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THE VARIATION OF THE FIRST EIGENVALUE OF THE LAPLACE OPERATOR AND THE PROBLEM OF LOCATING AN OBSTACLE

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Abstract: By using the derivation and variation of the first eigenvalue of the Laplace operator and the reflection properties we show that if B is a obstacle that moves inside Ω, then the first eigenvalue of the Laplace operator λ_1, is minimal when the obstacle touches the boundary of the domain Ω.

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1. Introduction

The problem of locating an obstacle to the fundamental eigenvalue value is to locate the setting up position of the barriers or wells in order to maximize or minimize the first eigenvalue of the considered operator.

In [6] , the authors studied this problem by considering the Laplace or Schrodinger operator. In this article we will study the variation of the fundamental value following the obstacle position.

Let D be an open bounded set in $\mathbb{R}^N$ and B an obstacle moving inside D.

We will study the variation of λ_1, the first eigenvalue of the operator $-\Delta$

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if the obstacle B moves inside D. The approach to the study of the problem is as follows. We will first state the problem, study the derivation and the variation in λ the first eigenvalue of the Laplace operator.

The variation of Ω is explained by the fact that B moves in D without going out. If B is a hard obstacle,the movement of B in D is done either by translation or by rotation or combining these two types of movement. If B is considered a soft obstacle, B can be transformed by homothecy.

We provide the derivative of the first eigenvalue of the Laplace operator for a hard obstacle and also in the case of a soft obstacle or a well. We will study the variation of the first eigenvalue of the Laplace operator λ, and we also state a theorem on the variation of λ that will give us the obstacle position for λ to be minimal.

2. Presentation of Different Obstacle and First Eigenvalue of the Laplace Operator

Let D an open fixed set in $\mathbb{R}^N$ and B an obstacle moving inside D. In this work we study the minimization of the first eigenvalue of the Laplace Dirichlet operator. Specifically, placing B inside D with zero boundary conditions of Dirichlet on the border of $\Omega = D/B$. We want to determine the position of B in D for λ_1 to be minimal.

$$\begin{cases} -\Delta u = \lambda_1 u & \text{in } D\backslash B = \Omega, \\ u = 0 & \text{on } \partial(D\backslash B) = \partial\Omega. \end{cases} \tag{1}$$

Define a vector field

$$\begin{aligned} V : \mathbb{R}^N & \longmapsto \mathbb{R}^N, \\ x & \longmapsto (V_1(x),\ V_2(x),\ V_3(x), ..., V_N(x)), \end{aligned}$$

for every real t small enough, we define disturbed areas:

$$\Omega_t = (Id + tV)(\Omega) = \{x + tV(x), \quad x \in \Omega\}.$$

The variation of Ω is explained by the fact that B moves in D without going out. If B is hard obstacle,the movement of B in D is done either by translation or by rotation or combining these two types of movement. On the other hand if B is considered a soft obstacle, B can be transformed by homothecy.

After the disturbance problem (1) becomes:

$$\begin{cases} -\Delta u_t &= \lambda\ u_t \text{ in } \Omega_t \\ u_t &= 0 \text{ on } \partial\Omega_t \end{cases}$$

With $\Omega_t = (Id + tV)(\Omega) = \{x + tV(x),\quad x \in \Omega.\}$

The derivative of is given by

$$\begin{cases} -\Delta u' = \lambda_k u' + \lambda_k' u \text{ in } \Omega \\ \quad u' = -\dfrac{\partial u}{\partial n} V.n \text{ on } \ \Gamma : \displaystyle\int_\Omega u u' dx = 0, \end{cases} \tag{2}$$

with $n(.)$ external unit normal to $\partial\Omega$.

We will give the definition of the spectrum and first eigenvalue of the Laplace Dirichlet operator.

Definition 2.1. Let A square symmetric matrix of order N of $M(\alpha,\beta,\Omega)$ constant. λ is an eigenvalue of the operator $\mathcal{A} = -div(A\nabla)$ with Dirichlet boundary conditions Ω if $u \neq 0$ is solution of problem

$$\begin{cases} \mathcal{A}u = \lambda u \ \text{ dans } \Omega \\ u = 0 \ \text{ sur } \partial\Omega \end{cases} \tag{3}$$

The function u is called proper function of $\mathcal{A}$ Associated with the eigenvalue λ. The set of eigenvalues is called the spectrum of $\mathcal{A}$. Let $\sum(\mathcal{A})$ this set.

Definition 2.2. Let $\alpha, \beta \in \mathbb{R}$ such as $0 < \alpha < \beta$. We denote by $M(\alpha,\beta,\Omega)$ all square matrices of order N as

$$A = (a_{ij})_{1 \le i,j \le n} \in (L^\infty(\Omega))^{N \times N}$$

satisfying

$$(i)\ A(x)\xi \,.\, \xi \ge \alpha|\xi|^2 \ \ \forall \xi \in \mathbb{R}^N \ \text{ in } \Omega \tag{4}$$

$$(ii)\ |A(x)\xi| \le \beta|\xi| \tag{5}$$

Proposition 2.1. *The following problem of minimization:*

$$\lambda_1 = min\left\{\int_\Omega A\nabla u \nabla u,\ u \in H_o^1(\Omega) \setminus \{0\} \int_\Omega u^2 dx = 1\right\}.$$

has a solution.

Proof of Proposition 2.1. The set of eigenvalues of $\mathcal{A}$ is the set

$$G=\left\{a\in\mathbb{R}\text{ such as }a=\int_\Omega A\nabla u\nabla u\text{ avec }u_o\in H^1_o(\Omega)\setminus\{0\}\text{ et }||u||_{L^2(\Omega)}=1\right\}$$

let

$$J(u)=\int_\Omega A\nabla u\nabla u\;dx.$$

We show that the function $J(u)$ admits a lower bound K with

$$K=\left\{u\in H^1_o(\Omega),\;||u||_{L^2(\Omega)}=1\right\}$$

$$A\in\mathcal{M}(\alpha,\beta,\Omega)\Longrightarrow(A\;\nabla u\,.\,\nabla u)\geq\alpha|\nabla u|^2\geq 0$$

$$\Longrightarrow\int A\nabla u\,.\,\nabla u)\geq 0\;\;\forall u\in H^1_o(\Omega)$$

(i) $J(u)$ admits a lower bound. so G admits a lower bound.
Let

$$\alpha=inf\;G=\inf_{\substack{u^\varepsilon\in H^1_o(\Omega)\\ ||u||_{L^2(\Omega)}\neq 0}}J(u)=\inf_{u\in K}J(u)$$

(ii) $\lambda_1\in G\Longrightarrow\alpha\leq\lambda_1$
Using the definition of lower bound $\exists u_n\in H^1_o(\Omega)/\{0\}$ such as

$$J(u_n)\longrightarrow\alpha.$$

$J(u_n)=\displaystyle\int(A^{\mathcal{E}}\nabla u\,.\,\nabla u)$ convex, continuous weakly sequentially s.c.i.

u_n admits a lower bound $H^1_o(\Omega)$, moreover $H^1_o(\Omega)$ is reflexive $\Longrightarrow\exists u^\epsilon_n$ a subsequence of u_n such as $u^\epsilon_n\longrightarrow u$ or K closed J weakly sequentially s.c.i.

$$\Longrightarrow J(u^\epsilon_n)\leq\underline{lim}\;J(u^\epsilon_n)=\alpha$$

$$u^\epsilon_n\in K\Longrightarrow J(u^\epsilon_n)\geq\inf_{u^\epsilon_n\in K}J(u)=\alpha$$

$$J(u^\epsilon_n)=\inf_{u^\epsilon_n\in K}J(u)=\min_{u^\epsilon_n\in K}J(u^\epsilon_n)$$

then $J(u^\epsilon_n)=\alpha=min\;G$

$$\alpha=min\;G=J(u^\epsilon_n)=\frac{\displaystyle\int_\Omega A\nabla u\;\nabla u}{\displaystyle\int_\Omega(u_o)^2\;du}$$

thus α is a eigenvalue of A. λ_1 this is the smallest eigenvalue A.

(iii) $\implies$ $\lambda_1 \leq \alpha$ Using (i), (ii) and (iii)

$$\lambda_1 = \alpha = \min_{\substack{u \in H_o^1(\Omega) \\ u \neq 0}} J(u)$$

thus

$$\lambda_1 = \min_{\substack{u \in H_o^1(\Omega) \\ u \neq 0}} \left(\frac{\int_\Omega A\nabla u \ \nabla u \ dx}{\int_\Omega u^2 \ dx} \right). \qquad \square$$

We will give some definitions before formulating the problem more precisely.

Definition 2.3. It said that the obstacle B is soft,if the operator we are going to consider is of the following form

$$-\Delta + \alpha \chi_B \quad \text{ou} \quad \alpha > 0$$

and χ_B is the indicator function of the region B.

A hard obstacle corresponds to $\alpha = +\infty$ and B is considered a well if α is negative.

In the case of a hard obstacle: Define for any real t small enough $T_t(B)$ as translation, rotation or a combination of both.

Let $J_2(\Omega_t) = \int_{\Omega_t} dx - v_o$ with $v_o > 0$ and

$$\Theta_\varepsilon = \Big\{ \Omega_t = D \backslash T_t(B), \text{ open set in } \mathbb{R}^N \text{ and verifying}$$
$$\varepsilon - \text{cone condition and } \int_{\Omega_t} dx = v_o \Big\}.$$

So the problem becomes determining the position of B such than

$$\min_{\Omega_1 \in \Theta_\varepsilon} \lambda_1(\Omega_t) \text{ is reached}$$

$$\text{or } \lambda_1(\Omega_t) = \min_{u \in H_o^1(\Omega)} \left\{ \int_{\Omega_t} |\nabla u|^2 dx \ : \ \int_{\Omega_t} u^2 dx = 1, \right\}$$

In the case of a soft obstacle we remove the constraint on the volume of the eligible set Θ_ε. Let $V_\varepsilon = \Big\{ \Omega_t = D \backslash T_t(B)$ open from $\mathbb{R}^N$ verifying

$\left.\varepsilon - \text{cone condition}\right\}$ where T_t may be a homothecy or a composition of a translation and a homothecy, then the problem is to determine the position of Ω_t so that $\min\limits_{\Omega_t \in V_\varepsilon} \lambda_1(\Omega_t)$ is reached where

$$\lambda_1(\Omega_t) = \min_{u \in H_o^1(\Omega)} \left\{ \int_D |\nabla u|^2 + \alpha \int_D \chi_B u^2 \ , \ \int_D u^2 = 1 \text{ or } \alpha \in \mathbb{R} \right\}.$$

3. The Variation of the First Eigenvalue and Locating of Obstacle

We gave the derivative of the first eigenvalue of the Laplace operator for a hard obstacle as well as in the case of a soft obstacle or well. Now we will study the variation of the first eigenvalue λ of the Laplace operator. We also state a theorem on the variation of λ.

Definition 3.1. Let J a functional on Ω. We define the derivative (Gateaux) of J at point Ω, in the direction of deformation V the limit denoted $dJ(\Omega, V)$, if it exists

$$dJ(\Omega, V) = \lim_{t \longmapsto 0} \frac{J(\Omega_t) - J(\Omega)}{t}$$

Definition 3.2. We define the form derivative of λ the limit denoted λ' if it exists

$$\lambda' = \lim_{t \longrightarrow 0} \frac{\lambda(\Omega_t) - \lambda(\Omega)}{t}$$

This definition can be found in [12].

Proposition 3.1. *Let Ω open bounded of class C^2 , we suppose that $\lambda_k(t)$ is a simple eigenvalue. So the functions*

$$t \longmapsto \lambda_k(t) \ et \ t \longmapsto u_t \in L^2(\mathbb{R}^N)$$

are differentiable at $t = 0$ and $u' \in H^1(\Omega)$ is the only solution to (2) with

$$\lambda_k'(0) = -\int_{\partial\Omega} \left(\frac{\partial u}{\partial n}\right)^2 V.n \tag{6}$$

Proof. We are going to give a sketch of proof by giving some hints to the desired result. To prove this we use in part the implicit functions theorem. We also use the shape derivative techniques see for instance pionner works of M. Schiffer [**?**] or [12], [15]. Let us give now some hints for the proof.

Let us consider the problem

$$\begin{cases} -\Delta u_\Omega &= \lambda_\Omega u_\Omega \quad \text{in} \quad \Omega \\ u &= 0 \qquad \text{on} \quad \partial\Omega \end{cases} \tag{7}$$

Using the shape derivative, we get

$$\begin{cases} -\Delta u' &= \lambda' u + \lambda u' \quad \text{in} \quad \mathcal{D}(\Omega) \\ u' &= -\frac{\partial u}{\partial n} V(0).n \quad \text{on} \quad \partial\Omega \end{cases} \tag{8}$$

Multiplying by u the first equation of the above system, using the Green formula and finally replacing u' by its value on the boundary of Ω, we get:

$$\lambda'_k(\Omega, V) = -\int_{\partial K} (\frac{\partial u}{\partial n})^2 V(0).n d\sigma.$$

□

We gave the derivative of the first eigenvalue of the Laplace operator for a hard obstacle and in the case of a soft obstacle or a well. We shall study in the following proposition:

Proposition 3.2. *Consider the case of a soft obstacle or well so using [5] we have the following problems*

$$\begin{cases} -\Delta u + \alpha \chi_B(x) u &= \lambda u \text{ in } D \\ u = 0 & \text{on } \partial D \end{cases}$$

With $\alpha \in \mathbb{R}$ and χ_B is the indicator function of the region B. Assuming that boundary of B is smooth by piece.

Suppose that B can be displaced by a distance in the direction of a vector field V. So we have:

$$\frac{d\lambda}{dV} = \alpha \int_{\partial B} |u|^2 n.V \; ds \tag{9}$$

Proof. (see [6]) We will study the variation of the eigenvalue λ. We recall a useful definition which will allow us to make constructions on the domain Ω, and we enunciate a theorem on the variation of λ.

Definition 3.3. Let P a hyperplane of dimension $N-1$ which intersects Ω. For any connected set S not intersecting P.

We call S^P it's symmetrical with respect to P. We say the domain Ω verifies the property of inner reflection with respect to P if there is a connected component Ω_S of $\Omega\backslash P$ such that Ω_S^P is a proper subset of another connected component Ω_b of $\Omega\backslash P$. Such P will be called an inner reflection hyperplane for Ω , while Ω_S will be called the small side of Ω (and Ω_b will be called the big side for Ω).

We enunciate a theorem on the variation of the first eigenvalue of the Laplace operator in the different cases of obstacles.

This theory was demonstrated in [6] we'll show it using other technical tools.

Theorem 3.1. *suppose that Ω has the inner reflection property with respect to P with B a ball. Suppose that B moves with a translational movement following a particular field vector V in P in the same direction as V and pointing from the small side towards the big side.*

Let λ_1 be the first eigenvalue of the Laplace Dirichlet operator:

$$\begin{cases} -\Delta u = \lambda_1 u & \text{in } D\backslash B = \Omega \\ u = 0 & \text{on } \partial(D\backslash B) = \partial\Omega \end{cases} \tag{10}$$

So in the case of a hard or soft obstacle

$$d\lambda_1(\Omega, V) > 0$$

in the case of a well

$$d\lambda_1(\Omega, V) < 0$$

Before giving the proof of this theorem, we will give some useful results for the different stages of the proof.

Proposition 3.3 (Maximum principle for self-adjoint operators)**.** *Let Ω an open regular domain of $\mathbb{R}^N$. Consider an elliptic operator of second order in Ω*

$$L = \partial_i\ (a_{ij}(x_i\ \partial_j) + c(x)$$

such that

$$c_o|\xi|^2 \leq a_{ij}(x)\ \xi_i\xi_j \leq C_o|\xi|^2 \ \ c_o, C_o > 0\ ,\ \forall\xi \in \mathbb{R}^N$$

with $a_{ij}(x) \in C(\Omega)$, $c(x) \in L^\infty(\Omega)$.
Then we have the maximum principle by :

$$(PM)' : \begin{cases} u \in H^1(\Omega) \\ Lu \leq 0 \\ u|_{\partial\Omega} \geq 0 \end{cases} \quad \text{is verifie} \implies u \geq 0 \ \ pp \ \text{ in } \ \Omega$$

Proposition 3.4. *Let λ_1 the first eigenvalue of the operator $-L$. So $(PM)'$ is satisfied if and only if $\lambda_1 > 0$.*

Proof of Theorem 3.1. There are three cases to consider, a hard/soft obstacle and a well. We will consider the hard obstacle as a last case, for the other two cases we explain that for any point x of ∂B located on the small side $\Omega = D|K$ such that $u(x) < u(x^p)$. For the case of a hard obstacle we use on the Hadamard formula see [6]. We only need to prove that $|\nabla u(x)| < |\nabla u(x^p)|$.

Ω verifying the reflection property with respect to a hyperplane T_λ given by equation $(x_N = \lambda)$ in the direction of the axis x_N suppose $B = K$.

- D admits an axis of symmetry of equation $(x_N = 0)$.
- K_λ^+ the part of K located further above T_λ.
- $\sigma_\lambda(K_\lambda^+)$ the symmetrical of K_λ^+ with respect to T and $K_\lambda = K|K_\lambda^+$
- D_λ^+ the part of Ω completely located above T_λ
- $\sigma_\lambda(D_\lambda^+)$ the symmetrical of D_λ^+ with respect to T_λ and $D_\lambda^- = \Omega \setminus \{D_\lambda^+ \cup \sigma_\lambda(D_\lambda^+)\}$

One may encounter the following two cases of figures: the figure that explains the notation

- (i) $\sigma(K_\lambda^+)$ is inner tangent to $\partial\Omega$ at a point y_o, with $y_o \notin T_\lambda$
- (ii) T is orthogonal to $\partial\Omega$ at a point x_o that is there $\exists\lambda$ such as $\sigma_\lambda(\Omega_\lambda^+) \subset \Omega$.

- Either we have case (i).
- or case (ii).

Suppose $\lambda \leq 0$ to fix ideas, let $\Sigma_\lambda = K_\lambda^+ \cup \sigma(K_\lambda^+)$. Let v the function defined on Σ_λ by $\forall x \in \Sigma_\lambda$

$v(x) = u(x^p)$ with $x^p = \sigma_{\lambda_o}(x)$. so on Σ_λ we have

$$\begin{cases} -\Delta u = \lambda_1 u & in \quad K_\lambda^+ \cup \sigma(K_\lambda^+) \\ u = 0 & on \quad \partial K_\lambda \setminus T_{\lambda_o} \\ u(x) = u(x^p) & on \quad T_{\lambda_o} \\ u(x) = 0 & on \quad \partial\sigma(K_\lambda^+) \setminus T_{\lambda_o} \end{cases}$$

$$\begin{cases} -\Delta(u(x) - v(x) = \lambda(u(x) - v(x)) & in \quad K_\lambda^+ \cup \sigma(K_\lambda^+) \\ u(x) - v(x) = -u(x^p) & on \quad \partial K_\lambda^+ \,/\, T_{\lambda_o} \\ u(x) - v(x) = 0 & on \quad T_{\lambda_o} \\ u(x) - v(x) = -u(x^p) & on \quad \partial\sigma(K_\lambda^+) \setminus T_{\lambda_o} \end{cases} \tag{11}$$

In order to determine sign u in Ω, we will determine use the maximum principle (PM) and proposition (3.4).

Let $a_{ij}(x) = I \quad c(x) = 0$

a_{ij} elliptical $\quad a_{ij} \in C(\Omega) \;\; c(x) \in L^{\infty}(\Omega)$.

Let $L = \partial i \;\; (a_{ij}(x)\, \partial_j) + c(x)$.

So $L(u) = \Delta u$. Hence the first eigenvalue of $-L$ is equal to λ_1 and $\lambda_1 > 0$. Thus using proposition (3.4) $(PM)'$ is satisfied $\Longrightarrow u \geq 0$ ae in Ω.

Consequently equation (11) becomes

$$\begin{cases} \Delta(v(x) - u(x)) + \lambda(v(x) - u(x)) = 0 & in \quad \Sigma_\lambda \\ v(x) - u(x) = u(x^p) \geq 0 & sur \quad \partial K_\lambda^+ \setminus T_{\lambda_o} \\ v(x) - u(x) = 0 & sur \quad T_{\lambda_o} \\ v(x) - u(x) = u(x^p) \geq 0 & sur \quad \partial\sigma(K_\lambda^+) \setminus T_{\lambda_o} \end{cases}.$$

Let $\omega = v(x) - u(x)$.

So Using to the maximum principle if we define

$c(x) = \lambda$

$L\omega = \Delta\omega + \lambda\omega$ our $a_{ij} = I$

$L = \partial_i(a_{ij}(x)\, \partial_j) + \lambda$

I elliptical $c(x) \in L^{\infty}(\Omega)$

$\Longrightarrow \begin{cases} L\omega = 0 & on \;\; \Sigma_\lambda \\ \omega \geq 0 & on \;\; \partial\Sigma_\lambda \end{cases}.$

Using $(PM) \Longrightarrow \omega \geq 0$ ae in $\sum_\lambda \Longrightarrow \omega(x) = u(x^p) - u(x) \geq 0$ pp in $\sum_\lambda$ we will show that

$$\omega(x) \neq 0 \;\; \forall x \in \partial K_\lambda^+ \backslash T_\lambda.$$

Now, if $x \in K_\lambda^+ \backslash T_\lambda$. let $u(x) = u(x^p) \Longrightarrow \omega(x) = 0$.

So $\begin{cases} L\omega = 0 & in \; \Sigma_\lambda \\ \omega = 0 & \text{on } \partial\Sigma_\lambda \end{cases} \Longrightarrow$ using $(PM)'$ $\omega \geq 0$ in Σ_λ.

Let $h = -\omega$

$\begin{cases} L(h) = 0 & in \; \Sigma_\lambda \\ h = 0 & on \; \partial\Sigma_\lambda \end{cases} \Longrightarrow$ using $(PM)'$ $h \geq 0$ $over$ $\partial\Sigma_\lambda \Longrightarrow -\omega \geq 0$.

$\Longrightarrow \omega \leq 0 \Longrightarrow \omega = 0$ ae in Σ_λ.

Let $x \in \Sigma_\lambda$

recall that $\dfrac{\partial\omega(x)}{\partial n} = \lim\limits_{h \to 0} \dfrac{\omega(x - nh) - \omega(x)}{h} = 0.$

Hence $\dfrac{\partial\omega(x)}{\partial n} = 0$

Again, recall that $u(x) - v(x) = 0$ we apply Lemma (1) page (308) [11].

Ze get $\dfrac{\partial \omega(x)}{\partial n} > 0$ which is contradictory.

So if $x \in \partial K_\lambda^+ \setminus T_{\lambda_o} \quad u(x^p) - u(x) > 0$
$\Longrightarrow u(x^p) > u(x)$.

In the case of an hard obstacle we use the formula of Hadamard, this time we consider the function $\omega(x) = u(x) - u(x^p)$ according to $(PM)'$ we have $\omega(x) < 0$ inside of $K_\lambda^+$to complete the proof in this case we use the lemma (1) page (308) [11]. So either the normal derivative of $\omega(x)$, or the second derivative of $\omega(x)$ in that direction is positive. The latter is impossible considering the equation of eigenvalue. Hence $|\nabla u(x)| < |\nabla u(x^p)|$ for any $x \in \partial\ K_\lambda^+ \setminus T_{\lambda_o}$. □

Conclusion

Using the property of reflection, concepts of small and big side of the domain Ω and the variation of λ_1.

We find that λ_1 is strictly increasing when the obstacle B is placed in contact with the border towards the large side. Thus λ_1 is minimal when the obstacle touches the boundary of the domain Ω. □

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References

[1] G. Allaire, *Shape Optimization by the Homogenization Method*, Springer, 2001.

[2] G. Allaire, F. Jouve, A.M. Toader, A level-set method for shop optimization, *C.R. Acad. Sci. Paris*, Ser. I, **334** (2002), 1125-1130.

[3] O.M. Badahi, I. Ly, D. Seck, Analysis of stability of the exterior and interior Bernoulli's free boundary problems, *Applied Mathematical Sciences*, **10**, No. 12 (2016), 591-612.

[4] Cioranescu, Doïna, Patrizia, Donato, *An Introduction to Homogeneization* Oxford Lecture Series in Mathematics and Applications, **17**, Oxford, 1999.

[5] Ph. Guillaume, Dérivées d'ordre supérieur en conception optimal de forme, Thése de l'Université Paul Sabatier, 1994.

[6] E.M. Harrell II, P. Kroger, K. Kurata, On the placement of an obstacle nor well so as to optimize the fundamental eigenvalue, *SIAM J. Math Anal.*, **33**, No. 1, 240-259.

[7] J. Hass, M. Hutchings, R. Schlafty, The bubble conjecture, electron, Res, Announc, *Amer. Math. Soc.*, **1**, No. 3 (1995), 98-102.

[8] O. Kavian, Introduction à la théorie des points critiques et applications aux problémes elliptiques Nancy, 1993.

[9] F. Murat, J. Simon, *Sur le Contrôle par un domaine géométrique*, Thése d'Etat, Paris, 1976.

[10] M.H. Protter, H.F. Weinberger, *Maximum Principles in Differential Equations*, Springer-Verlag, New York, 1985.

[11] J. Serrin, A symmetry problem in potentiel theory, *Arch. Rational Mech. Anal.*, **43** (1971), 304-318.

[12] A. Henrot, M. Pierre, *Variations et Optimisation de Formes. Une Analyse Géométrique*, Mathématiques et Applications, **48**, Springer, 2005.

[13] A. Henrot, H. Shahgholian, Existence of classical solutions to a free boundary problem for the p-Laplace operator, the exterior convex case, *J. Reine Angew.*, **521** (2000), 85-97.

[14] A. Henrot, H. Shahgholian, Existence of classical solutions to a free boundary problem for the p-Laplace operator II: The interior convex case, *Indiana Univ. Math. Journal*, **49**, No. 1 (2000), 301-323.

[15] F. Murat, J. Simon, *Derivation de Fonctionnelles par à un Domaine Géométrique*, Numer. Prépublications de Paris 6.

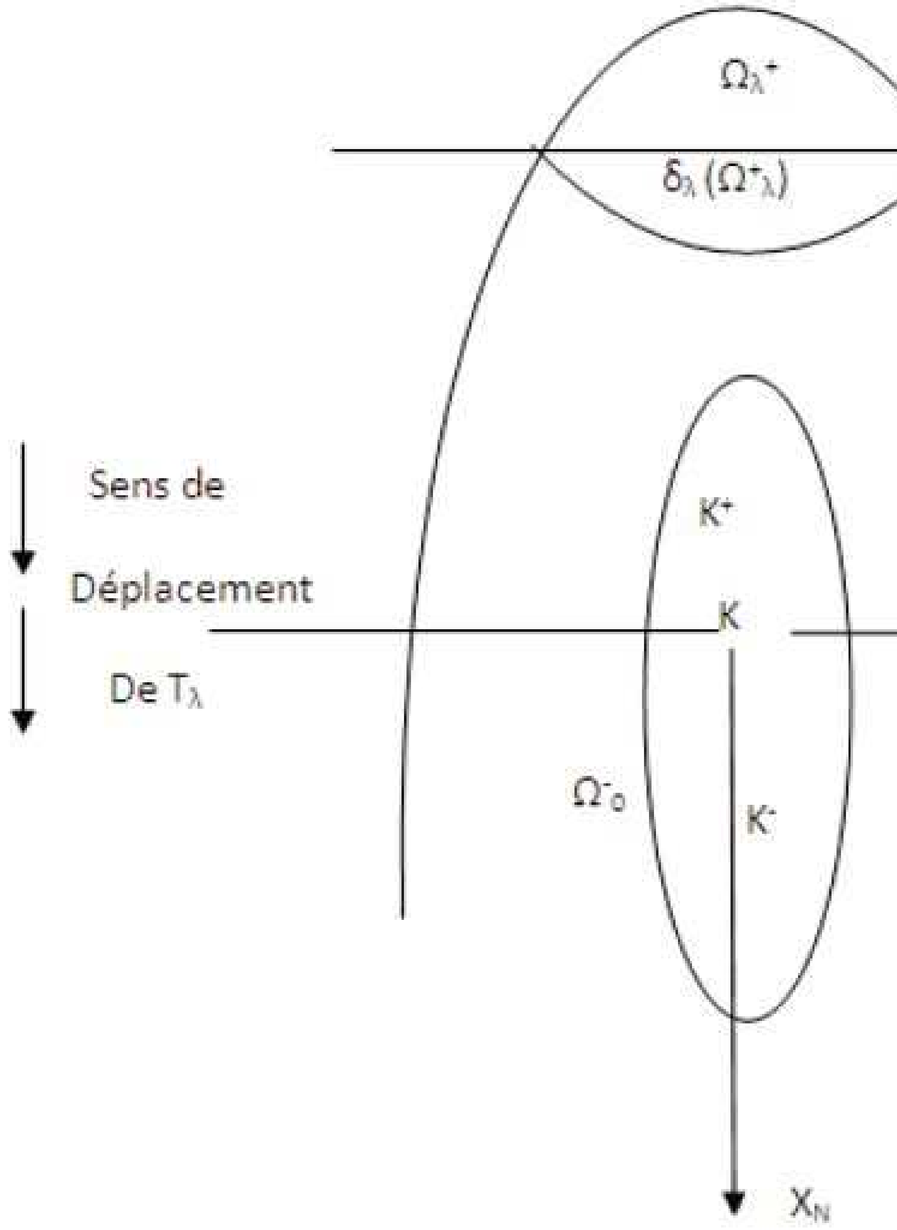

Figure 1