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ON FUZZY SOFT EDGE DOMINATION

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Abstract: Let $G_{A,V}$ be a fuzzy soft graph (FSG). A subset of T is E said to be a *fuzzy soft edge dominating (FSED) set* if for every edge in $E - T$ is adjacent to at least one edge in T for each parameter $e \in A$. The minimum fuzzy soft cardinality of an edge dominating set is called a fuzzy soft edge domination number and is denoted by $\gamma^1(G_{A,V})$.

Key Words: Edge domination, fuzzy soft edge domination, fuzzy soft edge domination number, fuzzy soft t-edge domination and fuzzy soft t-edge domination number

1. Introduction

A Somasundaram and S Somasundaram [8] first introduced the concept of domination in fuzzy graphs. C.Y ponnappan, S. Basheer Ahamed and P. Surulinathan [5] discussed edge domination in fuzzy graphs. Sumit Mohinta and Samanta [7] introduced the notion of fuzzy soft graphs and some operations in fuzzy soft graphs and later on Muhammed Akram and Saira Nawas [6] introduced different types of fuzzy soft graphs and their properties.

In this paper we introduce the concepts of edge domination in fuzzy soft graphs, fuzzy soft edge domination number, fuzzy soft t-edge domination and fuzzy soft t-edge domination number.

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2. Preliminaries

Definition 2.1. [6]. Let be a graph with V be the set of all vertices and E be the set of all edges. If there exist two functions $\rho : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ such that $\mu(x,y) \leq \rho(x) \wedge \rho(y) \forall x, y \in V$. Then $G(\rho,\mu)$ is called a fuzzy graph.

Definition 2.2. [7]. Let $V = \{x_1, x_2, x_3, \ldots, x_n\}$ (non empty set) E (parameters set) and $A \subseteq E$. Also let

(i) $\rho : A \to F(V)$ collection of all fuzzy subsets V in and each element e of A is mapped to $\rho(e) = \rho_e$ (say) and $\rho_e : V \to [0,1]$, each element x_i is mapped to $\rho_e(x_i)$ and we call (A,ρ), a fuzzy soft vertex.

(ii) $\mu : A \to F(V \times V)$ collection of all fuzzy subsets in $V \times V$, which mapped each element e to $\mu(e) = \mu_e$ (say) and $\mu_e : V \times V \to [0,1]$, which mapped each element (x_i, x_j) to $\mu_e(x_i, x_j)$ and we call (A,μ) as a fuzzy soft edge.

Then $((A,\rho),(A,\mu))$, is called fuzzy soft graph if and only if $\mu_e(x_i,x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j) \forall e \in A$ and $\forall i,j = 1,2,3,\ldots,n$, this fuzzy soft graph is denoted by $G_{A,V}$.

Definition 2.3. [7]. The underlying crisp graph of a fuzzy soft graph $G_{A,V} = ((A,\rho),(A,\mu))$ is denoted by $G^* = (\rho^*, \mu^*)$, where $\rho^* \{x_i \in V; \rho_e(x_i) > 0$ for some $e \in E\}$ and

$$\mu^* = \{(x_i, x_j) \in V \times V; \mu_e(x_i,x_j) > 0 \text{ for some } e \in E\}.$$

Definition 2.4. [6]. A fuzzy soft graph $G_{A,V}$ is called a ***strong fuzzy soft graph*** if $\mu_e(x_i,x_j) = \rho_e(x_i) \wedge \rho_e(x_j) \forall (x_i,x_j) \in \mu^*, e \in A$ and is called a ***complete fuzzy soft graph*** if $\mu_e(x_i,x_j) = \rho_e(x_i) \wedge \rho_e(x_j) \forall x_i, x_j \in \rho^*, e \in A$ and is denoted by $K_{G_{A,V}}$.

Definition 2.5. [6]. Let $G_{A,V}$ be a fuzzy soft graph. Then the ***order*** of $G_{A,V}$ is defined as $O(G_{A,V}) = \sum_{e \in A} \left(\sum_{x_i \in V} \rho_e(x_i) \right)$ and ***size*** of $G_{A,V}$ is defined as $S(G_{A,V}) = \sum_{e \in A} \left(\sum_{x_i, x_j \in V} \mu_e(x_i,x_j) \right)$ and the ***degree*** of a vertex x_i is defined as $d_{(G_{A,V})}(x_i) = \sum_{e \in A} \left(\sum_{x_j \in V x_i \neq x_j} \mu_e(x_i,x_j) \right)$.

Definition 2.6. *Degree* of a fuzzy soft graph $G_{A,V}$ is defined as $D_{G_{A,V}} = \max\{d_{G_{A,V}}(x_i) : \forall x_i \in V\}$.

Definition 2.7. A fuzzy soft graph $G_{A,V}$ is said to be *regular fuzzy soft graph* if the fuzzy graph corresponding to each parameter $e \in A$ is a regular fuzzy graph.

Definition 2.8. A fuzzy soft graph $G_{A,V}$ without fuzzy soft loops and fuzzy soft multiple edges is called a *fuzzy soft simple graph.*

Definition 2.9. An FSG is said to be *isolated* if $\mu_e(x_i, x_j) = 0 \forall (x_i, x_j) \in E, e \in A$.

Definition 2.10. Two fuzzy soft vertices are said to be adjacent if they are connecting by a fuzzy soft edge and two fuzzy soft edges are said to be adjacent if they are incident on a common fuzzy soft vertex.

Definition 2.11. The *complement* of a FSG $G_{A,V}$ is defined as $\overline{G}_{A,V} = ((A, \rho), (\overline{A, \mu}))$ where $\overline{\mu}_e(x_i, x_j) = \rho_e(x_i) \wedge \rho_e(x_j) - \mu_e(x_i, x_j) \forall (x_i, x_j) \in E \,\&\, e \in A\}$.

Definition 2.12. *Neighborhood of a vertex* is defined as $N(x_i) = \{x_j \in V \mu_e(x_i, x_j) = \rho_e(x_i) \wedge \rho_e(x_j) \forall e \in A\}$.

Definition 2.13. Let $G_{A,V}$ be an FSG. $\delta^1(G_{A,V}) = \min\{d_{(G_{A,V})}(x_i) | \forall x_i \in V\}$ & $\Delta^1(G_{A,V}) = \max\{d_{(G_{A,V})}(x_i) | \forall x_i \in V\}$.

Definition 2.14. The *edge degree* of a FSG is defined as $d^1_{(G_{A,V})}(x_i, x_j) = d_{(G_{A,V})}(x_i) + d_{(G_{A,V})}(x_j)$. $N(x_i, x_j)$ is the set of all edges incident with the vertices and x_i and x_j.

3. Fuzzy Soft Edge Dominating (FSED) Set

Definition 3.1. Let $G_{A,V}$ be a fuzzy soft graph. A subset T of E is said to be a *fuzzy soft edge dominating (FSED) set* if for every edge in $E - T$ is adjacent to at least one edge in T for each parameter $e \in A$. The minimum cardinality of fuzzy soft edge dominating set is called *fuzzy soft edge domination number* and is denoted by $\gamma^1(G_{A,V})$.

Definition 3.2. Suppose $G_{A,V}$ is a FSG and T be an FSED. Then T is said to be *minimal FSED* set if deletion of an edge from T is not a FSED set of $G_{A,V}$.

Theorem 3.3. *If $G_{A,V}$ is any fuzzy soft graph, then $\gamma^1(G_{A,V}) + \overline{\gamma}^1(\overline{G}_{A,V} \leq 2S(G_{A,V})$. And $\gamma^1(G_{A,V}) + \overline{\gamma}^1(\overline{G}_{A,V}) = 2S(G_{A,V})$ if and only if $0 < \mu_e(x_i, x_j) < \rho_e(x_i) \wedge \rho_e(x_j) \forall (x_i, x_j) \in E \,\&\, e \in A\}$.*

Proof. Suppose $G_{A,V}$ is any FSG. Then $\gamma^1(G_{A,V}) + \overline{\gamma}^1(\overline{G}_{A,V}) < 2S(G_{A,V})$ is trivial. Further $\gamma^1(G_{A,V}) = S(G_{A,V})$ if and only if $\mu_e(x_i, x_j) < \rho_e(x_i) \wedge \rho_e(x_j) \forall (x_i, x_j) \in E \,\&\, e \in A\}$ and $\overline{\gamma}^1(\overline{G}_{A,V}) = S(G_{A,V})$ if and only if $\rho_e(x_i) \wedge$

$\rho_e(x_j) - \mu_e(x_i, x_j) < \rho_e(x_i) \wedge \rho_e(x_j) \forall (x_i, x_j) \in E \,\&\, e \in A\}$. This implies $\mu_e(x_i, x_j) > 0 \forall (x_i, x_j) \in E \,\&\, e \in A$.

Hence $\gamma^1(G_{A,V}) + \overline{\gamma}^1(\overline{G}_{A,V}) = 2S(G_{A,V})$ if and only if $0 < \mu_e(x_i, x_j) < \rho_e(x_i) \wedge \rho_e(x_j) \forall (x_i, x_j) \in E \,\&\, e \in A\}$. □

Theorem 3.4. *If T is minimal FSED set then for each edge in T, one of the following two conditions holds.*

(a) $N(x_i, x_j) \cap T = \phi$.

(b) There exist an edge $(x_m, x_n) \in E - T$ such that $N(x_m, x_n) \cap T = \{(x_i, x_j)\}$.

Proof. Suppose T is a minimal FSED set and $(x_i, x_j) \in T$. Then $T - \{(x_i, x_j)\}$ is not an FSED set and hence there exist $(x_m, x_n) \in E - [T - \{(x_i, x_j)\}]$ such that (x_m, x_n) is not dominated by any element of $T - \{(x_i, x_j)\}$. If $\{(x_m, x_n)\} = \{(x_i, x_j)\}$ then we get $N(x_i, x_j) \cap T = \phi$ and if $\{(x_m, x_n)\} \neq \{(x_i, x_j)\}$, then we get $N(x_m, x_n) \cap T = \{(x_i, x_j)\}$. □

Definition 3.5. Let $G_{A,V}$ be a fuzzy soft graph. An edge (x_i, x_j) is said to be an isolated fuzzy soft edge if no edge incident on x_i and x_j for each parameter $e \in A$.

Theorem 3.6. *If $G_{A,V}$ be a fuzzy soft graph without isolated edges then for every minimal FSED set T, its complement is also an FSED.*

Proof. Suppose $G_{A,V}$ be an edge in T. Since $G_{A,V}$ has no isolated edges, there exist an edge $(x_s, x_t) \in N(x_m, x_n)$. By above theorem we get $(x_m, x_n) \in E - T$. Thus every element of T is dominated by some element of $E - T$. □

Theorem 3.7. *Suppose (x_m, x_n) is an FSG without isolated edges then* $\frac{S(G_{A,V})}{\Delta^1(G_{A,V}+1)} \geq \gamma^1_{fs}(G_{A,V})$.

Proof. Let P be a FSED set. We have

$$|p|\Delta^1(G_{A,V}) \leq \sum_{(x_i,x_j)\in p} d(x_i, x_j)$$

$$= \sum_{(x_i,x_j)\in p} |N(x_i, x_j)| \leq \left| \bigcup_{(x_i,x_j)\in P} N(x_i, x_j) \right| \leq S(G_{A,V}) - |p|,$$

$$\because \qquad |p|\Delta^1(G_{A,V}) + |P| \leq S(G_{A,V}) \Rightarrow |p| \leq \frac{S(G_{A,V})}{\Delta^1(G_{A,V}) + 1}$$

$$\Rightarrow \gamma^1_{fs}(G_{A,V})\frac{S(G_{A,V})}{\Delta^1(G_{A,V})+1}$$

Hence the theorem. □

4. Fuzzy Soft *t*-Edge Dominating (FSED-*t*) Set

Definition 4.1. An FSED set T of an FSG $G_{A,V}$ is said to be a *fuzzy soft t-edge dominating (FSED-t) set* if for every edge in $E-T$ is adjacent to at least t-edges in T for each parameter $e \in A$. The minimum fuzzy soft cardinality of an FSED-t set is called *fuzzy soft t-edge domination number* and is denoted by $\gamma^1_t(G_{A,V})$.

Definition 4.2. Suppose $G_{A,V}$ is an FSG and T be an FSED-t. Then T is said to be *minimal FSED-t* set if deletion of an edge from T is not a FSED set of $G_{A,V}$.

Theorem 4.3. *If $G_{A,V}$ is a complete FSG with even number of vertices, then there must exists fuzzy soft t-edge dominating sets and the minimum value of t is $\frac{n}{2}$ and $n \geq 4$.*

Proof. Suppose $G_{A,V}$ is a complete fuzzy soft graph with even number of vertices.

First assume $n = 4$. Choose an edge which is adjacent to exactly $2(4-2) = 4$ edges. Next choose one edge from the remaining 2 edges so that it will become an FSED-t set and the minimum value of t is $\frac{4}{2} = 2$.

Hence the theorem is true for $n = 4$.

Next assume $n = 6$. Choose an edge which is adjacent to exactly $2(6-2) = 8$ edges. Now there are 6 edges remaining. Then as in the earlier case there are 2 edges in FSED-t. Hence totally FSED set containing 3 elements so that the minimum value of t is $\frac{6}{2} = 3$.

Continuing like this let us assume that the result is true for $< n$ vertices.

Now we prove the case for n. Choose an edge so that it will be adjacent to exactly edges $2(n-2)$. Then the remaining number of edges is equal to

$$\frac{n(n-1)}{2} - 1 - 2(n-2) = \frac{n^2 - n - 2 - 4n + 8}{2}$$
$$= \frac{n^2 - 5n + 6}{2} = \frac{(n-3)(n-2)}{2} = (n-2)C_2.$$

But the result is true for $< n$ vertices. Hence for $(n-2)C_2$ edges the minimum value of t in FSED-t is $\frac{(n-1)}{2}$. So the total number of edges in FSED-t is $\frac{(n-2)}{2} + 1 = \frac{n}{2}$.

Hence the minimum value of t in FSED-t is $\frac{n}{2}$.

□

Theorem 4.4. *If $G_{A,V}$ is a complete FSG with even number of vertices, then there must exists fuzzy soft t-edge dominating sets and the minimum value of t is $\frac{n}{2}$ and $n \geq 4$.*

Proof. The proof is on the same way as above. □

5. Conclusion

In this paper we defined new concepts such as fuzzy soft edge domination (FSED), fuzzy soft edge domination number, fuzzy soft t-edge domination and fuzzy soft t-edge domination number and proved some theorems related to this. The fuzzy soft edge domination and fuzzy soft edge domination number are very useful for solving wide range of problems.

References

[1] E. J. Cockayne and S. T. Hedetniemi, Towards a theory of domination in graphs, Networks. (1977), 247–261.

[2] Anas Al-Masarwah.Majdoleen Abu Qamar, some New Concepts of Fuzzy Soft Graphs, *Fuzzy Inf. Eng.* **8**, (2016), 427–438.

[3] S. Ramya and S. Lavanya, Perfect Vertex(Edge) domination in Fuzzy Graphs, *Indian Journal of Science and Technology*, **9**, No. 4. DOI: 10.17485/ijst/2016, January 2016.

[4] C. Y. Ponnappan, S. Basheer Ahamed and P. Surulinathan, Edge domination in Fuzzy Graphs- New approach, *IJIEASR*, **4**, No. 1. January 2015.

[5] A. Nagoor Gani and V.T.Chandra Sekaran, Domination in Fuzzy Graph, *Advances in fuzzy Sets and Systems*, **1**, No. 1 (2006), 17–26

[6] Muhammad Akram and Saira Nawaz, On Fuzzy Soft Graphs, *Italian Journal of Pure and Applied Mathematics- N*, **34**, (2015), 497–514.

[7] Sumit Mohinta and T.K.Samanta, An Introduction to Fuzzy Soft Graph, *Mathematica Morarica*, **19**, No. 2 (2015), 35–48.

[8] A Somasundaram and S.Somasundaram, Domination of Fuzzy Graphs-I, *Elsevier Science*, **19** (1988), 787–791.