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MHD FLOW IN A POROUS CHANNEL WITH CONSTANT SUCTION/INJECTION AT THE WALLS

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Abstract: In this paper our study is concern with the steady state fully developed MHD flow of a viscous incompressible conducting fluid within a channel filled with a porous medium and bounded by two infinite walls. Poiseuille and Couette-Poiseuille flow with constant suction and injection through the walls have been considered under the transverse applied magnetic field of uniform strength. The Brinkman equation has been used for the flow in the porous channel. Exact solutions are obtained for the flow velocity, volumetric flow rate and skin friction. The effects of various parameters on the flow are discussed and presented graphically.

AMS Subject Classification: 76Dxx, 76Sxx
Key Words: MHD flow, Brinkman equation, permeability parameter, Hartman number, suction/injection

1. Introduction

Magnetohydrodynamic flow of an electrically conducting fluid through a porous channel having uniform suction/injection at the walls of channel has many applications such as MHD generators, air filters, water filters, water coolers, artery blood flow and in petroleum engineering and chemical engineering. Absorption of digested food in human body and blood flow through an artery with permeable wall are examples of flow with suction/ injection at the walls.

Many authors investigated flow in the channels of various geometry hav-

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ing suction/ injection at the walls. Berman (1953) investigated the effect of wall porosity on the two-dimensional steady-state incompressible laminar flow of a fluid in a channel having rectangular cross section with the assumption of uniform suction at the porous walls of the channel. He obtained an exact solution of the considered problem by using Navier-Stoke's equation of motion. Verma and Bansal (1966) discussed the motion of viscous incompressible fluid between two parallel plates, upper plate is moving with uniform velocity and lower plate is stationary with uniform suction. In the limiting case the obtained results reduces to the classical plane Couette flow without suction. Muhuri (1963) considered formation of MHD Couette flow between two parallel walls due to impulsive and uniformly accelerated motion of one of the walls with suction at the walls. He use Navier-Stoke's equation to solve the problem for small magnetic Reynold number by applying Laplace transformation technique. Muhuri presented their analysis by taking the magnetic lines of force fixed relative to the fluid. Singh and Kumar (1983) investigated the same problem when the magnetic lines of force are fixed relative to the moving plate. Yih (1998) investigated the uniform suction/blowing effect on the forced MHD Hiemenz flow of electrically conducting fluids over a flat plate with power-law wall temperature in a porous medium. The nonlinear boundary-layer equation were transformed and the resulting ordinary differential equations were solved numerically by Keller box method. Yih used Darcy's law to analyze the flow in porous medium. Bujurke et. al. (2000) considered steady laminar flow of a viscous incompressible fluid through a uniformly porous tube with constant suction or injection at the porous walls. Bujurke obtain solution of the problem by perturbation analysis in terms of power series of Reynold number. Seth et. al. (2011) studied the unsteady MHD Couette flow between two parallel porous plates induced due to the impulsive and uniformly accelerated motion of the lower plate under the transverse magnetic field with uniform suction and injunction at the plates. They have found the asymptotic solution valid for large time for the velocity field using Laplace transform technique. Attia et. al. (2014) investigated the transient flow with heat transfer through a porous medium of an incompressible viscous fluid between two infinite horizontal porous plates. Uniform suction and injection is imposed in the direction normal to the plates. The flow through a porous medium deals with the analysis in which the differential equation governing the fluid motion is based on the Darcys law. The two plates are maintained at two different but constant temperatures. He investigated the effect of the porosity of the medium and the suction and injection velocity on both the velocity and temperature distributions. Falade et. al. (2016) investigate the effect of suction/injection on the unsteady oscillatory flow through a vertical channel subjected to a transverse magnetic field. Exact solution of the governing equation has been obtained under the usual Bousinesq approximation and the effects of the flow parameters on temperature, velocity profiles, skin friction and rate of heat transfer are discussed.

In the present paper we have considered a steady flow of a viscous, incompressible, conducting fluid through a horizontal channel filled with a porous material and bounded by two infinite parallel plates with constant suction and injection at the walls under the transverse applied magnetic field. The flow is due to the constant pressure gradient in the X-direction. Two cases of interest are considered; (i) Poiseuille flow, when both the plates are stationary and (ii) Couette flow, when the upper plate is moving and lower is stationary. The Brinkman (1947) equation is used to analyze the flow in the porous channel.

2. Mathematical Formulation

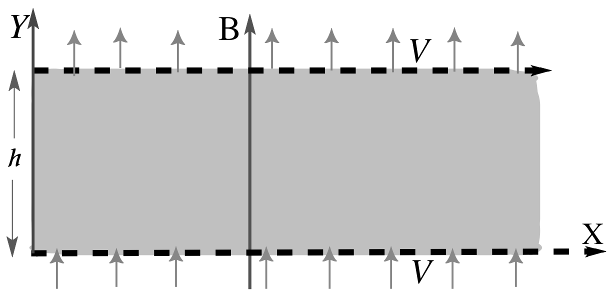

Fig. 1: Schematic diagram of the problem.

We have considered steady flow of a viscous incompressible electrically conducting fluid in fully developed situation in a parallel plate channel filled with a porous medium of permeability k. The fluid flow in the channel is driven by constant applied pressure gradient $\partial p^*/\partial x^*$ in the X-direction and the uniform magnetic field B is applied in the Y-direction normal to the plates. We assume that the magnetic Reynolds number is small and there is no external electric field. Therefore, induced current can be neglected and internal causes, such as separation of charges or polarization, do not rise to induced electric field. The fluid is being injected into the channel through the lower plate with uniform velocity V along Y-axis and fluid is being sucked out of the channel at the same velocity V through the upper plate as shown in Fig.1. We assume that the induced magnetic field can be neglected in comparison to the applied one. The governing Brinkman equation for the present flow in the porous channel is given by

$$\mu_e \frac{d^2u^*}{dy^{*2}} - \rho V \frac{du^*}{dy^*} - \frac{\mu}{k}u^* - \sigma B^2 u^* = \frac{\partial p^*}{\partial x^*} \tag{1}$$

where u^*, μ, ρ, σ are velocity, viscosity, density and conductivity of the fluid, respectively. k is the permeability of the porous medium, μ_e is the effective viscosity and V is the constant suction/injection velocity. We follow Brinkman(1947) and Chikh et al.(1995) and assume that $\mu_e = \mu$ for high porosity. With this assumption equation (1) becomes

$$\frac{d^2u^*}{dy^{*2}} - \frac{\rho V}{\mu}\frac{du^*}{dy^*} - \frac{u^*}{k} - \frac{\sigma B^2}{\mu}u^* = \frac{1}{\mu}\frac{\partial p^*}{\partial x^*} \tag{2}$$

Now, we introduce dimensionless variables u, y and P as follows

$$u = \frac{u^*}{U}, \qquad y = \frac{y^*}{h} \quad \text{and } P = -\frac{h^2}{\mu U}\frac{\partial p^*}{\partial x^*}$$

Here U is the characteristic velocity and h is the distance between the plates of the channel. Using the dimensionless variables in eq.(2), we get

$$\frac{d^2u}{dy^2} - S\frac{du}{dy} - (\alpha^2 + M^2)u = -P \tag{3}$$

Where $S = \rho h V/\mu$ is suction/injection parameter, $\alpha = h/\sqrt{k}$ is Permeability parameter and $M = \sqrt{\sigma B^2 h^2/\mu}$ is the Hartmaan number.

3. Solution and Discussion

We will consider two most applicable cases of channel flow namely, Poiseuille flow and Couette- Poiseuille flow within the porous channel with constant suction/injection through the walls of the channel under uniform transverse applied magnetic field.

3.1. Poiseuille Flow

For Poiseuille flow upper and lower plates of the channel are kept stationary and flow in the channel is due to the constant applied pressure gradient. We assume no-slip condition at the plates of the channel, which can be expressed as

$$u(0) = 0 \qquad \text{and} \qquad u(1) = 0. \tag{4}$$

Solution of equation (3) together with the boundary condition (4) is given by

$$\begin{aligned} u(y) &= e^{\frac{S}{2}y}[C_1 \sinh(\frac{1}{2}\sqrt{4M^2+S^2+4\alpha^2})y + \\ &\quad C_2 \cosh(\frac{1}{2}\sqrt{4M^2+S^2+4\alpha^2})y] + \frac{P}{M^2+\alpha^2} \end{aligned} \tag{5}$$

where C_1 and C_2 are constants of integration and are given by

$$C_1 = \frac{\cosh(\frac{1}{2}\sqrt{4M^2+S^2+4\alpha^2}) - e^{-\frac{S}{2}}}{(M^2+\alpha^2)\sinh(\frac{1}{2}\sqrt{4M^2+S^2+4\alpha^2})} \quad \text{and} \quad C_2 = \frac{-P}{M^2+\alpha^2}. \tag{6}$$

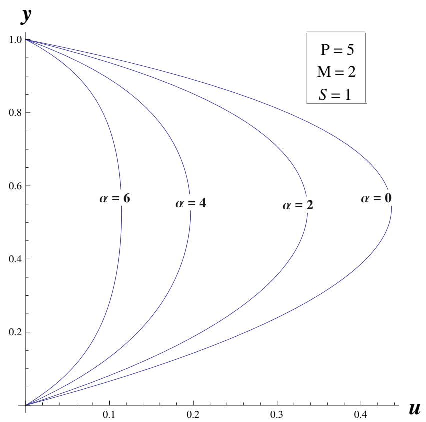

Fig. 2: Velocity profiles for Poiseuille flow for different values of α.

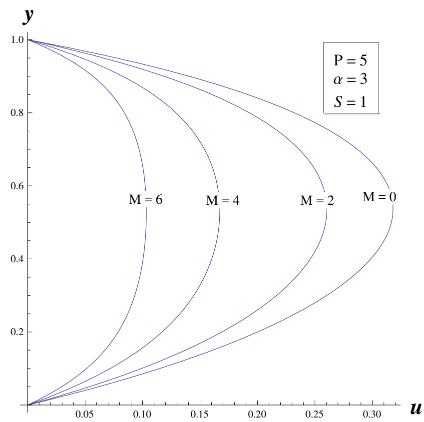

Fig. 3: Velocity profiles for Poiseuille flow for different values of M.

In the limiting case, for clear fluid flow in the absence of magnetic field and without suction/injection on the plates i.e. when $\alpha \to 0, S \to 0$ and $M \to 0$, we have by equation (5)

$$u(y) = \frac{P}{2}(y - y^2) \tag{7}$$

which is the velocity profile for the Classical Poiseuille flow of clear fluid flow between two plates. Fig.(2) shows the velocity profile u within the channel

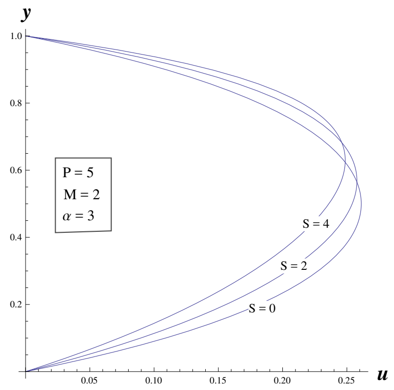

Fig. 4: Velocity profiles for Poiseuille flow for different values of S.

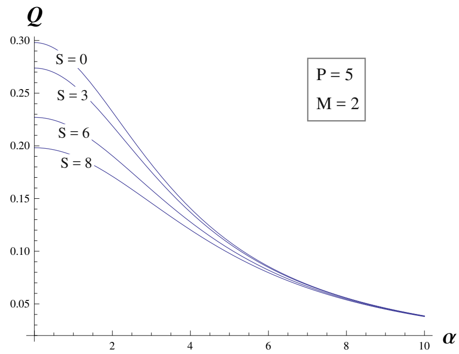

Fig. 5: Volume flow rate Q with α for Poiseuille flow for different values of S.

for different values of permeability parameter α when $P = 5$, $M = 2$ and $S = 1$. Figure reveals that velocity decreases as α increases. This is because increase in α ($\alpha^2 = h^2/k$) means decrease in permeability of the porous channel. Fig.(3) represent the velocity profile u within the channel for different values of Hartmaan number M when $P = 5$, $\alpha = 3$ and $S = 1$. Figure reveals that velocity decreases as M increases. This is because increase in M means increase in Lorentz force in the direction opposite to flow. Fig.(4) shows the effect of suction/injection parameter S on the velocity profile for $P = 5$, $M = 2$ and $\alpha = 3$. It is found that as suction on the upper plate of the porous channel with simultaneous injection on the lower plate increases, velocity decreases. Fig.(5) represents the variation of volume flow rate Q with α for different values of suction/injection parameter ($S = 0, 3, 6$ and 8). It is found that rate of volume flow Q decrease with permeability parameter α for all the values of suction/injection parameter S. Also, increase in S results decrease in Q.

3.1.1. Volumetric Flow Rate

The dimensionless volumetric flow rate Q through the cross section of the channel of unit width for the Poiseuille flow is given by

$$Q = \int_0^1 u(y)dy$$

substituting velocity u from eq.(5) in the above equation and integrating the resulting equation, we get

$$Q = \frac{P}{2(M^2+\alpha^2)^2}[2(M^2+\alpha^2) + e^{-S/2}\sqrt{4M^2+S^2+4\alpha^2}\{1+e^S - 2e^{S/2}\cosh\frac{1}{2}(\sqrt{4M^2+S^2+4\alpha^2})\}\csc h\frac{1}{2}(\sqrt{4M^2+S^2+4\alpha^2})] \tag{8}$$

In the limiting case when $\alpha \to 0$, $S \to 0$ and $M \to 0$ in equations (8), we get

$$Q_0 = \frac{P}{12}$$

which is the dimensionless volumetric flow rate for the Classical Poiseuille flow of clear fluid through the channel of unit width. Variation of dimensionless volume flow rate Q with α is shown in figure (5).

3.1.2. Skin Friction

The dimensionless skin friction τ at any point within the channel is given by

$$\tau(y) = \frac{du}{dy} = \frac{1}{2}e^{\frac{Sy}{2}}[(C_1 S + C_2\sqrt{4\alpha^2+4M^2+S^2})\sinh(\frac{1}{2}y\sqrt{4\alpha^2+4M^2+S^2}) + (C_1\sqrt{4\alpha^2+4M^2+S^2} + C_2 S)\cosh(\frac{1}{2}y\sqrt{4\alpha^2+4M^2+S^2})] \tag{9}$$

where C_1 and C_2 are constants given by eq.(6). Skin friction at lower plate $(y=0)$ and upper plate $(y=1)$ of the channel are given by

$$|\tau(0)| = |\frac{1}{2}(C_1\sqrt{4\alpha^2+4M^2+S^2} + C_2 S)| \tag{10}$$

and

$$|\tau(1)| = |\frac{1}{2}e^{\frac{S}{2}}[(C_1 S + C_2\sqrt{4\alpha^2+4M^2+S^2})\sinh(\frac{1}{2}\sqrt{4\alpha^2+4M^2+S^2}) +$$

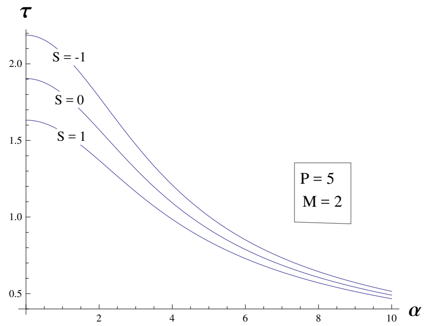

Fig. 6: Skin Friction τ with α on the lower plate for Poiseuille flow for different values of S.

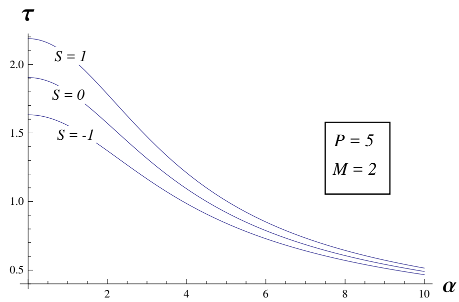

Fig. 7: Skin Friction τ with α on the upper plate for Poiseuille flow for different values of S.

$$(C_1\sqrt{4\alpha^2+4M^2+S^2}+C_2S)\cosh(\frac{1}{2}\sqrt{4\alpha^2+4M^2+S^2})]| \tag{11}$$

In the limiting case when $\alpha \to 0$, $M \to 0$ and $S \to 0$ in Eq.(10) and (11), we get

$$|\tau_0| = |\tau_1| = |\frac{P}{2}|$$

which is skin friction on the walls for the Classical Poiseuille flow of clear fluid in the channel bounded by two parallel plates. Fig.(6) and (7) represents variation of skin friction on the lower and upper plates of the channel, respectively for the poiseuille flow. Figure reveal that on both the plates skin friction τ decreases with increase in α. This is because increase in α results decrease in flow velocity in the channel. Also, it is found that on the lower plate, where injection is taking place, τ decreases as suction/injection parameter S increases whereas on the upper plate, where suction is taking place, τ increases as S increases. Therefore effect of injection on the plate is to decrease the skin friction and suction on the plate increases the skin fraction there.

3.2. Couette-Poiseuille Flow

For Couette- Poiseuille flow we have consider that the lower plate is fixed and the upper plate is moving with constant velocity U in the X-direction. No-slip condition at the walls of the channel in non dimensional variables is given by

$$u(0) = 0 \quad \text{and} \quad u(1) = 1 \tag{12}$$

Solution of equation (3) together with boundary condition (12) is given by

$$u(y) = e^{\frac{S}{2}y}[C_3 \sinh(\frac{\sqrt{4M^2+S^2+4\alpha^2}}{2})y + C_4 \cosh(\frac{\sqrt{4M^2+S^2+4\alpha^2}}{2})y] + \frac{P}{M^2+\alpha^2} \tag{13}$$

where C_3 and C_4 are constants of integration and are given as

$$C_3 = \frac{1}{\sinh(\frac{1}{2}\sqrt{4M^2+S^2+4\alpha^2})}[\frac{\cosh(\frac{\sqrt{4M^2+S^2+4\alpha^2}}{2})}{M^2+\alpha^2} + e^{-\frac{S}{2}}(1-\frac{P}{M^2+\alpha^2})], C_4 = \frac{-P}{M^2+\alpha^2} \tag{14}$$

In the limiting case when $\alpha \to 0$, $S \to 0$ and $M \to 0$ in Eq. (13) (i.e. for clear fluid flow in the absence of magnetic field and without suction/injection on the plates), we get

$$u(y) = y + \frac{P}{2}(y - y^2)$$

which is the velocity profile for the classical Couette - Poiseuille flow of clear fluid between two parallel infinite plates.

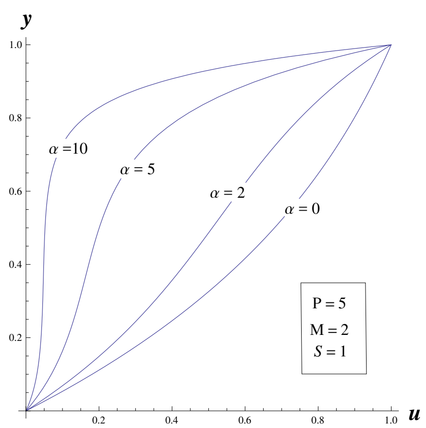

Fig. 8: Velocity profiles for Couette-Poiseuille flow for different values of α.

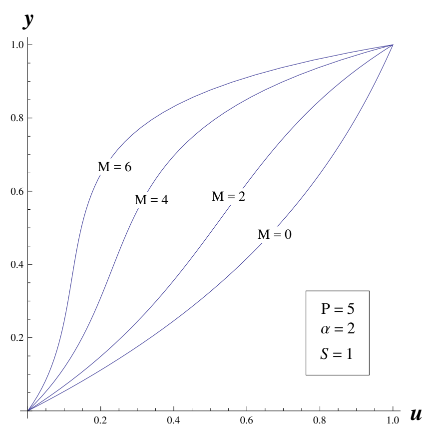

Fig. 9: Velocity profiles for Couette-Poiseuille flow for different M.

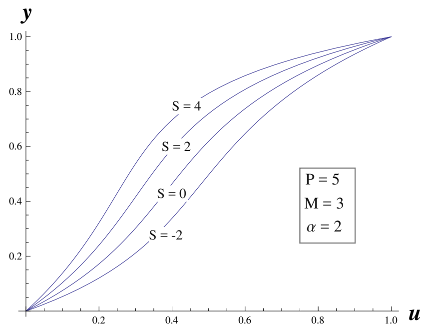

Fig. 10: Velocity profiles for Couette-Poiseuille flow for different S.

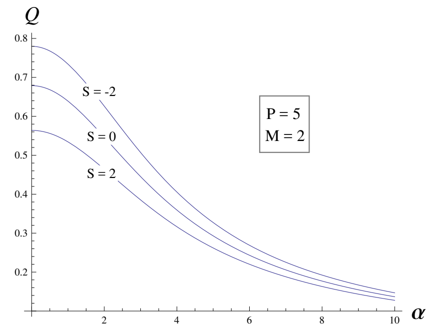

Fig. 11: Volume flow rate Q with α for Couette-Poiseuille flow for different S.

Fig.(8) shows the velocity profile u within the channel for different values of permeability parameter α when $P = 5$, $M = 2$ and $S = 1$. Figure reveals that velocity decreases as α increases. This is because increase in α ($\alpha^2 = h^2/k$) means decrease in permeability of the porous channel. Fig.(9) represent the velocity profile u within the channel for different values of Hartmaan number M when $P = 5$, $\alpha = 2$ and $S = 1$. It is found that velocity decreases as M increases. This is because increase in M means increase in Lorentz force in the direction opposite to flow. Fig.(10) shows the effect of suction/injection parameter S on the velocity profile for $P = 5$, $M = 3$ and $\alpha = 2$. It is found that as S increases, velocity decreases. Fig.(11) represents the variation of volume flow rate Q with α for different values of suction/injection parameter ($S = -2, 0, 2$). It is found that rate of volume flow Q decrease with permeability parameter α for all the values of suction/injection parameter S. Also, increase in S results decrease in Q.

3.2.1. Volumetric Flow Rate

The dimensionless volumetric flow rate Q through cross section of the channel of unit width is given by

$$Q = \int_0^1 u(y)dy.$$

Substituting velocity u from Eq. (13) in the above equation, we get after integration

$$Q = \frac{1}{2(M^2+\alpha^2)^2}[2P + (C_4S - C_1\sqrt{2M^2+S^2+4\alpha^2})(1 - e^{S/2}\cosh\frac{1}{2}\sqrt{4M^2+S^2+4\alpha^2}) - (C_3S - C_2\sqrt{4M^2+S^2+4\alpha^2})e^{S/2}\sinh\frac{1}{2}\sqrt{4M^2+S^2+4\alpha^2}] \quad (15)$$

In the limiting case when $\alpha \to 0, S \to 0$ and $M \to 0$ in (15), we can get

$$Q_0 = \frac{1}{2} + \frac{P}{12}$$

which is non dimensional volumetric flow rate for the Classical Couette - Poiseuille flow of clear fluid between two parallel plates.

3.2.2. Skin Friction

Dimensionless skin friction τ at any point within the channel for Couette-Poiseuille flow is given by

$$\tau(y) = \frac{du}{dy} \quad (16)$$

where velocity u is given by eq. (9), substituting u in the above equation, we get

$$\tau(y) = \frac{1}{2}e^{\frac{S}{2}y}[\sinh(\frac{y}{2}\sqrt{4\alpha^2+4M^2+S^2})(C_3S + C_4\sqrt{4\alpha^2+4M^2+S^2}) + \cosh\frac{y}{2}\sqrt{4\alpha^2+4M^2+S^2}) (C_3\sqrt{4\alpha^2+4M^2+S^2} + C_4S)]. \quad (17)$$

Skin friction at the lower fixed plate $(y = 0)$ of the channel is

$$|\tau(0)| = |\frac{1}{2}(C_3\sqrt{4M^2+S^2+4\alpha^2} + C_4S)| \quad (18)$$

and skin friction at upper moving plate $(y = 1)$ of the channel is

$$|\tau(1)| = |\frac{1}{2}e^{S/2}[\sinh(\frac{1}{2}\sqrt{4\alpha^2+4M^2+S^2})(C_3S + C_4\sqrt{4\alpha^2+4M^2+S^2}) + \cosh(\frac{1}{2}\sqrt{4\alpha^2+4M^2+S^2})$$

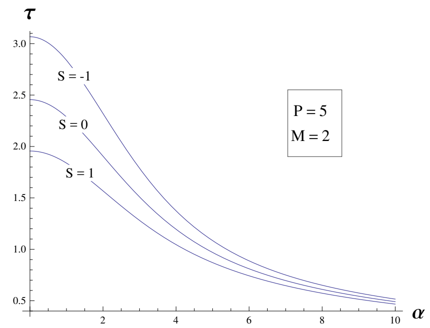

Fig. 12: Skin Friction τ with α at the fixed plate for Couette flow for different S.

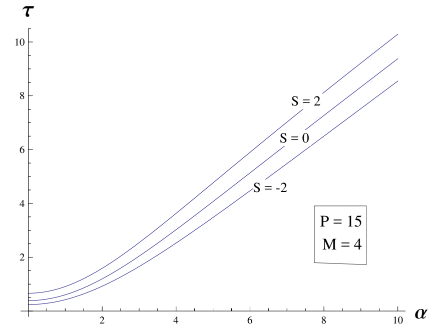

Fig. 13: Skin Friction τ with α at the moving plate for Couette flow for different S.

$$(C_3\sqrt{4\alpha^2+4M^2+S^2}+C_4S)]| \tag{19}$$

In the limiting case when $\alpha \to 0$, $M \to 0$ and $S \to 0$ in Eq.(14) and (15), we get

$$|\tau(0)| = |1+\frac{P}{2}| \quad \text{and} \quad |\tau(1)| = |1-\frac{P}{2}|$$

which is skin friction on the stationary and moving plate for the classical Couette-Poiseuille flow of clear fluid between two parallel plates.

Fig.(12) and (13) represents variation of skin friction on the lower and upper plates of the channel, respectively for the Couette-Poiseuille flow. We observe that on the plate, which is fixed, skin friction τ decreases with increase in α. On the upper moving plate τ increases with increase in α. Also, increase in S results decrease in τ on the lower plate whereas increase in τ on the upper plate. Therefore effect of injection on the plate is to decrease the skin friction and suction on the plate increases the skin fraction there.

4. Conclusion

Fluid flow through saturated porous material between two parallel infinite plates with uniform suction/inection at the plates under the transversely applied uniform magnetic field is studied for two cases (i) Poiseuille and (ii) Couette-Poiseuille flow. The analytical solution for the velocity, volume flow rate and the skin friction at the lower/uppper plates have been evaluated. The effect of various parameters such as permeability parameter α, Hartmaan number M and suction/injection parameter S on the flow is investigated. It is found that these parameters have strong effects on the flow characteristics. Increasing injection at lower stationary plate of the porous channel and simultaneous suction at the upper (stationary/moving) plate of the porous channel decreases volume flow rate through the channel for both Poiseuille and Couette-Poiseuille flow. We found that effect of injection on the plate is to decrease the skin friction and suction on the plate increases the skin fraction on that plate. Therefore injection has accelerating whereas suction has retarding effect on the flow.

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