



ON GENERALISED p -CLOSED SETS

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Abstract: This paper introduces a new class of sets called generalised p -closed sets in topological spaces. Moreover we present several new separation axioms using the concept of generalised p -closed sets and prime open sets and identified the spaces in which all this separation axioms coincide.

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1. Introduction

It was Gierz et.al.[3] introduced the concept of prime/irreducible elements in a continuous lattice. Later many authors [4], [5], [7] studied irreducible open and closed sets in many different contexts. The concept of generalised closed sets introduced by Norman Levine [6] plays a significant role in general topology. The investigation of generalised closed sets has led to the study of several new weaker separation axioms and in [6] Levine proved that for a symmetric space the separation axioms T_0, T_1 and $T_{1/2}$ are equivalent. Motivated by the definition of prime element by [3], [4], [7] we studied some notions in general topological spaces using the concept of prime open sets. Also we introduce a

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new class of sets called generalised p-closed sets and several new separation axioms $p-T_0$, $p-T_1$, $p-T_{1/2}$ has also been discussed using the concept of prime open sets and generalised p-closed sets. We proved that T_1 and $p-T_1$ axioms always coincide and for prime symmetric spaces $T_0, T_1, T_{1/2}$, $p-T_0$ and $p-T_{1/2}$ separation axioms are all equivalent.

2. Preliminaries

Definition 1. [5] Let $(P, \leq)$ be any partially ordered set or briefly a poset. For any subset A of a poset $(P, \leq)$ define $\uparrow A = \{x/x \geq y; y \in A\}$ and $\downarrow A = \{x/x \leq y; y \in A\}$.

Lemma 2. [5] Let $(X, \leq)$ be any poset. Then $L \subseteq X$ is prime if and only if $F = X/\downarrow L$ is a filter.

Definition 3. [2] Let $(L, \leq)$ be any lattice an element $a \in L$ is said to be an *atom* if there is no 'x' in L between 0 and a where 0 is the smallest element of the lattice.

Definition 4. [6] Let (X, T) be any topological space and $A \subseteq X$. A is generalised closed briefly g-closed iff $\overline{A} \subseteq O$ whenever $A \subseteq O$ and O is open.

Definition 5. [6] A topological space (X, T) is said to be $T_{1/2}$ iff every g-closed set is closed.

Definition 6. [6] A topological space (X, T) is said to be *symmetric* iff for $x, y \in X$; $x \in \overline{\{y\}}$ implies that $y \in \{x\}$

Theorem 7. [6] If (X, T) is a symmetric space , then (X, T) is T_0 iff $T_{1/2}$ iff T_1 .

3. p-Open Sets in the Lattice of Open Sets of any Arbitrary Topological Space

Definition 8. Let (X, T) be any arbitrary topological space. The open sets in T forms a complete lattice with smallest element 0 and largest element 1 ; where $0 = \phi$ and $1 = X$. We define an open set $G \neq 1$ in T to be *prime open set* if $H \cap K \subseteq G \Rightarrow H \subseteq G$ or $K \subseteq G$; where H, K are open sets in T such that $H \cap K \neq \phi$. Clearly 0 and 1 are prime in T . Prime open sets are denoted by *p-open* sets. Complements of p-open sets are called *p-closed sets*.

Remark 9. p-open implies open but converse not true.

Example 10. Let $X = \{a, b, c\}$ and $T = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$ be a topology on X , then $\{a, b\}, \{a, c\}$ are p -open sets in (X, T) but $\{a\}$ is not p -open.

Remark 11. Intersection of two p -open sets need not be a p -open set, for example in example 10 $\{a\}$ is not p -open where as $\{a, b\}, \{a, c\}$ are p -open sets.

Proposition 12. Let (X, T) be any arbitrary topological space. Intersection of p -open sets is a p -open set if and only if it forms a chain.

Proof. Let $\{G_i : i \in I\}$ be a collection of p -open sets in T . Assume that $G_i \cap G_j$ is p -open for some i, j , to prove that $G_i \subseteq G_j$ or $G_j \subseteq G_i$. On contradiction assume that it does not form a chain then $G_i \cap G_j = G_k$ for some ' k '; $k \neq i, j$ and G_k is a p -open set but since $G_k \subset G_i$ and $G_k \subset G_j$; G_k cannot be prime which is a contradiction. Hence either $G_i \subseteq G_j$ or $G_j \subseteq G_i$ and proof of sufficiency part is obvious. $\square$

Proposition 13. Let (X, T) be any arbitrary topological space. Union of p -open sets is a p -open set if it forms a chain.

Remark 14. Converse of above proposition is not true, for example let $X = \{a, b, c\}$ and $T = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ be a topology on X , then all open sets are p -open but not a chain.

Remark 15. Since the collection of all open sets of any arbitrary topological space is a lattice, results in Lemma 2 can be extended to any arbitrary topological space (X, T) by defining upper set and lower set of any $G \subseteq X$ as : *upper set of G* denoted by $U(G)$ is defined as $U(G) = \{A \in T/A \supseteq G\}$ and *lower set of G* denoted by $L(G)$ is defined as $L(G) = \{A \in T/A \subseteq G\}$ and we obtain the result in lemma 2 as : Let (X, T) be any arbitrary topological space and $G \in T$. G is p -open if and only if $T/L(G)$ is a filter ; where $T/L(G)$ is the collection of open sets which does not belong to $L(G)$.

Theorem 16. Let (X, T) be a hausdorff space and $x \in X$ then the only p -open sets are $X - \{x\}$.

Proof. Let $G \in T, G \neq X - \{x\}$ for any $x \in X$ and G be prime. There exists at least two elements $x_1, x_2 \in X$ such that $x_1 \neq x_2$ and $x_1, x_2 \notin G$. Since X is T_2 there exists disjoint open sets U_1, U_2 such that $x_1 \in U_1$ and $x_2 \in U_2$. Clearly $U_1 \in T/L(G)$ and $U_2 \in T/L(G)$ implies $U_1 \cap U_2 = \phi \in T/L(G)$ which is not possible since G is prime. Hence $G = X - \{x\}$ always. $\square$

4. p -Closure, p -Interior and p -Open Sets in Relative Topology

Definition 17. Let (X, T) be a topological space and let $A \subseteq X$, then the p -closure of A with respect to T is defined as the minimal p -closed super set of A in X and is denoted as $p-cl(A)$.

Example 18. Let $X = \{a, b, c, d\}$ and

$$T = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$$

be a topology on X then $p-cl(\{c\}) = \{b, c, d\}$ and $\{a, c, d\}$.

Remark 19. Let (X, T) be a topological space for any $A \subseteq X, \bar{A} \subseteq p-cl(A)$ where $\bar{A}$ is the closure of A with respect to T .

Proposition 20. Let (X, T) be a topological space, then for every p -open set there always exists a unique p -closed set containing A .

Proof. Let A be a p -open set and B^c be a p -closed super set of A . On contradiction let C^c be another p -closed super set of A , then clearly B, C are p -open sets of X . If $B \cap C = \phi$, then A will not be prime since A can be written as intersection of open sets $A \cup B$ and $A \cup C$ which is not possible. If $B \cap C \neq \phi$, then B can be written as the intersection of open sets $B \cup C$ and $A \cup B$, that is B is not prime which is a contradiction. Hence there does not exist more than one p -closed super set of A . $\square$

Remark 21. For a p -open set definition of p -closure becomes smallest p -closed set containing it.

Proposition 22. Let (X, T) be a topological space, $A \subseteq X$ and $x \in X$. Then $x \in p-cl(A)$ if and only if every p -open set containing 'x' intersects A .

Proof. If $x \notin p-cl(A)$, then $U = X - p-cl(A)$ is a p -open set containing 'x' which does not intersect A . Conversely, if there exists a p -open set U containing 'x' which does not intersect A , then $X - U$ is a p -closed set containing A then either $p-cl(A) \subset X - U$ or $X - U$ itself happens to be $p-cl(A)$ since $p-cl(A)$ is defined as the minimal p -closed superset of A which implies $x \notin p-cl(A)$ in both cases. $\square$

Definition 23. Let (X, T) be a topological space and $A \subseteq X$; an element $x \in X$ is called a p -limit point/ p -cluster point of $A \subseteq X$ if every p -open set containing 'x' intersects A .

Remark 24. Every limit point is a p -limit point but converse not true.

Proposition 25. *Let A be a subset of a topological space X , then A is p -closed if and only if A contains all its p -limit points.*

Definition 26. Let (X, T) be a topological space and let $A \subseteq X$, then the p -interior of A with respect to T is defined as the maximal p -open subset of A in X and is denoted as $p\text{-int}(A)$.

Proposition 27. *Let (X, T) be a topological space, then for every p -closed set there always exists a unique p -open set contained in A .*

Proof. Proof is similar to that of proposition :14 by taking dual. $\square$

Theorem 28. *Let (X, T) be a topological space and $Y \subseteq X$. U p -open in X implies $U \cap Y$ p -open in Y .*

Proof. Let X be any set with topology T , $Y \subseteq X$ and let T/Y be the relative topology on Y . Also let U be a p -open set in X then

$$H \cap K \subseteq U \Rightarrow H \subseteq U \text{ or } K \subseteq U; H, K \in T \quad (1)$$

We have to prove that $U \cap Y$ prime in Y ; that is to prove that $H' \cap K' \subseteq U \cap Y \Rightarrow H' \subseteq U \cap Y$ or $K' \subseteq U \cap Y$ where $H', K' \in T/Y$; but $H', K' \in T/Y$ implies $H' = H \cap Y$ for some $H \in T$ and $K' = K \cap Y$ for some $K \in T$. From (1) we have $(H \cap K) \cap Y \subseteq U \cap Y \Rightarrow H \cap Y \subseteq U \cap Y$ or $K \cap Y \subseteq U \cap Y$ that is $H' \cap K' \subseteq U \cap Y \Rightarrow H' \subseteq U \cap Y$ or $K' \subseteq U \cap Y$ where $H', K' \in T/Y$ hence $U \cap Y$ is prime in T/Y . $\square$

Proposition 29. *Let (X, T) be a topological space and Y be a subspace of X . Then a set A is p -closed in Y if and only if it equals the intersection of a p -closed subset of X with Y .*

Proof. For the necessary part assume that A is p -closed in Y , then $Y - A$ is p -open in Y that is $Y - A = U \cap Y$ where U is p -open in X which implies $A = Y \cap (X - U)$. Hence A equals intersection of a p -closed set in X with Y . For sufficiency assume that A equals intersection of a p -closed set in X with Y ; that is $A = C \cap Y$ where C is p -closed in X which implies $Y - A = (X - C) \cap Y$. Hence $Y - A$ is p -open in Y so that A is p -closed in Y . $\square$

Proposition 30. *Let (X, T) be a topological space, Y be a subspace of X and A be a p -open subset of Y . Let $p\text{-cl}(A)_X$ denote the p -closure of A in X . Then $p\text{-cl}(A)_Y$ in Y equals $p\text{-cl}(A)_X \cap Y$.*

Proof. Let B denote the $p\text{-cl}(A)_Y$ in Y . $p\text{-cl}(A)_X$ is p -closed in X so $p\text{-cl}(A)_X \cap Y$ is p -closed in Y . Clearly $A \subseteq p\text{-cl}(A)_X \cap Y$ which implies

$$B \subseteq p\text{-cl}(A)_X \cap Y \quad (1)$$

On the other hand, B is p -closed in Y . Hence by above proposition $B = C \cap Y$ for some p -closed set C in X , then C is a p -closed set in X containing A then $p-cl(A)_X \subset C$ which implies

$$p-cl(A)_X \cap Y \subseteq C \cap Y = B \quad (2)$$

Hence (1) and (2) implies $B = p-cl(A)_X \cap Y$. $\square$

5. Generalised p -Closed Sets

Definition 31. Let (X, T) be a topological space and $A \subseteq X$ then A is said to be *generalised p -closed* shortly *g - p -closed* if $p-cl(A) \subseteq O$ whenever $A \subseteq O$; O p -open in X .

Remark 32. Any p -closed set is generalised p -closed and converse need not be true.

Theorem 33. Let (X, T) be a topological space and $A \subseteq X$ then A is generalised p -closed if and only if $p-cl(A) - A$ contains no non-empty p -closed set.

Proof. Assume that $A \subseteq X$ is g - p -closed to prove that $p-cl(A) - A$ contains no non-empty p -closed set. Let F be a non-empty p -closed set such that $F \subseteq p-cl(A) - A$ then $A \subseteq F^c$ implies $p-cl(A) \subseteq F^c$ implies $F \subseteq X - p-cl(A)$ which is not possible since $F \subseteq p-cl(A)$ hence $F = \phi$. Conversely we assume that $p-cl(A) - A$ contains no non-empty p -closed set and let $A \subseteq O$, O p -open in X to prove that $p-cl(A) \subseteq O$. On contradiction let $p-cl(A) \cap O^c \neq \phi$ then $p-cl(A) \cap O^c$ is a closed subset of $p-cl(A) - A$. If $p-cl(A) - A$ is prime nothing more to prove, otherwise there exists closed subsets G, H such that $G \subseteq p-cl(A) \cap O^c$ and $H \subseteq p-cl(A) \cap O^c$ again if either G or H is prime proof is over otherwise continue until the atom element of T is reached and since all atoms are p -closed $p-cl(A) - A$ always contains a non-empty p -closed set which is not possible thus A is g - p -closed. $\square$

Theorem 34. Let (X, T) be a topological space and $A \subseteq X$ be such that A is generalised p -closed then A is g -closed.

Proof. Let A be not g -closed then $\overline{A} - A$ contains a non-empty closed set and $\overline{A} - A \subseteq p-cl(A) - A$ which implies $p-cl(A) - A$ contains non-empty closed

set and proceeding as in the proof of above theorem $p-cl(A) - A$ contains a non-empty p -closed set which implies A is not g - p -closed. Hence g - p -closed implies g -closed. $\square$

Remark 35. Converse of above theorem is not true for example ; let (X, D) be a discrete topological space, consider $A = \{x, y\}$ $x, y \in X$ then A is g -closed but not g - p -closed.

Theorem 36. Let (X, T) be a topological space and $A \subseteq X$ be a g - p -closed set, then A is p -closed if and only if $p-cl(A) - A$ is p -closed.

Proof. Assume that A is p -closed then $p-cl(A) - A$ is empty set which is always p -closed. Conversely, $p-cl(A) - A$ is p -closed then it is a p -closed subset of itself and $p-cl(A) - A$ is empty and hence A is p -closed. $\square$

Remark 37. Union of two g - p -closed sets need not be g - p -closed, for example let (X, D) be a discrete topological space $A = \{x_1\}$, $B = \{x_2\}$ both A and B are g - p -closed but $A \cup B$ is not g - p -closed.

Remark 38. Intersection of two g - p -closed sets need not be g - p -closed, for example let $X = \{a, b, c\}$ and $T = \{X, \phi, \{a\}\}$ be a topology on X . $A = \{a, b\}$ and $B = \{a, c\}$ are g - p -closed sets but $A \cap B$ is not g - p -closed.

Theorem 39. Let (X, T) be a topological space. $A \subseteq Y \subseteq X$ and A is g - p -closed in X . Then A is g - p -closed relative to Y .

Proof. Let $A \subseteq Y \cap O$ where O is p -open in X . Then $A \subseteq O$ implies $p-cl(A) \subseteq O$ implies $p-cl(A) \cap Y \subseteq O \cap Y$ which implies $p-cl(A)_Y \subseteq O \cap Y$. Hence A is g - p -closed with respect to Y . $\square$

Theorem 40. Let (X, T) be a topological space. If A is g - p -closed and $A \subseteq B \subseteq p-cl(A)$ then B is g - p -closed.

Proof. Clearly $p-cl(B) - B \subseteq p-cl(A) - A$ and given that A is g - p -closed hence $p-cl(B) - B$ contains no non empty p -closed set, that is B is g - p -closed. $\square$

Theorem 41. Let (X, T) be a topological space, $\mathbf{P}$ denote the collection of p -open sets in X and $\mathbf{F}$ the collection of p -closed sets in X then $\mathbf{P} = \mathbf{F}$ if and only if every subset of X is a g - p -closed set.

Proof. Assume $\mathbf{P} = \mathbf{F}$. Let A be any subset of X and $A \subseteq O$; O is p -open. Also O is p -closed by assumption hence A is g - p -closed. Since A is arbitrary every subset is g - p -closed. For sufficiency assume that every subset of X is g - p -closed and let $O \in \mathbf{P}$ which implies O is a subset of p -open set O itself and

since it is g-p.closed, $p-cl(O) \subseteq O$. Thus O is p-closed and $\mathbf{P} \subseteq \mathbf{F}$. Now let $O \in \mathbf{F}$ then $O^c \in \mathbf{P} \subseteq \mathbf{F}$. Hence $O \in \mathbf{P}$ and $\mathbf{P} \supseteq \mathbf{F}$. Therefore $\mathbf{P} = \mathbf{F}$. $\square$

6. Separation Axioms using p -Open Sets and g - p Closed Sets

Definition 42. Let (X, T) be any topological space. X is said to be $p-T_0$ if for every two distinct points $x, y \in X$ there exists p-open set U such that $x \in U, y \notin U$.

Remark 43. $p-T_0$ implies T_0 but converse not true for example let X be any set with overlapping interval topology then X is T_0 but not $p-T_0$.

Definition 44. Let (X, T) be any topological space. X is said to be $p-T_1$ if for every two distinct points $x, y \in X$ there exists p-open sets U, V such that $x \in U, y \notin U$ and $y \in V, x \notin V$.

Proposition 45. Let (X, T) be any topological space. X is $p-T_1$ if and only if $\{x\}$ is p-closed.

Proof. Necessary part trivially follows since $p-T_1$ implies T_1 . For sufficiency let x, y be two distinct points in X and assume that $\{x\}$ is p-closed which implies $X - \{x\}$ is a p-open set containing 'y' and not 'x'. Similarly by assuming $\{y\}$ to be p-closed we get a p-open set containing 'x' and not 'y'. Hence X is $p-T_1$. $\square$

Theorem 46. Let (X, T) be any topological space. X is T_1 if and only if it is $p-T_1$.

Proof. Proof is trivial by using the last proposition. $\square$

Definition 47. Let (X, T) be any topological space then X is $p-T_{1/2}$ if every g-p.closed set is p-closed.

Remark 48. $T_{1/2}$ does not implies $p-T_{1/2}$ for example, Let $X = \{a, b, c\}$ and $T = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$ be a topology on X then (X, T) is a $T_{1/2}$ space but not $p-T_{1/2}$.

Theorem 49. Let (X, T) be a $p-T_{1/2}$ topological space then it is always $p-T_0$.

Proof. Let (X, T) be a non $p-T_0$ topological space and let $x, y \in X$; $x \neq y$, then any p-open set containing 'x' contains 'y' and vice versa. Consider $A = p-cl(\{x\}) \cap \{x\}^c$

Claim : A is g-p.closed but not p-closed.

First we will prove that $O \cap A \neq \phi$ for every p -open set O containing 'x' which in turn implies 'x' is a p -limit point of A and since $x \notin A$ by definition of A ; A becomes a non p -closed set. In order to prove that $O \cap A \neq \phi$ for every p -open set O containing 'x' we will prove that $\{y\} \subseteq O \cap A$; that is to prove that $\{y\} \subseteq O \cap p-cl(\{x\}) \cap \{x\}^c$.

$$\text{clearly}; y \in \{x\}^c \quad (3)$$

Since X is non $p-T_0$

$$y \in O \quad (4)$$

again non $p-T_0$ implies 'y' is a p -limit point of $\{x\}$ and hence

$$y \in p-cl(\{x\}) \quad (5)$$

Thus (3), (4) and (5) implies $\{y\} \subseteq O \cap A$ and A becomes a non p -closed set. Again we claim that A is g - p -closed ; for that we assume that $A \subseteq O^* \in T$ where O^* is p -open in X to prove that $p-cl(A) \subseteq O^*$. But $p-cl(A) = p-cl(p-cl(\{x\}) \cap \{x\}^c) \subseteq p-cl(\{x\})$. Now it is enough to prove that $p-cl(\{x\}) \subseteq O^*$. But if $x \in O^*$, $A \cup \{x\} \subseteq O^*$ which implies $p-cl(\{x\}) \subseteq O^*$. Thus it is enough to prove that $x \in O^*$. On contradiction if $x \notin O^*$ then $(O^*)^c$ is a p -closed super set of $\{x\}$ which implies $p-cl(\{x\}) \subseteq (O^*)^c$, but $y \in p-cl(\{x\})$ by (5) which implies $y \in (O^*)^c$. Also $y \in A \subseteq O^*$ that is $y \in O^*$ and $y \in (O^*)^c$ which is not possible. Hence $x \in O^*$ and therefore there exists a g - p -closed but not p -closed set in X . Thus X is not $p-T_{1/2}$ if it is not $p-T_0$ and the theorem is proved. $\square$

Theorem 50. Let (X, T) be a $p-T_1$ topological space then it is always $p-T_{1/2}$.

Proof. In order to prove that X is $p-T_{1/2}$ it is enough to prove that every g - p -closed set is p -closed. Let A be a non p -closed set then $p-cl(A) \neq A$ then there exists an $x \in X$ such that $\{x\} \subseteq p-cl(A) - A$ which implies $p-cl(A) - A$ contains a p -closed set since the given space is $p-T_1$ which implies A is not g - p -closed. Hence all g - p -closed sets are p -closed that is X is $p-T_{1/2}$. $\square$

Definition 51. A topological space (X, T) is said to be *prime symmetric* if $x \in p-cl(\{y\}) \implies y \in p-cl(\{x\})$.

Theorem 52. A topological space (X, T) is *prime symmetric* if and only if $\{x\}$ is g - p -closed for every $x \in X$.

Proof. Assume (X, T) to be prime symmetric to prove that $\{x\}$ is g-p.closed. Let $\{x\} \subseteq O$ where O is p-open in X to prove that $p-cl(\{x\}) \subseteq O$. On contradiction $O^c \cap p-cl(\{x\}) \neq \phi$ and let $y \in O^c \cap p-cl(\{x\}) \subseteq p-cl(\{x\})$ but then $x \in p-cl(\{y\}) \subseteq O^c$, that is $x \in O^c$ which is not possible. Hence $\{x\}$ is g-p.closed. For sufficiency assume that $\{x\}$ is g-p.closed for every $x \in X$ and $x \in p-cl(\{y\})$ but $y \notin p-cl(\{x\})$ which implies $\{y\} \subseteq (p-cl(\{x\}))^c$ implies $p-cl(\{y\}) \subseteq (p-cl(\{x\}))^c$ since singleton sets are g-p.closed ; that is $x \in p-cl(\{y\}) \subseteq (p-cl(\{x\}))^c$ which is not possible. Hence X is prime symmetric. $\square$

Corollary 53. *Let (X, T) be a prime symmetric topological space, then it is always a symmetric space.*

Proof. Proof is trivial by last theorem since g-p.closed sets are always g-closed. $\square$

Proposition 54. *Let (X, T) be a $p-T_1$ topological space then its a prime symmetric one.*

Proof. Since the space is $p-T_1$ singleton sets are all p-closed and hence g-p.closed which in turn implies prime symmetry. $\square$

Proposition 55. *A topological space (X, T) is prime symmetric and $p-T_0$ if and only if (X, T) is $p-T_1$.*

Proof. Let $x, y \in X; x \neq y$ then by $p-T_0$,

$$x \in O \subseteq (\{y\})^c \quad (6)$$

for some O p-open in X which implies $x \notin p-cl(\{y\})$ hence $y \notin p-cl(\{x\})$. Thus there exists O^* such that O^* is p-open in X and

$$y \in O^* \subseteq (\{x\})^c \quad (7)$$

Thus (6) and (7) implies (X, T) is $p-T_1$ and the converse part is trivial. $\square$

Proposition 56. *Let (X, T) be a prime symmetric topological space then (X, T) is $p-T_0 \Leftrightarrow p-T_{1/2} \Leftrightarrow p-T_1$.*

Proof. Proof is trivial by Theorem 49, Theorem 50 and Proposition 55. $\square$

Theorem 57. *Let (X, T) be a prime symmetric topological space then (X, T) is $p-T_0 \Leftrightarrow p-T_{1/2} \Leftrightarrow T_1 \Leftrightarrow T_{1/2} \Leftrightarrow T_0$.*

Proof. Proof directly follows from Theorem 7, Theorem 46, Corollary 53 and Proposition 56. $\square$

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