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TWO NEW WEIGHTED DIVERGENCE MEASURES IN PROBABILITY SPACES

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Abstract: It is known that the measures of distance in metric spaces are not adequate to find applications in all disciplines of mathematical sciences. Thus, the inevitability arises for the development of a variety of generalized measures of distance in probability spaces so as to extend the scale of their applications. The present communication is a step in this direction and deals with the growth of two new measures of distance for the discrete probability distributions so as to provide their applications in furtherance of our research findings.

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1. Introduction

The idea of probabilistic distances, also called divergence measures, which in some sense assess how close two probability distributions are from one another, has been widely employed in probability, statistics, information theory and other related fields. In information theory, the Kullback-Leiblers [8] divergence measure, also known as information divergence or information gain or relative entropy, is a non-symmetric measure of the difference between two probability

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distributions P and Q. Specifically, the Kullback-Leibler (KL) divergence of Q from P is a measure of the information lost when Q is used to approximate P. Although, it is often intuited as a metric or distance, the KL divergence is not a true metric- for example, it is not symmetric: the KL divergence from P to Q is generally not the same as the KL divergence from Q to P. This most important and useful measure of directed divergence is due to Kullback and Leibler [8] is given by the following expression:

$$D(P:Q) = \sum_{i=1}^{n} p_i \ln \frac{p_i}{q_i}. \tag{1.1}$$

Some other measures of divergence are:

$$D_R(P:Q) = \frac{1}{\alpha - 1} \ln \sum_{i=1}^{n} p_i^{\alpha} q_i^{1-\alpha}, \quad \alpha \neq 1,\ \alpha > 0, \tag{1.2}$$

which is Renyi's [1] probabilistic measure of directed divergence.

$$D_{HC}(P:Q) = \frac{1}{\alpha - 1} \left[\sum_{i=1}^{n} p_i^{\alpha} q_i^{1-\alpha} - 1 \right], \quad \alpha \neq 1,\ \alpha > 0, \tag{1.3}$$

which is Havrda and Charvats [3] probabilistic measure of divergence.

$$D_{\lambda}(P:Q) = \frac{1}{\lambda} \sum_{i=1}^{n} (1 + \lambda p_i) \ln \frac{1 + \lambda p_i}{1 + \lambda q_i}, \ \lambda > 0, \tag{1.4}$$

which is Ferreri's [2] probabilistic measure of divergence.

Kapur in [4] introduced the following measures of directed divergence:

$$D_1(P:Q) = \sum_{i=1}^{n} p_i \ln \frac{p_i}{q_i} - \frac{1}{a} \sum_{i=1}^{n} (q_i + a p_i) \ln \frac{(q_i + a p_i)}{q_i (1 + a)}, \quad a \geq -1, \tag{1.5}$$

$$D_2(P:Q) = \sum_{i=1}^{n} p_i \ln \frac{p_i}{q_i} - \frac{1}{a} \sum_{i=1}^{n} (1 + a p_i) \ln \frac{(1 + a p_i)}{1 + a q_i}, \quad a \geq -1. \tag{1.6}$$

Parkash and Mukesh [5] and [6] have investigated and developed following generalized parametric measures of directed divergence:

$$S(P:Q) = \sum_{i=1}^{n} \left[\left(\frac{1}{\sqrt{p_i}} + \frac{1}{\sqrt{q_i}} \right) \left(\frac{p_i + q_i}{2} \right)^{\frac{3}{2}} - 2 p_i \right], \tag{1.7}$$

$$G^{\alpha}(P:Q) = \frac{\sum_{i=1}^{n} p_i \left(\alpha + \frac{1}{2}\right)^{\log \frac{p_i}{q_i}} - 1}{\alpha - \frac{1}{2}}, \quad \alpha > \frac{1}{2}, \tag{1.8}$$

where α is a real parameter.

$$D_{\alpha,\beta}(P:Q) = \frac{1}{\alpha - \beta}\left(\sum_{i=1}^{n} p_i^{\alpha-\beta+1} q_i^{\beta-\alpha} - 1\right),$$
$$\alpha \neq \beta,\ \beta < \alpha + 1,\ \alpha > 0,\ \beta > 0. \tag{1.9}$$

Today, it is well known that in different disciplines of science and engineering, the concept of distance has been proved to be very useful but its application areas can be extended to other emerging disciplines of social, economic, physical and biological sciences by the modification of the concept of distance. To explain the necessity for a new concept of distance for the disciplines other than that of science and engineering, Kapur [4] considered many distinctive problems usually encountered in these promising fields.

But, in many real life situations dealing with random events, it becomes the necessity to take into account both the equally important aspects, that is, quantity as well as quality. Inspired by this idea, it was Guiasu[7] who customized the concept and converted it into weighted information. Thus, upon attaching weights to the probability distributions, the well known divergence measure $D(P:Q)$ due to Kullback-Leibler [8] may generate a family of weighted divergence measures. Further, taking into consideration the limitations of the weighted divergence measures, Kapur [4] stressed that a weighted divergence measure denoted by $D(P;Q:W)$ will be an appropriate measure of weighted directed divergence if

(i) It is continuous function of $p_1, p_2, \ldots, p_n; q_1, q_2, \ldots, q_n; w_1, w_2, \ldots, w_n$.

(ii) It is permutationally symmetric function of $p_1, p_2, \ldots, p_n; q_1, q_2, \ldots, q_n; w_1, w_2, \ldots, w_n$, that is, it does not change when the triplets (p_1, q_1, w_1), $(p_2, q_2, w_2), \ldots, (p_n, q_n, w_n)$ are permuted among themselves.

(iii) It is always greater than or equal to zero and vanishes when $p_i = q_i$ for each i.

(iv) It is a convex function of $p_1, p_2, \ldots, p_n$ which has its minimum value zero when$p_i = q_i$ for each i.

(v) It is a convex function of $q_1, q_2, \ldots, q_n$.

(vi) It reduces to an ordinary measure of directed divergence upon ignoring weights.

With the above mentioned properties, Kapur[4] discussed, investigated and proved the validity of many measures of weighted directed divergence so as to make them appropriate. Some of these measures are:

$$D_1(P;Q:W)=\sum_{i=1}^{n} w_i\left[p_i\log\frac{p_i}{q_i}-p_i+q_i\right], \tag{1.10}$$

$$D_2(P;Q:W)=\frac{1}{\alpha(\alpha-1)}\ln\sum_{i=1}^{n} w_i\left[p_i^{\alpha}q_i^{1-\alpha}-\alpha p_i+\alpha q_i-q_i\right],$$
$$\alpha>0,\alpha\neq 1, \tag{1.11}$$

$$D_3(P;Q:W)=\sum_{i=1}^{n} w_i\left[\frac{p_i\ln q_i-p_i\ln p_i-q_i+p_i}{\ln q_i}\right], \tag{1.12}$$

$$D_4(P;Q:W)=\sum_{i=1}^{n} w_i\left[\frac{p_i-p_i\ln p_i-q_i+q_i\ln q_i}{\ln q_i}+p_i-q_1\right], \tag{1.13}$$

$$D_5(P;Q:W)=\frac{\sum_{i=1}^{n} w_iq_i}{\alpha(\alpha-1)}\log\frac{\sum_{i=1}^{n} w_i\left[p_i^{\alpha}q_i^{1-\alpha}-\alpha p_i+\alpha q_i\right]}{\sum_{i=1}^{n} w_iq_i},$$
$$\alpha>0,\ \alpha\neq 1, \tag{1.14}$$

$$D_6(P;Q:W)=\sum_{i=1}^{n} w_ip_i\log\frac{p_i}{q_i}+\sum_{i=1}^{n} w_i(p_i-q_i)\log(1+a)$$
$$+\frac{1}{a}(1+a)\log(1+a)\sum_{i=1}^{n} w_iq_i$$
$$-\frac{1}{a}\sum_{i=1}^{n} w_i(q_i+ap_i)\log\left(1+\frac{ap_i}{q_i}\right),\quad a\geq 0. \tag{1.15}$$

Using the construction criteria discussed above, many more measures of weighted divergence can be developed so as to provide their applications.

2. Two New Weighted Measures of Divergence

In this section, we introduce two new parametric measures of weighted divergence given by

$$I.D_\alpha(P;Q:W) = \frac{1}{2^{1-\alpha}-1}\sum_{i=1}^{n} w_i\left[(\alpha-1)p_i - p_i^\alpha q_i^{1-\alpha} + \alpha p_i^{\frac{1}{\alpha}} q_i^{1-\frac{1}{\alpha}} + 2(1-\alpha)q_i\right], \quad \alpha > 1 \quad (2.1)$$

We observe that

$$\begin{aligned}\lim_{\alpha\to 1} D_\alpha(P;Q:W) &= -\sum_{i=1}^{n} w_i\left[2p_i - 2q_i + 2p_i\log q_i - 2p_i\log p_i\right]\\ &= -2\sum_{i=1}^{n} w_i\left[p_i\log\frac{q_i}{p_i} + p_i - q_i\right]\\ &= 2\sum_{i=1}^{n} w_i\left[p_i\log\frac{p_i}{q_i} - p_i + q_i\right],\end{aligned}$$

which is Kapur's [4] measure of directed divergence except a multiplicative constant. Thus, (2.1) can be taken as a generalization of Kapurs [4] measure of weighted divergence.

To prove the authenticity of the above measure, we study its properties as follows:

(i) When $p_i = q_i$, $D_\alpha(P;Q:W) = 0$;

(ii) Convexity: We have

$$\begin{aligned}\frac{\partial^2 D_\alpha(P;Q:W)}{\partial p_i^2} &= \frac{w_i}{2^{1-\alpha}-1}\left[-\alpha(\alpha-1)p_i^{\alpha-2}q_i^{1-\alpha} + \left(\frac{1}{\alpha}-1\right)p_i^{\frac{1}{\alpha}-2}q_i^{1-\frac{1}{\alpha}}\right]\\ &= \frac{(1-\alpha)w_i}{2^{1-\alpha}-1}\left[\alpha p_i^{\alpha-2}q_i^{1-\alpha} + \frac{1}{\alpha}p_i^{\frac{1}{\alpha}-2}q_i^{1-\frac{1}{\alpha}}\right] > 0 \quad \text{as} \quad \alpha > 1.\end{aligned}$$

Similarly, we have

$$\frac{\partial^2 D_\alpha(P;Q:W)}{\partial q_i^2} = \frac{(1-\alpha)w_i}{2^{1-\alpha}-1}\left[\alpha p_i^\alpha q_i^{-(1+\alpha)} + \frac{1}{\alpha}p_i^{\frac{1}{\alpha}} q_i^{-(1+\frac{1}{\alpha})}\right] > 0.$$

Also

$$\frac{\partial^2 D_\alpha(P;Q:W)}{\partial p_i \partial p_j} = 0, \quad i \neq j,$$

and

$$\frac{\partial^2 D_\alpha(P;Q:W)}{\partial q_i \partial q_j} = 0, \quad i \neq j.$$

The Hessian matrix of $D_\alpha(P;Q:W)$ w.r.t $p_1, p_2, \ldots, p_n$ is given by

$$\begin{bmatrix} \frac{(1-\alpha)w_1}{2^{1-\alpha}-1}\left[\alpha p_1^{\alpha-2}q_1^{1-\alpha}+\frac{1}{\alpha}p_1^{\frac{1}{\alpha}-2}q_1^{1-\frac{1}{\alpha}}\right] & 0 & \cdots & 0 \\ 0 & \frac{(1-\alpha)w_2}{2^{1-\alpha}-1}\left[\alpha p_2^{\alpha-2}q_2^{1-\alpha}+\frac{1}{\alpha}p_2^{\frac{1}{\alpha}-2}q_2^{1-\frac{1}{\alpha}}\right] & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{(1-\alpha)w_n}{2^{1-\alpha}-1}\left[\alpha p_n^{\alpha-2}q_n^{1-\alpha}+\frac{1}{\alpha}p_n^{\frac{1}{\alpha}-2}q_n^{1-\frac{1}{\alpha}}\right] \end{bmatrix},$$

which is positive definite.

Similarly, Hessian matrix of $D_\alpha(P;Q:W)$ w.r.t $q_1, q_2, \ldots, q_n$ is given by

$$\begin{bmatrix} \frac{(1-\alpha)w_1}{2^{1-\alpha}-1}\left[\alpha p_1^{\alpha}q_1^{-(1+\alpha)}+\frac{1}{\alpha}p_1^{\frac{1}{\alpha}}q_1^{-(1+\frac{1}{\alpha})}\right] & 0 & \cdots & 0 \\ 0 & \frac{(1-\alpha)w_2}{2^{1-\alpha}-1}\left[\alpha p_2^{\alpha}q_2^{-(1+\alpha)}+\frac{1}{\alpha}p_2^{\frac{1}{\alpha}}q_2^{-(1+\frac{1}{\alpha})}\right] & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{(1-\alpha)w_n}{2^{1-\alpha}-1}\left[\alpha p_n^{\alpha}q_n^{-(1+\alpha)}+\frac{1}{\alpha}p_n^{\frac{1}{\alpha}}q_n^{-(1+\frac{1}{\alpha})}\right] \end{bmatrix},$$

which is again positive definite.

Thus, $D_\alpha(P;Q:W)$ is convex function of both P and Q.

Hence, $D_\alpha(P;Q:W)$ is a valid measure of directed divergence.

(iii) Non-negativity: We will find the extremum of $D_\alpha(P;Q:W)$ subject to the natural constraint $\sum_{i=1}^{n} p_i = 1$. Let us consider the Lagrangian given by

$$L = D_\alpha(P;Q:W) + \lambda(1 - \sum_{i=1}^{n} p_i).$$

p	q	$D_\alpha(P;Q:W)$
0.1	0.5	11.10422
0.2	0.5	5.538011
0.3	0.5	2.237824
0.4	0.5	0.524624
0.5	0.5	0
0.6	0.5	0.516266
0.7	0.5	2.170259
0.8	0.5	5.306252
0.9	0.5	10.54445

Table 2.1: $D_\alpha(P;Q:W)$ against p for $n=2$ and $\alpha=3$

Now we have

$$\frac{\partial L}{\partial p_i}=\frac{w_i}{2^{1-\alpha}-1}\left[(\alpha-1)-\alpha p_i^{\alpha-1}q_i^{1-\alpha}+p_i^{\frac{1}{\alpha}-1}q_i^{1-\frac{1}{\alpha}}\right]-\lambda.$$

Thus $\frac{\partial L}{\partial p_i}=0$ gives

$$\begin{aligned}&\frac{w_1}{2^{1-\alpha}-1}\left[(\alpha-1)-\alpha p_1^{\alpha-1}q_1^{1-\alpha}+p_1^{\frac{1}{\alpha}-1}q_1^{1-\frac{1}{\alpha}}\right]\\ =&\frac{w_2}{2^{1-\alpha}-1}\left[(\alpha-1)-\alpha p_2^{\alpha-1}q_2^{1-\alpha}+p_2^{\frac{1}{\alpha}-1}q_2^{1-\frac{1}{\alpha}}\right]\\ =&\dots\\ =&\frac{w_n}{2^{1-\alpha}-1}\left[(\alpha-1)-\alpha p_n^{\alpha-1}q_n^{1-\alpha}+p_n^{\frac{1}{\alpha}-1}q_n^{1-\frac{1}{\alpha}}\right],\end{aligned}$$

which is possible only if $p_i=q_i$ and at $p_i=q_i, D_\alpha(P;Q:W)=0$. Thus, the minimum value of $D_\alpha(P;Q:W)$ is 0. This implies $D_\alpha(P;Q:W)\geq 0$.

Thus, $D_\alpha(P;Q:W)$ is a correct measure of weighted divergence.

Next, for $\alpha=3$ and $W=\{w_1,w_2\}=\{1.1,1.2\}$, we have calculated various values for the divergence measure $D_\alpha(P;Q:W)$ as shown in the following Table 2.1 and obtained the Figure 2.1 which clearly shows $D_\alpha(P;Q:W)$ that is convex.

II. Next, we introduce another new parametric measure of weighted divergence given by

$$D^{\alpha,\beta}(P;Q:W)=\frac{\sum_{i=1}^n w_iq_i}{(\alpha-\beta)}\log\frac{\{\sum_{i=1}^n w_i\{p_i^\alpha q_i^{1-\alpha}-\alpha p_i+\alpha q_i\}\}}{\{\sum_{i=1}^n w_i\{p_i^\beta q_i^{1-\beta}-\beta p_i+\beta q_i\}\}},$$

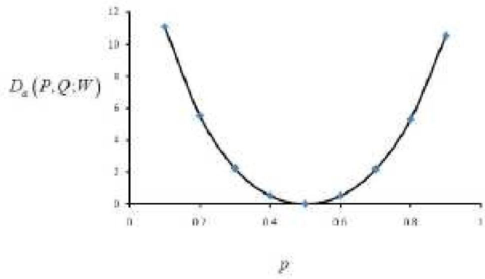

Figure 1

$$\alpha < 1, \beta > 1 \text{ or } \alpha > 1, \beta < 1 \quad (2.2)$$

To derive this measure, we make use of Kapur's [4] measure of weighted divergence given by

$$D^{\alpha}(P; Q : W) = \frac{\sum_{i=1}^{n} w_i q_i}{\alpha(\alpha - 1)} \log \frac{\{\sum_{i=1}^{n} w_i \{p_i^{\alpha} q_i^{1-\alpha} - \alpha p_i + \alpha q_i\}\}}{\sum_{i=1}^{n} w_i q_i}, \ \alpha > 1 \quad (2.3)$$

Similarly, we have

$$D^{\beta}(P; Q : W) = \frac{\sum_{i=1}^{n} w_i q_i}{\beta(\beta - 1)} \log \frac{\{\sum_{i=1}^{n} w_i \{p_i^{\beta} q_i^{1-\beta} - \beta p_i + \beta q_i\}\}}{\sum_{i=1}^{n} w_i q_i}, \ \beta > 1 \quad (2.4)$$

Now, let us consider a new function given by

$$\begin{aligned} D^{\alpha,\beta}(P; Q : W) = & \frac{\lambda(\sum_{i=1}^{n} w_i q_i)}{\alpha(\alpha - 1)} \log \frac{\{\sum_{i=1}^{n} w_i \{p_i^{\alpha} q_i^{1-\alpha} - \alpha p_i + \alpha q_i\}\}}{\sum_{i=1}^{n} w_i q_i} \\ & + \frac{\mu(\sum_{i=1}^{n} w_i q_i)}{\beta(\beta - 1)} \log \frac{\{\sum_{i=1}^{n} w_i \{p_i^{\beta} q_i^{1-\beta} - \beta p_i + \beta q_i\}\}}{\sum_{i=1}^{n} w_i q_i}, \quad (2.5) \end{aligned}$$

where $\lambda, \mu > 0$.

Next, we find the minimum value of $D^{\alpha}(P; Q : W)$. The corresponding Lagrangian is

$$L = D^{\alpha}(P; Q : W) - \lambda(\sum_{i=1}^{n} p_i - 1).$$

Now, we have

$$\frac{\partial L}{\partial p_i} = \frac{\sum_{i=1}^{n} w_i q_i}{\alpha - 1} \frac{\{p_i^{\alpha-1} q_i^{1-\alpha} - 1\}}{\{\sum_{i=1}^{n} w_i \{p_i^{\alpha} q_i^{1-\alpha} - \alpha p_i + \alpha q_i\}\}} - \lambda.$$

Thus, $\frac{\partial L}{\partial p_i} = 0$ implies that

$$\frac{w_i\{p_i^{\alpha-1}q_i^{1-\alpha}-1\}}{\sum_{i=1}^n w_i\{p_i^\alpha q_i^{1-\alpha}-\alpha p_i+\alpha q_i\}} = \frac{\lambda(\alpha-1)}{\sum_{i=1}^n w_i q_i}$$

or

$$w_i\{p_i^{\alpha-1}q_i^{1-\alpha}-1\} = \frac{\lambda(\alpha-1)}{\sum_{i=1}^n w_i q_i}\sum_{i=1}^n w_i\{p_i^\alpha q_i^{1-\alpha}-\alpha p_i+\alpha q_i\}$$

which is satisfied at $p_i = q_i, i = 1, 2, \ldots, n$.

Again,

$$\left.\frac{\partial^2 L}{\partial p_i^2}\right|_{p_i=q_i} = \frac{w_i}{p_i} > 0, \quad i = 1, 2, \ldots, n.$$

Thus, we see that the minimum value of $D^\alpha(P; Q : W)$ exists at $p_i = q_i, i = 1, 2, \ldots, n$ and at $p_i = q_i, D^\alpha(P; Q : W) = 0$. Similarly, we observe that the minimum value of $D^\beta(P; Q : W)$ exists at $p_i = q_i\ \forall i$ and $D^\beta(P; Q : W) = 0$. Thus, the minimum value of $D^{\alpha,\beta}(P; Q : W)$ exists at $p_i = q_i\ \forall i$.

Now, let

$$\frac{\mu}{\beta(\beta-1)} = -\frac{\lambda k}{\alpha(\alpha-1)}.$$

Thus, equation (2.5) can be written as

$$D^{\alpha,\beta}(P; Q : W) = \frac{\lambda}{\alpha(\alpha-1)}\left\{\sum_{i=1}^n w_i q_i\right\}\log\frac{\left\{\frac{\sum_{i=1}^n w_i\{p_i^\alpha q_i^{1-\alpha}-\alpha p_i+\alpha q_i\}}{\sum_{i=1}^n w_i q_i}\right\}}{\left\{\frac{\sum_{i=1}^n w_i\{p_i^\beta q_i^{1-\beta}-\beta p_i+\beta q_i\}}{\sum_{i=1}^n w_i q_i}\right\}^k}$$

Now, since $\lambda, \mu > 0$, $\alpha - 1$ and $\beta - 1$ must have opposite signs, that is, if $\alpha < 1, \beta > 1$ or if $\alpha > 1, \beta < 1$.

Further, we note that in any case $(\alpha-1)$ and $\alpha-\beta$ have same signs. Thus, we have

$$D^{\alpha,\beta}(P; Q : W) = \frac{\sum_{i=1}^n w_i q_i}{(\alpha-\beta)}\log\frac{\left\{\frac{\sum_{i=1}^n w_i\{p_i^\alpha q_i^{1-\alpha}-\alpha p_i+\alpha q_i\}}{\sum_{i=1}^n w_i q_i}\right\}}{\left\{\frac{\sum_{i=1}^n w_i\{p_i^\beta q_i^{1-\beta}-\beta p_i+\beta q_i\}}{\sum_{i=1}^n w_i q_i}\right\}^k},$$
$$\alpha < 1, \beta > 1 \quad \text{or} \quad \alpha > 1, \beta < 1,$$

which is a new measure of weighted divergence.

In particular, for $k = 1$, we get (2.2).

p	q	$D^{\alpha,\beta}(P;Q:W)$
0.1	0.5	1.352
.2	0.5	0.81509
.3	0.5	0.38675
.4	0.5	0.10127
.05	0.5	0
0.6	0.5	0.10135
0.7	0.5	0.38745
0.8	0.5	0.81776
0.9	0.5	1.35985

Table 2.2: $D^{\alpha,\beta}(P;Q:W)$ against p for $n=2$ and $\alpha=2, \beta=0.7$

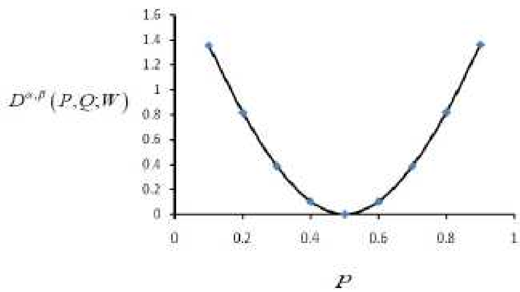

Figure 2

Next, for graphical presentation, we consider two cases as follows:

Case I: When $\alpha > 1, \beta < 1$, that is, $\alpha = 2, \beta = 0.7$ and $W = \{w_i, w_2\} = \{1.1, 1.2\}$, we have calculated various values for the divergence measure $D^{\alpha,\beta}(P;Q:W)$ as shown in the Table 2.2 and obtained the Figure 2.2 which clearly shows $D^{\alpha,\beta}(P;Q:W)$ that presented in (2.2) is convex.

Case-II: When $\alpha < 1, \beta > 1$, we again see that $D^{\alpha,\beta}(P;Q:W)$ is convex.

Proceeding on similar lines, one can generate many new weighted divergence measures.

3. Concluding Remarks

The measures of distance/directed divergence find tremendous applications in a variety of disciplines dealing with biological, economical, physical and engineering sciences. Since a single measure of distance cannot be adequate for each discipline, we need a variety of generalized measures to extend the scope of their applications. Further, these generalized measures induce flexibility and unbiasedness into the system, their preference towards optimization problems is highly recommendable. Keeping this idea in mind, we have generated two new measures of distance for the discrete probability distributions so as to provide their applications in furtherance of our research findings. With similar arguments, a variety of weighted and non-weighted divergence measures can be developed for continuous probability distributions.

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