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A SHORT NOTE ON RICCI TENSOR AND BIHARMONIC HYPERSURFACES

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Abstract: There is a conjecture due to Chen, that every complete biharmonic submanifold of a Euclidean space is minimal. Several papers gave some affirmative partial answers to this conjecture. We focus on the spacial case in which the submanifold is a hypersurface in Euclidean space that its Ricci tensor satisfies special identity. We also obtain some results for L_k-biharmonic hypersurfaces with the same property.

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1. Introduction

Let $x : M^n \to R^m$ be an isometric immersion from an n-dimensional Riemannian manifold M into the m-dimensional Euclidean space R^m. Then M is said to be biharmonic if $\triangle^2 f = 0$, where $\triangle$ is the Laplacian operator on M. This definition of biharmonic submanifolds is equivalent to say that f is a biharmonic map i.e. the critical point of bienergy functional (see [1]). A proper biharmonic is a non-harmonic biharmonic map, thus proper biharmonic submanifolds have proper biharmonic inclusion maps (see [1]).

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The manifold M is said of k-type if the position vector of x can be decomposed as follows

$$x = x_0 + x_{i_1} + ... + x_{i_K} \tag{1}$$

where $\triangle x_{i_j} = \lambda_{i_j} x_{i_j}$, $\lambda i_1 < ... < \lambda_{i_k}$, x_0 is a constant vector and $\triangle$ is the extension of Laplace operator. In special case, when some $\lambda_{i_j} = 0$, then M is called of null k-type (see [6]).

The first Chen conjecture saying that every biharmonic Riemannian submanifold is minimal [2]. There are some partial affirmative answers to this conjecture [2, 3] Dimitric gave one of these partial affirmative answers to Chen conjecture (see [5]). He proved that biharmonic hypersurfaces with at most two distinct principal curvatures are mininal. A concircular vector field ν on a Riemannian manifold M is a vector field which satisfy $\bigtriangledown_X \nu = \mu X$, where $\bigtriangledown$ denotes the Levi-Civita connection on M, X belongs to the tangent bundle TM of M and μ a nontrivial function on M. A concircular vector field ν is called a concurrent vector field if $\bigtriangledown_X \nu = X$.

A smooth vector field ζ on a Riemannian manifold (M, g) is said to define a Ricci soliton if it satisfies $\frac{1}{2} L_\zeta g + Ric = \lambda g$, where L_ζ is the Lie-derivative of the metric tensor g with respect to ζ, Ric is the Ricci tensor of (M, g) and λ is a constant. It is shown by theorem 6.1 of [4] that a submanifold M^n in N^m admits a Ricci soliton if and only if the Ricci tensor of M^n satisfies

$$Ric(X, Y) = (\lambda - \mu) g(X, Y) - \tilde{g}(h(X, y), v^\perp) \tag{2}$$

for smooth vector field v on $\mathbb{E}^{n+1}$, real valued functions λ and μ and any tangent vector fields X, Y. Let L_k be the linearized operator of the $(k+1)$-th mean curvature H_{k+1} of the Euclidean hypersurface $f : M \to \mathbb{R}^{n+1}$. If M satisfying the equation $L_k^2 x = 0$, then M is called a L_k-biharmonic hypersurface.

2. Some Results

Now, by ues of the corollary 6.5 in [4] we have the following results.

Theorem 1. *Let (M^n, g) be a biharmonic hypersurface of $(\mathbb{E}^{n+1}, \tilde{g})$ whit Ricci tensor that satisfies (1). Then:*

a) M^n cannot be proper.

b) M^n has constant mean curvature.

c) M^n cannot be of null 3-type.

Proof. By theorem 5.2 in [4], Euclidean spaces are the only Riemannian manifolds of constant sectional curvature that admit Ricci solitons with concircular potential fields. This and assumption yield that M^n has at most two distinct principal curvature (corollary 6.5 of [4]). So:

a) by theorem 5 of [1], M^n cannot be a proper biharmonic hypersurface.

b) M^n has constant curvature, by theorem 4 of [1].

c) the result is obvious, by part b and proposition 4.12 of [6] . □

The part b of above theorem can be sharpened in the following theorem.

Theorem 2. *If (M^n, g) is a biharmonic hypersurface of $(\mathbb{E}^{n+1}, \tilde{g})$ whit Ricci tensor that satisfies (1), then M is minimal. Moreover scalar curvature τ of M^n is equal to $\frac{n(\lambda-\mu)}{2}$.*

Proof. As a hypersurface of $\mathbb{E}^{n+1}$, M^n has at most two distinct principal curvature by proof of theorem 2.1. Therefore, M^n is minimal by [5]. Furthermore, the value of scalar curvature is determined by corollary 6.4 in [4]. □

Theorem 3. *For $n \geq 4$, Let M^n be a L_k-biharmonic hypersurface in Euclidean space $\mathbb{R}^{n+1}$ for some k, $1 \leq k \leq n-1$. If M^n endows with concircular vector field v which satisfy in (1), then M^n is k-minimal.*

Proof. Similar to proof of Theorem 2.1, M^n has at most two distinct principal curvatures.

So, we have $H_{k+1} = 0$ or M^n is k-minimal by theorems 5 and 6 of [7]. □

By using theorem 1.3 and 1.6 in [8] respectively, we also obtain the following result.

Corollary 4. *Let M^n be a hypersurface in $\mathbb{E}^{n+1}$ whit Ricci tensor that satisfies (1), then M^n cannot be of L_{n-1}-null-2-type. Furthermore, if M^n has constant $(k+1)$-mean curvature, then M^n cannot be of L_k-null-3-type.*

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