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QUASISTASTIC STABILITY OF A REBRY SHELL OF INTERACTING WITH MOBILE LOAD

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Abstract: The relevance of the research is that the development of technology requires the creation of strong, reliable, durable structures that are used in many mechanisms. Therefore, the main goal of the work is to study the quasi-static stability of the ribbed shell, which is affected by the mobile load. For a full study, the work was based on methods of analysis, review, modeling, experiment, comparison. All of them allowed to decompose the investigated problem into elements and accordingly study each of them. As a result, the authors in a quasistatic formulation solved the problem of the stability of a shallow shell with a discrete arrangement of stiffeners under the action of a moving linear radial load. In this case, the spectrum of the critical velocities of the load is determined. The work is of practical importance in the construction of designs of lethal devices, for which a stable ribbed shell is used.

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Key Words: shell conduit, discrete arrangement of ribs, movable radial load, quasistatic solution, critical speed

1. Introduction

Scientific and technological progress requires the creation of new highly efficient

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and economical models of technology that would facilitate the operation conditions. All this requires accurate calculations, in which the strength, reliability, durability of structures increases. Correct calculations of such structures are possible only on the basis of modern advances in the mechanics of a deformed solid and computer technology.

The development of the aerospace industry leads to the need to develop new efficient methods for calculating structures operating under conditions of complex thermal force loading. At the same time, the problems of nonlinear deformation of structures, for which it is very important in calculations, is to take into account the behavior of the material at high loading and temperature levels [1]. Under these conditions, deformation is accompanied by the creep of metals and alloys, which has an unlimited character and can lead to the destruction of the structure during operation.

Creep of materials is a slow deformation that occurs over time under the influence of a constant load or mechanical stress. Creep is characteristic of all solids, both crystalline and amorphous [2].

Creep leads to all kinds of deformations, while their strength decreases, which leads to the destruction of mechanisms. Therefore, the task of creating methods for calculating the strength of designs of lethal devices that take into account the phenomenon of creep and which allow us to develop valid recommendations to designers and designers for improving the operational reliability of the created models of equipment is therefore topical in scientific and practical terms. The airborne apparatus is a generalized name for the device for flights in the atmosphere or in general, in outer space.

Calculations that are necessary to calculate the properties of the material of flying machines are especially important for thin-walled structures, which are an integral part of most types of equipment. Using shells and plates, such structures have been created in which strength and rigidity are combined with low weight. This is of great importance for aviation and missile technology. A significant contribution to the development of the theory of shells was made by A.L.Goldenweiser, S.A. Ambartsumian, V.Z. Vlasov, H.M.Mushtari, V.V. Bolotin, A.I. Lurie, V.V. Novozhilov and others.

2. Materials and Methods

The study of the stability of the ribbed shell under a mobile load has a variety of approaches. Therefore, the methods of analysis, review, comparison, modeling, and experiment were used in the work.

The method of analysis made it possible to divide the problem of the deformation of a material into many elements, which enabled them to learn their properties, relationships and relationships. All this contributed to a more detailed structuring of the investigated problem. Then, by means of synthesis, the decomposed particles were combined into a single whole.

The analogy method was used to study the stability properties of the ribbed shell under different moving loads. Based on these data, quasistatic stability was studied. The analogy method is similar to the comparison method [1].

With the help of experiment and modeling, the question of thin-walled structures that consist of shells has been studied, which find their application in different fields of technology, including those used in lethal devices. To give considerable rigidity, the plates and the shell are equipped with stiffening ribs.

With the help of the analysis it was also found that the structure of the shell is mesh, smooth, ribbed wavy. In this paper, it is the ribbed shells, in which the thin curvilinear wall is reinforced with ribs.

3. Results and Discussion

The loss of stability of thin-walled structural elements of aircraft is the most common form of their destruction. In this paper, the determination of the critical velocity of a pressure wave of a smooth, ribbed cylindrical shell moving along the surface is considered, which is the most common constructive element of all aviation and missile systems. The article takes into account the discreteness of the location of the stiffeners (stringers) in contrast to other works, where the ribbing is taken into account only in the framework of the constructively orthotropic model [3].

Consider a flat panel of a cylindrical shell, related to a curvilinear orthogonal coordinate system $0xyz$, in which a number of elastic one-dimensional reinforcing elements are parallel to the x-axis. Figure 1 shows only one stringer.

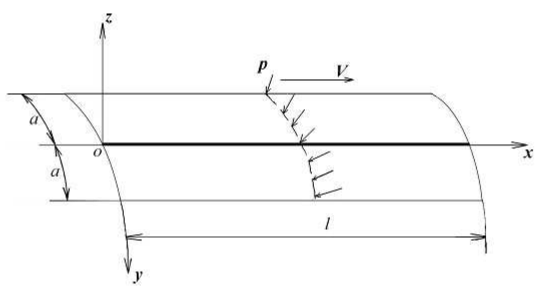

Figure 1: A sloping panel of a cylindrical shell

In this figure, an infinite uniform inertial load of intensity p moving at a constant speed V is conditionally shown in the form of linear forces distributed along a line perpendicular to the x axis. The neutral axis of the stringer lies in the middle surface of the shell, and therefore they can be considered as one-dimensional elastic inclusions [4]. When solving the problem, we neglect the mass and inertia of the movement of the proper structural elements. Its deformed state is considered quasi-static, in which the curved surface of the shell $w(x, y)$ does not depend on time. The element of the mobile load for a time interval t will pass the path $x = Vt$, and the total gravitational and inertial load developed by it in the direction normal to the shell surface is determined by the formula [1]

$$p - \frac{p}{g}\frac{\partial^2 w}{\partial t^2} = p - \frac{p}{g}V^2\frac{\partial^2 w}{\partial x^2} \quad (1)$$

To solve the problem, we mentally separate the stringers from the shell and replace their effect with unknown contact reactions normal to its surface $q_i(x)$, Distributed along the contact line of the bodies in the middle surface of the panel. Since the rigidity of the shell in the tangential directions is much larger than in the radial, then the tangential components of the interaction forces in the direction of the axes x and y in what follows we neglect. To describe the dynamic deformed state of a shell, we use its equations in mixed form with respect to the radial displacement (deflection) w and the stress function F, based on the hypotheses of the technical theory of shells [1]. In the case under consideration, the quasistatic shell deformation, taking into account the action of the contact reactions $q_i(x)$ they take the form:

$$\frac{D}{h}\nabla^2\nabla^2 w - \frac{1}{R}\frac{\partial^2 F}{\partial x^2} = \frac{p}{h} - \frac{p}{gh}V^2\frac{\partial^2 w}{\partial x^2} - \sum_{i=1}^{C} q_i\delta(y - y_i) \quad (2)$$

$$\frac{1}{E}\nabla^2\nabla^2 F + \frac{1}{R}\frac{\partial^2 w}{\partial x^2} = 0,$$

where ∇^2 – Laplace operator, D and h – cylindrical rigidity and shell thickness, respectively, g – gravitational acceleration, C – number of stringers, delta-function of Dirac $\delta(y - y_i)$ specify the coordinates of their location y_i along the axis y. Each of the stringers is assigned to a rectangular coordinate system $0xz$ (fig. 1). Equations of its equilibrium in the projection on the axis z has the form:

$$EJ_i\frac{d^4 z_i}{dx^4} = q_i, \ (i = 1, 2, \ldots, C), \quad (3)$$

where EJ_i and z_i – their flexural stiffness and deflections of stringer, respectively.

On each of the contact lines the condition of equality of deflections of both bodies $w(x, y_i) = z_i$, but we assume that the deformed states of the shell, caused by neighboring contact reactions, do not interfere with each other. Then substituting the contact reactions q_i of (3) into the bending equations of the shell (1), we obtain a resolving system of equations:

$$\frac{D}{h}\nabla^2\nabla^2 w - \frac{1}{R}\frac{\partial^2 F}{\partial x^2} + \frac{p}{gh}V^2\frac{\partial^2 w}{\partial x^2} + \sum_i^C \delta(y - y_i)(EJ_i\frac{\partial^4 w}{\partial x^4}) = \frac{p}{h}, \tag{4}$$

$$\frac{1}{E}\nabla^2\nabla^2 F + \frac{1}{R}\frac{\partial^2 w}{\partial x^2} = 0.$$

Equations (4) are a system of two partial differential equations with coefficients discontinuous with respect to the coordinate y, which is due to the presence in the first of the equations (4) of the Dirac delta functions that determine the position of the stringers on the shell surface [5]. In addition, in these equations the velocity of the load V enters as a parameter. For their approximate solution, we use the Bubnov method, in accordance with which we represent the deflection of the shell w and the stress function F in the form of expansions

$$w = \sum_m^K \sum_n^L w_{mn}\phi_{mn}(x, y), \;\; F = \sum_m^K \sum_n^L F_{mn}\psi_{mn}(x, y), \tag{5}$$

where w_{mn} and F_{mn} – unknown coefficients,

$\phi_{mn}(x, y)$ and $\psi_{mn}(x, y)$ – orthogonal forms of natural oscillations of a smooth shell panel.

Applying the procedure of the Bubnov method to equations (4), we arrive at a system of coupled linear algebraic equations of order $2 \times K \times L$ relatively unknown coefficients w_{mn} and F_{mn} in expansions (5). However, the algebraic equations corresponding to the second of the relations (4) due to the orthogonality of the coordinate functions in the expansions (5) break up into separate independent equations for each pair of values m and n. Express in them F_{mn} across w_{mn}. Taking this into account, after a number of transformations, the first equations (4) in the matrix form of the recording become:

$$\left[[K] - V^2 [M]\right]\{W\} = \{P\}, \tag{6}$$

where

$$K = [k_{mn}^{kl}], \;\; M = [m_{mn}^{kl}], \;\; W = \{w_{kl}\}, \;\; P = \{p_{kl}\}. \tag{7}$$

Dimension of stiffness matrices , Masses and vectors W and . Is determined by the number of terms of the series stored in the expansion (5). Matrix elements , and vectors F have the form:

$$k_{mn}^{kl} = \frac{D}{h}\int\limits_S \nabla^2\nabla^2\phi_{mn}\cdot\phi_{kl}dS + \frac{E}{R^2}\frac{\int\limits_S \frac{\partial^2\phi_{mn}}{\partial x^2}\psi_{kl}dS\cdot\int\limits_S \frac{\partial^2\psi_{mn}}{\partial x^2}\phi_{kl}dS}{\int\limits_S \nabla^2\nabla^2\psi_{mn}\cdot\psi_{mn}dS} +$$

$$+\sum_{i=1}^{C} EJ_i\int\limits_S \delta(y-y_i)\frac{\partial^4\phi_{mn}}{\partial x^4}\phi_{kl}dS,$$

$$m_{mn}^{kl} = \frac{p}{g}\left|\int\limits_S \frac{\partial^2\phi_{mn}}{\partial x^2}\phi_{kl}dS\right|,\ p_{kl} = \frac{p}{hg}\int\limits_S \phi_{kl}dS. \tag{8}$$

The stiffness matrix K has a block diagonal character, since the Dirac delta functions are located only on the y axis. In addition, in each of its blocks the first two terms in the stiffness coefficient k_{mn}^{kl} and all the mass coefficients are diagonal. At a given load speed V The solution of the coupled system of equations (6) is carried out by the usual methods [6]. If for some values of the speed V The deflections of the shell begin to increase indefinitely, this means that we are approaching its critical value, at which the loss of stability of the ribbed panel occurs [7]. If we assume in addition that the gravitational component of the load Is small in comparison with the inertial one, the lower part of the spectrum of critical velocities can be found from the condition that the determinant of the system of equations (6)

$$\det|K - V^2M| = 0. \tag{9}$$

Example. Let the shell be supported by one stringer located along the axis y ($y_1 = 0$) (fig.1). In the first approximation, we use a simplified approach based on the one-term approximation in the series (5) for arbitrary wave numbers m and n. We believe that all edges of the shell panel have free support on flexible inextensible edges, and it can deform with respect to the axis (fig.1) Or symmetrically or antisymmetrically [8]. In the case of symmetric deformation, the approximating function in the expansions (5) is taken in the form (10a), and for the antisymmetric function in the form (10b)

$$\phi_{mn} = \psi_{mn} = \sin\mu_m x\cos\lambda_n y, \tag{10a}$$

$$\phi_{mn} = \psi_{mn} = \sin\mu_m x\sin\lambda_n y, \tag{10b}$$

where $\mu_m = m\pi/l$, $\lambda_n = n\pi/ka$, ($k = 2$ or $k = 1$ under symmetric or antisymmetric deformation of the shell, respectively). In both cases, the square of the critical velocity of the load in the form $m \times n$ is determined by formula (11) obtained in accordance with condition (9).

$$V_{KP}^2 = \frac{g}{p\mu_m^2}\left[D(\mu_m^2+\lambda_n^2)^2 + \frac{Eh}{R^2}\frac{\mu_m^4}{(\mu_m^2+\lambda_n^2)^2} + \frac{2}{a}EJ\mu_m^4\right] \tag{11}$$

The first term of formula (11) corresponding to the critical velocity for a smooth plate completely coincides with the value of the velocity from the book [2]. From the point of view of practical applications, the minimum critical speed is most important, realized probably with the most simple form of stability loss $m = n = 1$ in the formula (11). The shell has the following dimensionless parameters: $l/R = 2,\ a/R = 0,5,\ v = 0,3$. In Fig. 2 shows the dependences of the dimensionless critical velocity of the load $V_{KP}^{2*} = V_{KP}^2 p/ghE$ of the relative thickness of the shell R/h at the dimensionless moment of inertia of the stringer $J/h^4 = 10^3$.

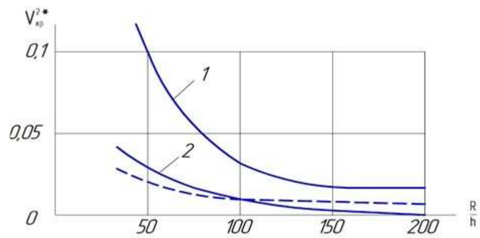

Figure 2: Dependence of the dimensionless critical velocity of the load on the relative thickness of the shell

Curve 1 corresponds to the symmetric deformation of the panel, and curve 2 corresponds to the antisymmetric deformation. The dotted line shows the dependence V_{KP}^{2*}. For a smooth shell without a stringer in the case of a symmetrical deformation of the panel.

Curves 1 and 2 confirm the fact that critical loads for an antisymmetric form of stability loss are lower than for a symmetric one, and the presence of even one stringer sharply increases the critical velocity of the load.

Conclusion

In the paper, the stability of a shallow shell with a discrete arrangement of stiffeners under the action of a moving linear radial load in quasi-static formulation was studied. Quasistatic processes play a role in thermodynamics in describing the system, which is in a state of thermodynamic equilibrium. Any factors can lead to a change in the state of equilibrium, which leads to processes that are difficult to describe. However, many processes in engineering with sufficient accuracy for practical purposes can be described as quasi-equilibrium. The model of quasistatic processes greatly simplifies the analysis of thermodynamic systems. When describing the current state of a system in which a quasistatic process occurs, the same parameters are required as for a macroscopic description of the equilibrium state.

Thus, the spectrum of the critical velocity of the load was determined in the work. It is established that for some values of the velocity the deflections of the shell increase. This process is critical, in which the loss of stability of the ribbed panel occurs. The given researches are considered on an example at which the shell is backed up by one stringer that is located along an axis.

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