International Journal of Pure and Applied Mathematics
Volume 117 No. 1 2017, 69-80
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v117i1.7
ijpam.eu

ISOMORPHISM ON ANTIFUZZY GRAPHS

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Abstract: The definition of complement of antifuzzy graphs is modified. The notion of isomorphism of antifuzzy graphs is introduced. Isomorphism between complement of join and union of two distinct antifuzzy graphs is discussed.

AMS Subject Classification: 05C72, 03E72

Key Words: antfuzzy graphs, complement of antifuzzy graphs, isomorphism on antifuzzy graphs

1. Introduction

The theory of fuzzy graphs was developed by Azriel Rosenfeld in 1975 [4]. Some operations of fuzzy graphs have been studied in [1] [2]. Sunitha and Vijayakumar [8] discussed about the complement of fuzzy graphs. Seethalakshmi and Gnanajothi [5] introduced the notion of antifuzzy graphs and discussed some of its properties. Nagoorgani and Malarvizhi [3] defined antipodal fuzzy graph and discussed its nature. Seethalakshmi and Gnanajothi [6], introduced antipodal antifuzzy graphs and discussed its properties. In this paper, isomorphism of antifuzzy graphs is introduced and it is discussed under various graph operations.

Received: 2017-06-06
Revised: 2017-07-21
Published: November 28, 2017

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2. Preliminaries

Definition 2.1. [5] Let S_1 and S_2 be two sets, σ_1 and σ_2 be fuzzy two sets of S_1 and S_2 respectively. That is $\sigma_1 : S_1 \to [0,1]$ and $\sigma_2 : S_2 \to [0,1]$. A function $\mu : S_1 \times S_2 \to [0,1]$ is called an anti-fuzzy relation on $\sigma_1 \times \sigma_2$ if

$$\mu(x,y) \geq \sigma(x) \vee \sigma(y), \forall x, y \in S_1 \times S_2,$$

where $\vee$ denote the maximum.

Definition 2.2. [5] An anti-fuzzy graph $\mathcal{A} = (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ with $\mu(u,v) \geq \sigma(u) \vee \sigma(v), \forall u, v \in V$, where V is a finite non-empty set and $\vee$ denote maximum.

Definition 2.3. [5] The graph $\mathcal{A}^* = (V, E)$ is called the underlying crisp graph of the anti-fuzzy graph $\mathcal{A}$ where $V = \{u/\sigma(u) < 1\}$ and $E = \{(u,v) \in V \times V/\mu(u,v) < 1\}$.

Definition 2.4. [5] An anti-fuzzy graph $\mathcal{A} = (\sigma, \mu)$ is said to be strong if $\mu(u,v) = \sigma(u) \vee \sigma(v), \forall (u,v) \in E$.

Definition 2.5. [5] An anti- fuzzy graph $\mathcal{A} = (\sigma, \mu)$ is said to be complete if the underlying graph $\mathcal{A}$ is complete and $\mu(u,v) = \sigma(u) \vee \sigma(v)$ for all $u, v \in V$.

Definition 2.6. [6] Let $\mathcal{A} = (\sigma, \mu)$ on (V, E) be an antifuzzy graph. The $\mu-$ distance $\delta(u,v)$ is the smallest length of any $u - v$ path where $\mu-$length of a path $p : u_0, u_1, u_2, \cdots, u_n$ is $\delta(p) = \sum_{i=1}^{n} \mu(u_{i-1}, u_i)$

The eccentricity of a node v is defined as $e(v) = max\{\delta(u,v)\}$.

The diameter of $\mathcal{A}$ is $diam\mathcal{A} = \vee \{e(v)/v \in V\}$.

The radius of $\mathcal{A}$ is $r(\mathcal{A}) = \wedge \{e(v)/v \in V\}$.

A node whose eccentricity if minimum in a connected antifuzzy graph is called a central node.

A connected antifuzzy graph is called self centered if each node is a central node.

Definition 2.7. [6] Let $\mathcal{A} = (\sigma, \mu)$ on (V, E) be an antifuzzy graph. Then the antipodal antifuzzy graph of $\mathcal{A}, A(\mathcal{A}) = (\sigma_A, \mu_A)$ on (V_A, E_A) is defined as follows:

The nodeset of $A(\mathcal{A})$ is the node set of $\mathcal{A}$.

Two nodes in $A(\mathcal{A})$ are made as neighbours if the $\mu-$distance between them is diam$\mathcal{A}$. Therefore, $\sigma_A(u) = \sigma(u), \forall u \in V$.

If $\delta(u,v) =$ diam $\mathcal{A}$, then

$$(\mu_{\mathcal{A}})(u,v) = \begin{cases} \mu(u,v) & \text{if } u \text{ } and \text{ } v \text{ } are \text{ } neighbours \text{ } in \text{ } \mathcal{A} \\ \sigma(u) \vee \sigma(v) & \text{if } u \text{ } and \text{ } v \text{ } are \text{ } not \text{ } neighbours \text{ } in \text{ } \mathcal{A} \end{cases}$$

If $\delta(u,v) \neq$ diam $\mathcal{A}$ then $\mu_A(u,v) = 1$

Definition 2.8. [5] Let $\mathcal{A}_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $\mathcal{A}_2 = (\sigma_2, \mu_2)$ on (V_2, E_2) be two anti fuzzy graphs. Then the union of $\mathcal{A}_1$ and $\mathcal{A}_2$ is defined as $\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2 = (\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2)$ on (V, E), where $V = V_1 \cup V_2, E = E_1 \cup E_2$.

$$(\sigma_1 \cup \sigma_2)(u) = \begin{cases} \sigma_1(u) & \text{if } u \in V_1 - V_2 \\ \sigma_2(u) & \text{if } u \in V_2 - V_1 \\ min\{\sigma_1(u), \sigma_2(u)\} & \text{if } u \in V_1 \cap V_2 \end{cases}$$

and

$$(\mu_1 \cup \mu_2)(u,v) = \begin{cases} \mu_1(u,v) & \text{if } (u,v) \in E_1 - E_2 \\ \mu_2(u,v) & \text{if } (u,v) \in E_2 - E_1 \\ min\{\mu_1(u,v), \mu_2(u,v)\} & \text{if } (u,v) \in E_1 \cap E_2 \end{cases}$$

Definition 2.9. [5] Let $\mathcal{A}_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $\mathcal{A}_2 = (\sigma_2, \mu_2)$ on (V_2, E_2) be two anti fuzzy graphs. Let $V = V_1 \cup V_2$ and $E = E_1 \cup E_2 \cup E'$ where E' is the set of edges joining of the nodes of V_1 and V_2. Assume that $V_1 \cap V_2 \neq \phi$. Then the join of $\mathcal{A}_1$ and $\mathcal{A}_2$ is $\mathcal{A} = (\sigma, \mu) = (\sigma_1 + \sigma_2, \mu_1 + \mu_2)$ on (V, E) is defined as

$$\begin{aligned} (\sigma_1 + \sigma_2)(u) &= (\sigma_1 \cup \sigma_2)(u), \forall u \in V_1 \cup V_2 \quad and \\ (\mu_1 + \mu_2)(u,v) &= \begin{cases} (\mu_1 \cup \mu_2)(u,v) & \text{if } (u,v) \in E_1 \cup E_2 \\ max\{\sigma_1(u), \sigma_2(u)\} & \text{if } (u,v) \in E' \end{cases} \end{aligned}$$

3. Complement of Antifuzzy graph

In our paper [5], Complement of antifuzzy graph was defined as:

Definition 3.1. Let $\mathcal{A} = (\sigma, \mu)$ on (V, E) be an antifuzzy graph. Then the complement $\overline{\mathcal{A}}$ of $\mathcal{A}$ is defined as $\overline{\mathcal{A}} = (\overline{\sigma}, \overline{\mu})$, where $\overline{\sigma} = \sigma$ and $\overline{\mu}(u,v) = 1 - \mu(u,v) + (\sigma(u) \vee \sigma(v))$, $\forall (u,v) \in E$.

Remark 3.2. In the above definition, $u, v \in V$ and $(u,v) \notin E$ are not taken care of. So, we modify the definition of complement as follows.

Definition 3.3. Let $\mathcal{A} = (\sigma, \mu)$ on (V, E) be an antifuzzy graph. Then the complement $\overline{\mathcal{A}}$ of $\mathcal{A}$ is defined as $\overline{\mathcal{A}} = (\overline{\sigma}, \overline{\mu})$, where $\overline{\sigma} = \sigma$ and

$$\overline{\mu}(u,v) = \begin{cases} 1 - \mu(u,v) + (\sigma(u) \vee \sigma(v)) & \text{for all } (u,v) \in E \\ \sigma(u) \vee \sigma(v) & \text{for } (u,v) \notin E \end{cases}$$

Theorem 3.4. If $\mathcal{A}$ is strong antifuzzy graph on a cycle, then $\mathcal{A} \cup \overline{\mathcal{A}}$ is complete antifuzzy graph.

Proof. Let $\mathcal{A} = (\sigma, \mu)$ be a strong antifuzzy graph on a cycle.
Then, $\mu(u,v) = \sigma(u) \vee \sigma(v), \forall (u,v) \in E$
Let $\overline{\mathcal{A}} = (\overline{\sigma}, \overline{\mu})$ be the complement of $\mathcal{A}$
Then, $\sigma = \overline{\sigma}$ and $\overline{\mu}(u,v) = 1 - \mu(u,v) + (\sigma(u) \vee \sigma(v))$
If, $(u,v) \in E$, then $\overline{\mu}(u,v) = 1$ and
if $(u,v) \notin E$, then $\overline{\mu}(u,v) = \sigma(u) \vee \sigma(v)$ as $\mu(u,v) = 1$.
Hence, the vertices which are not neighbours in $\mathcal{A}$ are neighbours in $\overline{\mathcal{A}}$
So, the underlying graph of $\mathcal{A} \cup \overline{\mathcal{A}}$ is complete and $\mathcal{A}$ is strong antifuzzy graph.

Since union of two disjoint strong antifuzzy graphs is again a strong antifuzzy graph, $\mathcal{A} \cup \overline{\mathcal{A}}$ is strong antifuzzy graph.

Hence, $\mathcal{A} \cup \overline{\mathcal{A}}$ is a complete antifuzzy graph.

Theorem 3.5. If $\mathcal{A}$ is any antifuzzy graph, the underlying graph of $\overline{\mathcal{A}}$ is complete if $\mu(u,v) \neq \sigma(u) \vee \sigma(v), \quad for\ any \quad u, v \in V$.

Proof. Let $\mathcal{A} = (\sigma, \mu)$ be an antifuzzy graph on (V, E) with $\mu(u,v) \neq \sigma(u) \vee \sigma(v),\ for\ any\ u, v \in V$.
Let $\overline{\mathcal{A}} = (\overline{\sigma}, \overline{\mu})$ be the complemet of $\mathcal{A}$
Case 1 Let $(u,v) \in E$.
Then $\mu(u,v) > \sigma(u) \vee \sigma(v)$. Therefore, $\sigma(u) \vee \sigma(v) - \mu(u,v) < 0$.
Hence, $\overline{\mu}(u,v) > 0$.
Case 2 Let $(u,v) \notin E$.
Then, $\mu(u,v) = 1$. Therefore, $\overline{\mu}(u,v) = \sigma(u) \vee \sigma(v) > 0$.
Hemce the underlying graph of $\overline{\mathcal{A}}$ is complete.

4. Isomorphism of Antifuzzy Graphs

Definition 4.1. Let $\mathcal{A}_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $\mathcal{A}_2 = (\sigma_2, \mu_2)$ on (V_2, E_2) be two antifuzzy graphs.

1. A homomorphism $h : \mathcal{A}_1 \to \mathcal{A}_2$ is a map $h : V_1 \to V_2$ which satisfies
   $$\sigma_1(u) \geq \sigma_2(h(u)), \forall u \in V \text{ and } \mu_1(u,v) \geq \mu_2(h(u), h(v)), \forall u, v \in V.$$
2. An isomorphism $h : \mathcal{A}_1 \to \mathcal{A}_2$ is a bijective map $h : V_1 \to V_2$ which satisfies $\sigma_1(u) \geq \sigma_2(h(u)), \forall u \in V$ and $\mu_1(u,v) \geq \mu_2(h(u), h(v)), \forall u, v \in V$.

   Then $\mathcal{A}_1$ is said to be isomorphic to $\mathcal{A}_2$

3. A wead isomorphism $h : \mathcal{A}_1 \to \mathcal{A}_2$ is a map $h : V_1 \to V_2$ which is a bijective homomorphism that satisfies $\sigma_1(u) = \sigma_2(h(u), \forall u \in V$.

4. A co-wead isomorphism $h : \mathcal{A}_1 \to \mathcal{A}_2$ is a map $h : V_1 \to V_2$ which is a bijective homomorphism that satisfies $\mu_1(u, v) = \mu_2(h(u), h(v)), \forall u, v \in V$

Definition 4.2. $\mathcal{A}_1$ is said to be isometric with $\mathcal{A}_2$ if for each $v \in \mathcal{A}_1$, there is a bijection $\phi : V_1 \to V_2$ such that $\delta_1(u, v) = \delta_2(\phi(u), \phi(v))$, for every $u \in V_1$.

Definition 4.3. An antifuzzy subgraph $\mathcal{B} = (\tau, \rho)$ is said to be a spanning antifuzzy subgraph of $\mathcal{A} = (\sigma, \mu)$ on (V, E) if $\tau(u) = \sigma(u), \forall u \in V$.

In this case, two antifuzzy graphs have the same antifuzzy node set. They differ only in the arc weight.

Theorem 4.4. Let $\mathcal{A}_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $\mathcal{A}_2 = (\sigma_2, \mu_2)$ on (V_2, E_2) be two antifuzzy graphs. If $\mathcal{A}_1$ is isomorphic to $\mathcal{A}_2$, then $\mathcal{A}_1$ is isometric with $\mathcal{A}_2$,

Proof. Since $\mathcal{A}_1$ is isomorphic to $\mathcal{A}_2$, there exist a bijection $h : V_1 \to V_2$ such that $\sigma_1(u) = \sigma_2(h(u)), \forall u \in V_1$ and $\mu_1(u, v) = \mu_2(h(u), h(v)), \forall u, v \in V_1$

Also, for all $u \in V_1$,

$$\begin{aligned} \delta_1(u, v) &= \wedge \left\{ \sum_{i=0}^{n} \mu_1(u_{i-1}, u_i) \right\}, \quad \textit{where} \;\; u_0 = u, u_n = v \\ &= \wedge \left\{ \sum_{i=0}^{n} \mu_2(h(u_{i-1}), h(u_i)) \right\} \\ &= \delta_2(h(u), h(v)), \quad \forall v \in V_1 \end{aligned}$$

Hence, $\mathcal{A}_1$ is isometric with $\mathcal{A}_2$.

Observation. The above result is true even when $\mathcal{A}_1$ is coweak isomorphic to $\mathcal{A}_2$.

Theorem 4.5. Let $\mathcal{A}_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $\mathcal{A}_2 = (\sigma_2, \mu_2)$ on (V_2, E_2) be two antifuzzy graphs. If $\mathcal{A}_1$ is isomorphic to $\mathcal{A}_2$, then $\overline{\mathcal{A}_1}$ is isomorphic with $\overline{\mathcal{A}_2}$,

Proof. Since $\mathcal{A}_1$ is isomorphic to $\mathcal{A}_2$, there exist a bijection $h : V_1 \to V_2$ such that $\sigma_1(u) = \sigma_2(h(u)), \forall u \in V_1$ and $\mu_1(u, v) = \mu_2(h(u), h(v)), \forall u, v \in V_1$

Now, $\overline{\sigma_1}(u) = \overline{\sigma_2}(h(u))$, from definition.

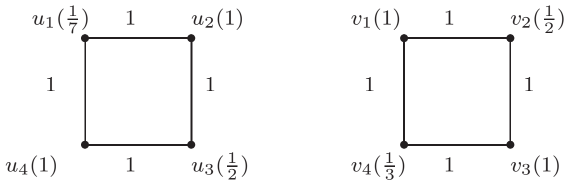
$$\textit{Also,} \;\; \overline{\mu_1}(u, v) \;=\; 1 - \mu_1(u, v) + (\sigma_1(u) \vee \sigma_1(v))$$

$$
\begin{aligned}
&= 1-\mu_2(h(u),h(v))+(\sigma_2(h(u)\vee\sigma_2(h(v)) \\
&= \overline{\mu_2}(h(u),h(v)), \quad \forall\, u,v\in V_1.
\end{aligned}
$$

Hence, $\overline{\mathcal{A}_1}$ is isomorphic with $\overline{\mathcal{A}_2}$.

Remark. If $\mathcal{A}_1$ is isometric with $\mathcal{A}_2$, then $\overline{\mathcal{A}_1}$ need not be isometric with $\overline{\mathcal{A}_2}$

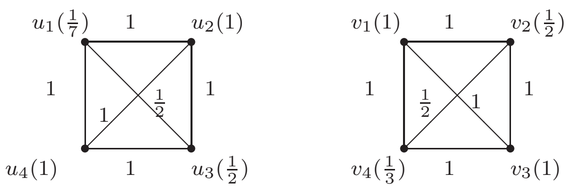
Example 4.6.



$\mathcal{A}_1$ $\mathcal{A}_2$

Define $\phi : V_1 \to V_2$ such that $\phi(u_1) = v_1, \phi(u_2) = v_2, \phi(u_3) = v_3$ and $\phi(u_4) = v_4$.

Then ϕ is one-one and onto.



$\overline{\mathcal{A}_1}$ $\overline{\mathcal{A}_2}$

Here, $\overline{\mathcal{A}_1}$ is not isometric with $\overline{\mathcal{A}_2}$.

Theorem 4.7. Let $\mathcal{A} = (\sigma,\mu)$ on (V,E) be a complete antifuzzy graph. Then $A(\mathcal{A})$ is a spanning subgraph of $\mathcal{A}$ and weight of each edge in $A(\mathcal{A})$ is $\vee\{\sigma(v)/v\in V\}$

Proof. Proof is analogous to theorem 4.5 in [3]

Theorem 4.8. Antipodal antifuzzy graphs of any two coweak isomorphic complete antifuzzy graphs are coweak isomorphic.

Proof. Let $\mathcal{A}_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $\mathcal{A}_2 = (\sigma_2, \mu_2)$ on (V_2, E_2) be two complete antifuzzy graphs. Let $\mathcal{A}_1$ is coweak isomorphic to $\mathcal{A}_2$.

Then there exists a bijection $h : \mathcal{A}_1 \to \mathcal{A}_2$ satisfying

$$\sigma_1(v_1) \geq \sigma_2(h(v_1)), \mu_1(v_i, v_j) = \mu_2(h(v_i), h(v_j)), \forall v_i, v_j \in V_1.$$

Let $V_1 = \{v_1, v_2, \cdots, v_n\}$.

Arrange these vertices of V_1 such that $\sigma_1(v_1) \leq \sigma_1(v_2) \leq \cdots \sigma_1(v_n)$.

Since $\mathcal{A}_1$ and $\mathcal{A}_2$ are complete coweak isomorphic antifuzzy graphs, we have $\sigma_1(v_i) = \sigma_2(h(v_i)),\ for\ i = 1, 2, \cdots, n-1.$ and $\sigma_1(v_n) \geq \sigma_2(h(v_n))$

Hence, $\mu_1(v_i, v_j) = \mu_2(h(v_i), h(v_j)),\ \forall v_i, v_j \in V_1$.

From theorem 4.7, $A(\mathcal{A}_i)$ is a spanning antifuzzy graph of $\mathcal{A}_i$ with weights of each edge in $A(\mathcal{A}_i)$ as $\wedge \{\sigma_i(v)/v \in V_i\}_{i=1,2,\cdots,n}$.

Hence the same bijection h is a coweak isomorphism between $A(\mathcal{A}_1)$ and $A(\mathcal{A}_2)$

Theorem 4.9. If $\mathcal{A}_1$ and $\mathcal{A}_2$ are connected antifuzzy graphs such that $\mathcal{A}_1$ is coweak isomorphic to $\mathcal{A}_2$, then $A(\mathcal{A}_1)$ is homomorphic to $A(\mathcal{A}_2)$

Proof. Let $\mathcal{A}_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $\mathcal{A}_2 = (\sigma_2, \mu_2)$ on (V_2, E_2) be connected antifuzzy graphs.

Suppose $\mathcal{A}_1$ is coweak isomorphic to $\mathcal{A}_2$ Then there exists a bijection $h : \mathcal{A}_1 \to \mathcal{A}_2$ satisfying $\sigma_1(v_i) \leq \sigma_2(h(v_i))$, and $\mu_1(v_i, v_j) = \mu_2(h(v_i), h(v_j))$, for $v_i, v_j \in V_1$

Also, μ distance will be preserved.

Let daim $\mathcal{A}_1$ =diam$\mathcal{A}_2 = k$ (say)

If $u, v \in V_1$, are at a distance k in $\mathcal{A}_1$, then they are made neighbours in $A(\mathcal{A}_1)$

So, $h(u), h(v)$ in $\mathcal{A}_2$ are also at a distance k in $\mathcal{A}_2$ and $h(u), h(v)$ are made as neighbours in $A(\mathcal{A}_2$, where σ attains its maximum at a unique vertex.

If u and v are neighbours in $\mathcal{A}_1$, then

$\mu_{\mathcal{A}_1}(u, v) = \mu_1(u, v) = \mu_2(h(u), h(v)) = \mu_{\mathcal{A}_2}(h(u), h(v))$

If u and v are not neighbours in $\mathcal{A}_1$, then

$\mu_{\mathcal{A}_1}(u, v) = \sigma_1(u) \vee \sigma_1(v) \leq \sigma_2(h(u)) \vee \sigma_2(h(v)) = \mu_{\mathcal{A}_2}(h(u), h(v))$

Hence, $A(\mathcal{A}_1)$ is homomorphic to $A(\mathcal{A}_2)$.

Theorem 4.10. Let $\mathcal{A} = (\sigma, \mu)$ on $(V.E)$ be an antifuzzy graph on a cycle C_n with $\mu(u, v) = c, \forall (u, v) \in E$, where $0 < c < 1$. Then

1. $\mathcal{A}$ is self centered antifuzzy graph.

2. $A(\mathcal{A})$ is strong antifuzzy graph.

3. $A(\mathcal{A})$ is a spanning subgraph of $\overline{\mathcal{A}}$.

Proof. Let $\mathcal{A} = (\sigma, \mu)$ be an antifuzzy graph on a cycle C_n.
1) Let $V = \{v_1, v_2, \cdots, v_n\}$ be the vertices of the cycle C_n.
Now, $\delta(v_i, v_{i-1}) = \delta(v_i, v_{i+1}) = c, \forall i = 1, 2, \cdots, n$.
$\delta(v_i, v_{i-2}) = \delta(v_i, v_{i+2}) = 2c, \forall i = 1, 2, \cdots, n$.
$\delta(v_i, v_{i-3}) = \delta(v_i, v_{i+3}) = 3c, \forall i = 1, 2, \cdots, n$.
$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$
$\delta(v_i, v_{i-\frac{n}{2}}) = \delta(v_i, v_{i+\frac{n}{2}}) = \frac{n}{2}c, .$ if n is even.
$\delta(v_i, v_{i-\lfloor\frac{n}{2}\rfloor}) = \delta(v_i, v_{i+\lceil\frac{n}{2}\rceil}) = [\frac{n}{2}]c$, if n is odd, where $\lfloor\frac{n}{2}\rfloor$ is flooring of $\frac{n}{2}$ and $\lceil\frac{n}{2}\rceil$ is ceiling of $\frac{n}{2}$.
Hence,

$$e(v_i) = max\,\{\delta(v_i, v_j)\} = [\frac{n}{2}]c, \ \forall\ i = 1, 2, \cdots, n.$$

Therefore, radius of $\mathcal{A} = [\frac{n}{2}]c =$ diam $\mathcal{A}$.
So, each node is a central node.
Hence, $\mathcal{A}$ is self centered antifuzzy graph.
2) Since diam$\mathcal{A} = [\frac{n}{2}]c$, the nodes v_i and $v_{i-\frac{n}{2}}$ and v_i and $v_{i+\frac{n}{2}}$ are neighbours in $A(\mathcal{A})$ which are not neighbours in $\mathcal{A}$, if n is even.
The nodes v_i and $v_{i-\lfloor\frac{n}{2}\rfloor}$ and v_i and $v_{i+\lceil\frac{n}{2}\rceil}$ are neighbours in $A(\mathcal{A})$ which are not neighbours in $\mathcal{A}$, if n is odd.
From the definition of antipodal antifuzzy graph,
$\mu(v_i, v_{i-\frac{n}{2}}) = \sigma(v_i) \vee \sigma(v_{i-\frac{n}{2}})$ and $\mu(v_i, v_{i+\frac{n}{2}}) = \sigma(v_i) \vee \sigma(v_{i+\frac{n}{2}})$, if n if even.
and
$\mu(v_i, v_{i-\lfloor\frac{n}{2}\rfloor}) = \sigma(v_i) \vee \sigma(v_{i-\lfloor\frac{n}{2}\rfloor})$ and $\mu(v_i, v_{i+\lceil\frac{n}{2}\rceil}) = \sigma(v_i) \vee \sigma(v_{i+\lceil\frac{n}{2}\rceil})$, if n if odd.
Hence, $A(\mathcal{A})$ is strong antifuzzy graph.
3) From the definition of complement and antipodal antifuzzy graph,
$\sigma(v) = \overline{\sigma}(v) = \sigma_A(v), \forall v \in V$.
Hence, $A(\mathcal{A})$ is spanning subgraph of $\overline{\mathcal{A}}$.

5. Self Complementary Antifuzzy Graph

Definition 5.1. An Antifuzzy graph is self complementary if $\mathcal{A} \cong \overline{\mathcal{A}}$

Theorem 5.2. Let $\mathcal{A} = (\sigma, \mu)$ be a self complementary antifuzzy graph on (V, E). Then

$$\sum_{u \neq v} \mu(u, v) = \frac{1}{2}(\binom{\gamma}{2} + \sum_{u \neq v} \sigma(u) \vee \sigma(v))$$

, where $\gamma = |V|$

Proof. As $\mathcal{A}$ is a self complementary antifuzzy graph, there exists an isomorphism $f : V \to V$ such that $\overline{\sigma}(f(u)) = \sigma(u), \forall u \in V$ and $\overline{\mu}(f(u), f(v)) = \mu(u, v), \forall u, v \in V$

As $\overline{\mathcal{A}}$ is complement of $\mathcal{A}, \sigma = \overline{\sigma}$ and
$\overline{\mu}(f(u), f(v)) = 1 - \mu(f(u), f(v)) + (\overline{\sigma}(u) \vee \overline{\sigma}(v)), \forall u, v \in V$
Therefore, $\mu(u, v) = 1 - \mu(f(u), f(v)) + (\sigma(u) \vee \sigma(v)), \forall u, v \in V$
$\Rightarrow \mu(u, v) + \mu(f(u), f(v)) = 1 + (\sigma(u) \vee \sigma(v))$ Hence,

$$\sum_{u \neq v} \mu(u, v) + \sum_{u \neq v} \mu(f(u), f(v)) = \sum_{u \neq v} 1 + \sum_{u \neq v} (\sigma(u) \vee \sigma(v))$$

Therefore

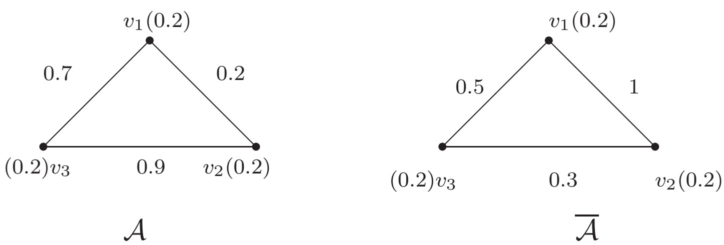
$$2 \sum_{u \neq v} \mu(u, v) = \binom{\gamma}{2} + \sum_{u \neq v} (\sigma(u) \vee \sigma(v)), \quad where \quad \gamma = |V|$$

Hence,

$$\sum_{u \neq v} \mu(u, v) = \frac{1}{2}(\binom{\gamma}{2} + \sum_{u \neq v} (\sigma(u) \vee \sigma(v)))$$

Remark 5.3. The condition of the above theorem is not sufficient.

Example 5.4.



Here,

$$\sum_{u \neq v} \mu(u, v) = 0.2 + 0.9 + 0.7 = 1.8.$$

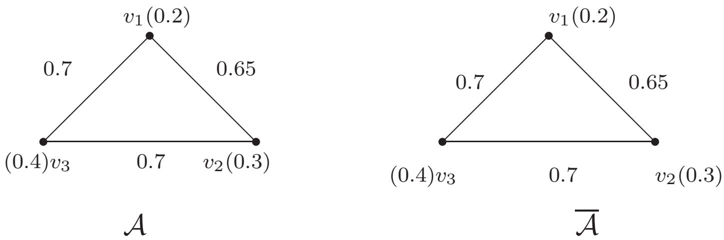
Also,

$$\frac{1}{2}(\binom{\gamma}{2} + \sum_{u \neq v} \sigma(u) \vee \sigma(v)) = \frac{1}{2}(3 + 0.2 + 0.2 + 0.2) = 1.8.$$

But $\mathcal{A}$ is not isomorphic to $\overline{\mathcal{A}}$.

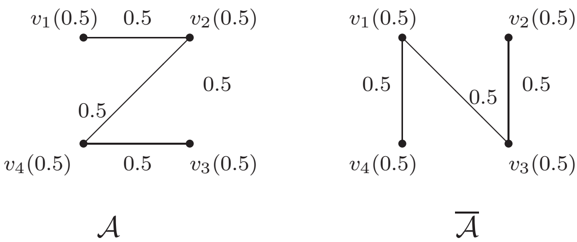
Theorem 5.5. Let $\mathcal{A} = (\sigma, \mu)$ be an antifuzzy graph on (V, E). If $\mu(u, v) = \frac{1}{2}(1 + (\sigma(u) \vee \sigma(v))), \forall u, v \in V$, then $\mathcal{A}$ is self complmentary.

Proof. Let $\mu(u, v) = \frac{1}{2}(1 + (\sigma(u) \vee \sigma(v))), \forall u, v \in V$.
Then $\mathcal{A}$ is isomorphic to $\overline{\mathcal{A}}$ under the identity map on V.
Hence, $\overline{\mu}(u, v) = 1 - \mu(u, v) + (\sigma(u) \vee \sigma(v)) = \mu(u, v)$.
$\Rightarrow \mathcal{A} \cong \overline{\mathcal{A}}$
Illustration of Theorem 5.5.



Remark 5.6. The condition in theorem 5.5 is not necessary.

Example 5.7.



Here, $\mathcal{A} \cong \overline{\mathcal{A}}$ under the isomorphism $f : V \to V$ given by
$f(v_1) = v_2; f(v_2) = v_3; f(v_3) = v_4; f(v_4) = v_1$
Also, $\mu(v_1, v_2) = 0.5$. But, $\frac{1}{2}(1 + (\sigma(v_1 \vee \sigma(v_2))) = \frac{1}{2}(1 + 0.5) = \frac{1.5}{2} = 0.75$.
So, $\mu(v_1, v_2) \neq \frac{1}{2}(1 + (\sigma(v_1 \vee \sigma(v_2)))$.

Theorem 5.8. Let $\mathcal{A}_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $\mathcal{A}_2 = (\sigma_2, \mu_2)$ on (V_2, E_2) be two disjoint antifuzzy graphs. Then

1. $\overline{G_1 + G_2} \cong \overline{G_1} \cup \overline{G_2}$
2. $\overline{G_1 \cup G_2} \cong \overline{G_1} + \overline{G_2}$

Proof. We shall prove that the identity map is the required isomorphism.
Let $I : V_1 \cup V_2 \to V_1 \cup V_2$ be the identity map.

1) We have to prove that $(\overline{\sigma_1+\sigma_2})(u) = (\overline{\sigma_1} \cup \overline{\sigma_2})(u)$ and
$(\overline{\mu_1+\mu_2})(u,v) = (\overline{\mu_1} \cup \overline{\mu_2})(u,v)$
Now,

$$\begin{aligned}
(\overline{\sigma_1+\sigma_2})(u) &= (\sigma_1+\sigma_2)(u), \quad from\ definition\ of\ complement \\
&= \begin{cases} \sigma_1(u) & \text{if} \quad u \in V_1 - V_2 \\ \sigma_2(u) & \text{if} \quad u \in V_2 - V_1 \end{cases} \\
&= \begin{cases} \overline{\sigma_1}(u) & \text{if} \quad u \in V_1 - V_2 \\ \overline{\sigma_2}(u) & \text{if} \quad u \in V_2 - V_1 \end{cases} \\
&= (\overline{\sigma_1} \cup \overline{\sigma_2})(u)
\end{aligned}$$

Also,

$$\begin{aligned}
(\overline{\mu_1+\mu_2})(u,v) &= 1-(\mu_1+\mu_2)(u,v)+((\sigma_1+\sigma_2)(u) \vee (\sigma_1+\sigma_2)(v)) \\
&= \begin{cases} 1-(\mu_1 \cup \mu_2)(u,v)+((\sigma_1 \cup \sigma_2)(u) \vee (\sigma_1 \cup \sigma_2)(v)), \\ \qquad\qquad \text{if} \quad (u,v) \in E_1 - E_2 \\ 1-(\sigma_1(u) \vee \sigma_2(v))+((\sigma_1 \cup \sigma_2)(u) \vee (\sigma_1 \cup \sigma_2)(v)), \\ \qquad\qquad \text{if} \quad (u,v) \in E_1' \end{cases} \\
&= \begin{cases} 1-\mu_1(u,v)+(\sigma_1(u) \vee \sigma_1(v)), \\ \qquad \text{if} \quad u \in V_1 - V_2 \ \text{and}(\ u,v) \in E_1 - E_2 \\ 1-\mu_2(u,v)+(\sigma_2(u) \vee \sigma_2(v)), \\ \qquad \text{if} \quad u \in V_2 - V_1 \ \text{and} \ (u,v) \in E_2 - E_1 \\ 1-(\sigma_1(u) \vee \sigma_2(v))+(\sigma_1(u) \vee \sigma_2(v)), \\ \qquad \text{if} \quad u \in V_1, v \in V_2 \text{and} \ (u,v) \notin E \end{cases} \\
&= \begin{cases} \overline{\mu_1}(u,v & \text{if} \quad (u,v) \in E_1 - E_2 \\ \overline{\mu_2}(u,v) & \text{if} \quad (u,v) \in E_2 - E_1 \\ 1 & \text{if} \quad (u,v) \in E' \end{cases} \\
&= (\overline{\mu_1} \cup \overline{\mu_2})(u,v)
\end{aligned}$$

2) We have to prove that $(\overline{\sigma_1 \cup \sigma_2})(u) = (\overline{\sigma_1} + \overline{\sigma_2})(u)$ and
$(\overline{\mu_1 \cup \mu_2})(u,v) = (\overline{\mu_1} + \overline{\mu_2})(u,v)$
Now,

$$\begin{aligned}
(\overline{\sigma_1 \cup \sigma_2})(u) &= (\sigma_1 \cup \sigma_2)(u) \\
&= \begin{cases} \sigma_1(u) & \text{if} \quad u \in V_1 - V_2 \\ \sigma_2(u) & \text{if} \quad u \in V_2 - V_1 \end{cases} \\
&= \begin{cases} \overline{\sigma_1}(u) & \text{if} \quad u \in V_1 - V_2 \\ \overline{\sigma_2}(u) & \text{if} \quad u \in V_2 - V_1 \end{cases}
\end{aligned}$$

$$= (\overline{\sigma_1} + \overline{\sigma_2})(u)$$

We have, if $(u.v) \notin E$, then

$$(\mu_1 \cup \mu_2)(u,v) = 1, (\sigma_1 \cup \sigma_2)(u) = \sigma_1(u) \quad and \quad (\sigma_1 \cup \sigma_2)(v) = \sigma_2(v). \tag{5.1}$$

Also,

$$\begin{aligned}
(\overline{\mu_1 \cup \mu_2})(u,v) &= 1 - (\mu_1 \cup \mu_2)(u,v) + ((\sigma_1 \cup \sigma_2)(u) \vee (\sigma_1 \cup \sigma_2)(v)) \\
&= \begin{cases} 1 - \mu_1(u,v) + (\sigma_1(u) \vee \sigma_1(v)), \\ \qquad\qquad \text{if} \quad u \in V_1 - V_2 \;\; \text{and}(\;\; u,v) \in E_1 - E_2 \\ 1 - \mu_2(u,v) + (\sigma_2(u) \vee \sigma_2(v)), \\ \qquad\qquad \text{if} \quad u \in V_2 - V_1 \;\; \text{and} \;\; (u,v) \in E_2 - E_1 \\ 1 - 1 + (\sigma_1(u) \vee \sigma_2(v)), \\ \qquad\qquad \text{if} \quad u \in V_1, v \in V_2 \;\; \text{and} \;\; (u,v) \notin E \text{ (by 5.1)} \end{cases} \\
&= \begin{cases} \overline{\mu_1}(u,v) & \text{if} \quad (u,v) \in E_1 - E_2 \\ \overline{\mu_2}(u,v) & \text{if} \quad (u,v) \in E_2 - E_1 \\ \sigma_1(u) \vee \sigma_2(v) & \text{if} \quad (u,v) \notin E' \end{cases} \\
&= (\overline{\mu_1} + \overline{\mu_2})(u,v)
\end{aligned}$$

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