2013, Rao et. al. [10] introduced the concept of complex valued b-metric space which was more general than the well known complex valued metric spaces[1]. In sequel, A.A. Mukheimer [8] proved the results of common fixed

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point satisfying a rational inequality on complex valued b-metric spaces. Fixed point theory has been studied in this space in a suitable number of papers, some of which we mention in ([2] to [7]).

In this paper, we continue the study of common fixed point theorems in complex valued b-metric spaces. Our results extend and generalize the results [3], [4], [5] & [11].

2. Preliminaries

We begin with a description of complex valued b-metric space.

Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$.Define a partial order $\precsim$ on $\mathbb{C}$ as follows:

$$z_1 \precsim z_2 \text{ if and only if } Re(z_1) \leq Re(z_2), Im(z_1) \leq Im(z_2).$$

Thus $z_1 \precsim z_2$ if one of the following holds:

(i) $Re(z_1) = Re(z_2), Im(z_1) = Im(z_2)$;

(ii) $Re(z_1) < Re(z_2), Im(z_1) = Im(z_2)$;

(iii) $Re(z_1) = Re(z_2), Im(z_1) < Im(z_2)$;

(iv) $Re(z_1) < Re(z_2), Im(z_1) < Im(z_2)$.

In particular, we write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of (ii),(iii) and (iv) is satisfied and we write $z_1 \prec z_2$ if only (iv) is satisfied. Notice that $0 \precsim z_1 \precnsim z_2 \Rightarrow |z_1| < |z_2|$ and $z_1 \precsim z_2$, $z_2 \prec z_3 \Rightarrow z_1 \prec z_3$.

The following definition is recently introduced by Rao et al. [10].

Definition 2.1. (see [10]) *Let X be a nonempty set and let $s \geq 1$ be a given real number. A function $d : X \times X \to \mathbb{C}$ is called complex valued b-metric on X if for all $x, y, z \in X$ the following conditions are satisfied :*

(i) $0 \precsim d(x, y)$ *and* $d(x, y) = 0$ *if and only if* $x = y$;

(ii) $d(x, y) = d(y, x)$;

(iii) $d(x, y) \precsim s[d(x, z) + d(z, y)]$.

The *pair (X, d) is called a complex valued b-metric space.*

Definition 2.2. (see [10]) Let $X = [0, 1]$. Define the mapping $d : X \times X \to \mathbb{C}$ by $d(x, y) = |x - y|^2 + i|x - y|^2$, for all $x, y \in X$. Then (X, d) is a complex valued b-metric space with $s = 2$.

Definition 2.3. (see [10]) Let (X, d) be a complex valued b-metric space.

(i) A point $x \in X$ is called interior point of a set $A \subseteq X$ whenever there exists $0 \prec r \in \mathbb{C}$ such that $B(x, r) = \{y \in X : d(x, y) \prec r\} \subseteq A$.

(ii) A point $x \in X$ is called a limit point of a set A whenever for every $0 \prec r \in \mathbb{C}, B(x, r) \cap (A - \{x\}) \neq \phi$.

(iii) A subset $A \subseteq X$ is called an open set whenever each element of A is an interior point of a set A.

(iv) A subset $A \subseteq X$ is called closed set whenever each limit point of A belongs to A.

(v) A sub-basis for Hausdorff topology τ on X is a family $F = \{B(x, r) : x \in X \, and \, 0 \prec r\}$.

Definition 2.4. (see [10]) Let (X, d) be a complex valued b-metric space and let $\{x_n\}$ be a sequence in X and $x \in X$.

(i) If for every $c \in \mathbb{C}$, with $0 \prec c$ there is $N \in \mathbb{N}$ such that for all $n > N$, $d(x_n, x) \prec c$, then $\{x_n\}$ is said to be convergent and converges to x. We denote this by $lim_{n\to\infty} x_n = x$ or $\{x_n\} \to x \, as \, n \to \infty$.

(ii) If for every $c \in \mathbb{C}$, with $0 \prec c$ there is $N \in \mathbb{N}$ such that for all $n > N$, $d(x_n, x_{n+m}) \prec c$, where $m \in \mathbb{N}$, then $\{x_n\}$is said to be Cauchy sequence.

(iii) If every Cauchy sequence in X is convergent in X, then (X, d) is said to be complete complex valued b-metric space.

Lemma 2.5. (see [10]) *Let (X, d) be a complex valued b-metric space and let $\{x_n\}$be a sequence in X. Then $\{x_n\}$converges to x if and only if $|d(x_n, x)| \to 0$ as $n \to \infty$.*

Lemma 2.6. (see [10]) *Let (X, d) be a complex valued b-metric space and let $\{x_n\}$be a sequence in X. Then $\{x_n\}$ is Cauchy sequence if and only if $|d(x_n, x_{n+m})| \to 0$ as $n \to \infty$, where $m \in \mathbb{N}$.*

3. Main Result

Theorem 3.1. *Let (X, d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let $S, T : X \to X$ be mappings satisfying:*

$$d(Sx, Ty) \precsim \alpha d(x, y) + \beta[d(x, Sx) + d(y, Ty)] + \gamma[d(y, Sx) + d(x, Ty)]$$

$$+ \delta \frac{d(y,Ty)[1+d(x,Sx)]}{1+d(x,y)} + \lambda \frac{d(y,Sx)[1+d(x,Ty)]}{1+d(x,y)}$$
$$+ \mu \frac{d(x,y)[1+d(x,Sx)+d(y,Sx)]}{1+d(x,y)} + Ld(y,Sx), \tag{3.1}$$

for all $x, y \in X$, where $\alpha, \beta, \gamma, \delta, \lambda, \mu$ and L are nonnegative reals with $\alpha + 2\beta + 2s\gamma + \delta + \mu < 1$. Then S and T have a unique common fixed point in X.

Proof. For any arbitrary point $x_0 \in X$, define sequence $\{x_n\}$ in X such that

$$\begin{aligned} x_{2n+1} =& Sx_{2n}, \\ x_{2n+2} =& Tx_{2n+1}, \text{ for } n = 0, 1, 2, 3. \end{aligned} \tag{3.2}$$

Now, we show that the sequence $\{x_n\}$ is a Cauchy sequence.

Let $x = x_{2n}$ and $y = x_{2n+1}$ in (3.1), we have

$$\begin{aligned} d(Sx_{2n}, Tx_{2n+1}) =& d(x_{2n+1}, x_{2n+2}) \\ \precsim& \alpha d(x_{2n}, x_{2n+1}) + \beta[d(x_{2n}, Sx_{2n}) + d(x_{2n+1}, Tx_{2n+1})] \\ &+ \gamma[d(x_{2n+1}, Sx_{2n}) + d(x_{2n}, Tx_{2n+1})] \\ &+ \delta \frac{d(x_{2n+1}, Tx_{2n+1})[1 + d(x_{2n,} Sx_{2n})]}{1 + d(x_{2n}, x_{2n+1})} \\ &+ \lambda \frac{d(x_{2n+1}, Sx_{2n})[1 + d(x_{2n,} Tx_{2n+1})]}{1 + d(x_{2n}, x_{2n+1})} \\ &+ \mu \frac{d(x_{2n}, x_{2n+1})[1 + d(x_{2n,} Sx_{2n}) + d(x_{2n+1}, Sx_{2n}]}{1 + d(x_{2n}, x_{2n+1})} \\ &+ Ld(x_{2n+1}, Sx_{2n}) \\ =& \alpha d(x_{2n}, x_{2n+1}) + \beta[d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})] \\ &+ \gamma[d(x_{2n+1}, x_{2n+1}) + d(x_{2n}, x_{2n+2})] \\ &+ \delta \frac{d(x_{2n+1}, x_{2n+2})[1 + d(x_{2n}, x_{2n+1})]}{1 + d(x_{2n}, x_{2n+1})} \\ &+ \lambda \frac{d(x_{2n+1}, x_{2n+1})[1 + d(x_{2n}, x_{2n+2})]}{1 + d(x_{2n}, x_{2n+1})} \\ &+ \mu \frac{d(x_{2n}, x_{2n+1})[1 + d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+1})]}{1 + d(x_{2n}, x_{2n+1})} \\ &+ Ld(x_{2n+1}, x_{2n+1}) \\ \precsim& \alpha d(x_{2n}, x_{2n+1}) + \beta[d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})] \\ &+ s\gamma[d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})] + \delta d(x_{2n+1}, x_{2n+2}) \\ &+ \mu d(x_{2n}, x_{2n+1}), \end{aligned}$$

which implies that

$$\begin{aligned}|d(x_{2n+1},x_{2n+2})| \leq & \alpha|d(x_{2n},x_{2n+1})| + \beta|d(x_{2n},x_{2n+1}) + d(x_{2n+1},x_{2n+2})| \\ & + s\gamma|d(x_{2n},x_{2n+1}) + d(x_{2n+1},x_{2n+2})| \\ & + \delta|d(x_{2n+1},x_{2n+2})| + \mu|d(x_{2n},x_{2n+1})| \\ & |d(x_{2n+1},x_{2n+2})| \leq h|d(x_{2n},x_{2n+1})|, \qquad (3.3)\end{aligned}$$

where $h = \frac{\alpha+\beta+s\gamma+\mu}{1-\beta-s\gamma-\delta} < 1$.

Similarly, we have

$$|d(x_{2n+2},x_{2n+3})| \leq h|d(x_{2n+1},x_{2n+2})|, \qquad (3.4)$$

where $h = \frac{\alpha+\beta+s\gamma+\mu}{1-\beta-s\gamma-\delta} < 1$.

Thus for all $n \geq 0$ and consequently, we get

$$\begin{aligned}|d(x_{n+1},x_{n+2})| \leq h|d(x_n,x_{n+1})| \leq h^2|d(x_{n-1},x_n)| \\ \leq \cdots \leq h^{n+1}|d(x_0,x_1)|. \qquad (3.5)\end{aligned}$$

Thus for any $m > n, m, n \in \mathbb{N}$,

$$\begin{aligned}|d(x_n,x_m)| \leq & s|d(x_n,x_{n+1})| + s|d(x_{n+1},x_m)| \\ \leq & s|d(x_n,x_{n+1})| + s^2|d(x_{n+1},x_{n+2})| + s^2|d(x_{n+2},x_m)| \\ \leq & s|d(x_n,x_{n+1})| + s^2|d(x_{n+1},x_{n+2})| + s^3|d(x_{n+2},x_{n+3})| \\ & + s^3|d(x_{n+3},x_m)| \\ \leq & s|d(x_n,x_{n+1})| + s^2|d(x_{n+1},x_{n+2})| + s^3|d(x_{n+2},x_{n+3})| \\ & + \cdots + s^{m-n-1}|d(x_{m-2},x_{m-1})| + s^{m-n}|d(x_{m-1},x_m)|.\end{aligned}$$

By (3.5), we get

$$\begin{aligned}|d(x_n,x_m)| \leq & sh^n|d(x_0,x_1)| + s^2h^{n+1}|d(x_0,x_1)| \\ & + s^3h^{n+2}|d(x_0,x_1)| + \cdots + s^{m-n-1}h^{m-2}|d(x_0,x_1)| \\ & + s^{m-n}h^{m-1}|d(x_0,x_1)| \\ = & \sum_{i=1}^{m-n} s^i h^{i+n-1}|d(x_0,x)|.\end{aligned}$$

Therefore,

$$|d(x_n,x_m)| \leq \sum_{i=1}^{m-n} s^{i+n-1}h^{i+n-1}|d(x_0,x_1)|$$

$$=\sum_{t=n}^{m-1} s^t h^t |d(x_0,x_1)|$$
$$\leq \sum_{t=n}^{\infty} (sh)^t |d(x_0,x_1)| = \frac{(sh)^n}{1-sh}|d(x_0,x_1)|$$

and hence

$$|d(x_n,x_m)| \leq \frac{(sh)^n}{1-sh}|d(x_0,x_1)| \to 0 \, as \, m,n \to \infty.$$

Thus $\{x_n\}$is a Cauchy sequence in X. Since X is complete, there exists some $u \in X$ such that $x_n \to u$ as $n \to \infty$.

Assume not, then there exists $z \in X$ such that

$$|d(u,Su)| = |z| > 0. \tag{3.6}$$

So by using the triangular inequality and (3.1), we receive

$$\begin{aligned}
z = d(u,Su) &\precsim sd(u,x_{2n+2}) + sd(x_{2n+2},Su) \\
&= sd(u,x_{2n+2}) + sd(Su,Tx_{2n+1}) \\
&\precsim sd(u,x_{2n+2}) + s \propto d(u,x_{2n+1}) + s\beta[d(u,Su) \\
&\quad + d(x_{2n+1},Tx_{2n+1})] \\
&\quad + s\gamma[d(x_{2n+1},Su) + d(u,Tx_{2n+1})] \\
&\quad + s\delta\frac{d(x_{2n+1},Tx_{2n+1})[1+d(u,Su)]}{1+d(u,x_{2n+1})} \\
&\quad + s\lambda\frac{d(x_{2n+1},Su)[1+d(u,Tx_{2n+1})]}{1+d(u,x_{2n+1})} \\
&\quad + s\mu\frac{d(u,x_{2n+1})[1+d(u,Su)+d(x_{2n+1},Su)]}{1+d(u,x_{2n+1})} + sLd(x_{2n+1},Su) \\
&= sd(u,x_{2n+2}) + s \propto d(u,x_{2n+1}) + s\beta[d(u,Su) + d(x_{2n+1},x_{2n+2})] \\
&\quad + s\gamma[d(x_{2n+1},Su) + d(u,x_{2n+2})] \\
&\quad + s\delta\frac{d(x_{2n+1},x_{2n+2})[1+d(u,Su)]}{1+d(u,x_{2n+1})} \\
&\quad + s\lambda\frac{d(x_{2n+1},Su)[1+d(u,x_{2n+2})]}{1+d(u,x_{2n+1})} \\
&\quad + s\mu\frac{d(u,x_{2n+1})[1+d(u,Su)+d(x_{2n+1},Su)]}{1+d(u,x_{2n+1})}
\end{aligned}$$

$$+ sLd(x_{2n+1}, Su),$$

which implies that

$$\begin{aligned}|z| =&|d(u,Su)|\\ \leq& s|d(u,x_{2n+2})| + s \propto |d(u,x_{2n+1})|\\ &+ s\beta|d(u,Su) + d(x_{2n+1},x_{2n+2})|\\ &+ s\gamma|d(x_{2n+1},Su) + d(u,x_{2n+2})|\\ &+ s\delta\frac{|d(x_{2n+1},x_{2n+2})||1+d(u,Su)|}{|1+d(u,x_{2n+1})|}\\ &+ s\lambda\frac{|d(x_{2n+1},Su)||1+d(u,x_{2n+2})|}{|1+d(u,x_{2n+1})|}\\ &+ s\mu\frac{|d(u,x_{2n+1})||1+d(u,Su)+d(x_{2n+1},Su)|}{|1+d(u,x_{2n+1})|}\\ &+ sL|d(x_{2n+1},Su)|. \end{aligned} \tag{3.7}$$

Taking the limit of (3.7) as $n \to \infty$, we get that $|z| = |d(u, Su)| \leq 0$, a contradiction with (3.6). So $|z| = 0$. Hence $Su = u$, Similarly, one can also show that $Tu = u$.

To prove the uniqueness of common fixed, let $u^* \in X$ be another common fixed point of S and T that is

$$u^* = Su^* = Tu^*.$$

Then

$$\begin{aligned}d(u,u^\star) =&d(Su,Tu^\star)\\ \precsim& \propto d(u,u^\star) + \beta[d(u,Su) + d(u^\star,Tu^\star)] + \gamma[d(u^*,Su) + d(u,Tu^\star)]\\ &+ \delta\frac{d(u^*,Tu^\star)[1+d(u,Su)]}{1+d(u,u^*)} + \lambda\frac{d(u^*,Su)[1+d(u,Tu^*)]}{1+d(u,u^*)}\\ &+ \mu\frac{d(u,u^*)[1+d(u,Su)+d(u^*,Su)]}{1+d(u,u^*)} + Ld(u^*,Su)\\ \precsim&\alpha d(u,u^*) + \gamma[d(u^*,u) + d(u,u^*)] + \lambda d(u^*,u)\\ &+ \mu d(u,u^*) + Ld(u,u^*)\\ =&(\alpha + 2\gamma + \lambda + \mu + L)d(u,u^*)\end{aligned}$$

so that $|d(u,u^\star)| \leq (\alpha + 2\gamma + \lambda + \mu + L)|d(u,u^*)|$, a contradiction, so that $u = u^*$. This completes the proof.

Corollary 3.2. *Let (X, d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let $T : X \to X$ be a mapping satisfying:*

$$\begin{aligned} d(Tx, Ty) \precsim & \alpha d(x, y) + \beta[d(x, Tx) + d(y, Ty)] + \gamma[d(y, Tx) + d(x, Ty)] \\ & + \delta \frac{d(y, Ty)[1 + d(x, Tx)]}{1 + d(x, y)} + \lambda \frac{d(y, Tx)[1 + d(x, Ty)]}{1 + d(x, y)} \\ & + \mu \frac{d(x, y)[1 + d(x, Tx) + d(y, Tx)]}{1 + d(x, y)} + Ld(y, Tx), \end{aligned} \tag{3.8}$$

for all $x, y \in X$, where $\propto, \beta, \gamma, \delta, \lambda, \mu$ and L are non-negative reals with $\propto +2\beta + 2s\gamma + \delta + \mu < 1$. Then the map T has a unique fixed point in X.

Proof. We can prove this result by applying Theorem 3.1 with $S = T$.

Corollary 3.3. *Let (X, d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let $T : X \to X$ be a mapping satisfying (for some fixed n):*

$$\begin{aligned} d(T^n x, T^n y) \precsim & \alpha d(x, y) + \beta[d(x, T^n x) + d(y, T^n y)] + \gamma[d(y, T^n x) + d(x, T^n y)] \\ & + \delta \frac{d(y, T^n y)[1 + d(x, T^n x)]}{1 + d(x, y)} + \lambda \frac{d(y, T^n x)[1 + d(x, T^n y)]}{1 + d(x, y)} \\ & + \mu \frac{d(x, y)[1 + d(x, T^n x) + d(y, T^n x)]}{1 + d(x, y)} + Ld(y, T^n x), \end{aligned} \tag{3.9}$$

for all $x, y \in X$, where $\propto, \beta, \gamma, \delta, \lambda, \mu$ and L are non-negative reals with $\propto +2\beta + 2s\gamma + \delta + \mu < 1$. Then the map T has a unique fixed point in X.

Proof. From Corollary 3.2, we obtain that $u \in X$ such that $T^n u = u$. The uniqueness follows from

$$\begin{aligned} d(Tu, u) =& d(TT^n u, T^n u) = d(T^n Tu, T^n u) \\ \precsim & \propto d(Tu, u) + \beta[d(Tu, T^n Tu) + d(u, T^n u)] \\ & + \gamma[d(u, T^n Tu) + d(Tu, T^n u)] \\ & + \delta \frac{d(u, T^n u)[1 + d(Tu, T^n Tu)]}{1 + d(Tu, u)} + \lambda \frac{d(u, T^n Tu)[1 + d(Tu, T^n u)]}{1 + d(Tu, u)} \\ & + \mu \frac{d(Tu, u)[1 + d(Tu, T^n Tu) + d(u, T^n Tu)]}{1 + d(Tu, u)} + Ld(u, T^n Tu) \\ = & \propto d(Tu, u) + \beta[d(Tu, Tu) + d(u, u)] + \gamma[d(u, Tu) + d(Tu, u)] \\ & + \delta \frac{d(u, u)[1 + d(Tu, Tu)]}{1 + d(Tu, u)} \end{aligned}$$

$$
\begin{aligned}
&+\lambda\frac{d(u,Tu)[1+d(Tu,u)]}{1+d(Tu,u)}\\
&+\mu\frac{d(Tu,u)[1+d(Tu,Tu)+d(u,Tu)]}{1+d(Tu,u)}+Ld(u,Tu)\\
\precsim&\propto d(Tu,u)+2\gamma d(Tu,u)+\lambda d(Tu,u)+\mu d(Tu,u)+Ld(Tu,u)\\
=&(\alpha+2\gamma+\lambda+\mu+L)d(Tu,u).
\end{aligned}
\tag{3.10}
$$

Taking modulus of (3.10) we get,

$$|d(Tu,u)| \leq (\alpha+2\gamma+\lambda+\mu+L)|d(Tu,u)|.$$

Since $|1+d(Tu,u)| \geq |d(Tu,u)|$, therefore,

$$|d(Tu,u)| \leq (\alpha+2\gamma+\lambda+\mu+L)|d(Tu,u)|,$$

a contradiction.

So $Tu=u$, Hence $Tu=T^n u=u$. Therefore, the fixed point of T is unique. This completes the proof.

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