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FUZZY GAMMA NEAR-ALGEBRAS OVER FUZZY FIELDS

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Abstract: The notion of fuzzy Γ-near-algebra over a fuzzy field is proposed to study in this paper. The main contribution of this work is that, the notion of fuzzy ideal in a Γ-near-algebra is initiated. Furthermore, pertaining to these notions some of important fundamental concepts with the illustrative examples are gained. Interestingly, this contribution opens up many enlightening theorems and propositions for future research in the field of fuzzy Γ-near-algebra.

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1. Introduction

The Γ-ring theory has been investigated in many directions and has evoked great interest among mathematicians working in different fields of mathematics. Research in the near-rings has had a long history. The concept of the generalization of a ring (or Γ-ring) has been studied by Nobusawa [5]. Further, Barnes [1] was analysed the generalized form of Γ-ring. In 1984, Satyanarayana [6] has been investigated generalization concepts of both the near-ring and the Γ-ring namely Γ-near-ring.

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Following that, the certain properties of a near-algebra in the topological sense was studied by many researchers and also available in the literature. The near-algebra concept was first to described by Brown [2], after that same concept was elaborated by Yamamuro [9] in the field of Banach spaces. The notion of a fuzzy sets was first introduced by Zadeh [10].

In addition, many attempts has been made in the field of near-algebra according to the physical situation was studied in the finite dimensional continuous field by Irish [3] and Yamamuro [9]. The applications of near-algebra was studied by Srinivas [7] and Narasimha Swamy [4]. It's worth mentioning that the Γ-near-algebra is a generalization of both the concepts of near-algebra and Γ-near-ring was proposed by Srinivas *et al.*[8].

In this paper, we studied the two concepts, such as first one is notion of fuzzy Γ-near-algebra, and second one is fuzzy ideal of a Γ-near-algebra. Further, we obtained fundamental results of this notion. Throughout this work mentioned, under "near-algebra" by mean a "right near-algebra", and X denotes a field.

2. Preliminaries

A *right near-algebra* Y over a field X is a linear space Y over X on which a multiplication is defined such that (i) Y forms a semigroup under multiplication, (ii) multiplication is right distributive over addition and (iii) $\lambda_1(ab) = (\lambda_1 a)b$ for all $a, b \in Y$, $\lambda_1 \in X$. Let N be a linear space over a field X and Γ be a non-empty set. Then N is said to be a Γ-near-algebra over a field X if there exist a mapping $N \times \Gamma \times N \to N$ (the image of (a, α, b) is denoted by $a\alpha b$) satisfying the following three conditions:(i) $(a\alpha b)\beta c = a\alpha(b\beta c)$, (ii) $(a+b)\alpha c = a\alpha c + b\alpha c$ and (iii) $(\lambda_1 a)\alpha b = \lambda_1(a\alpha b)$ for every $a, b, c \in N$,$\alpha, \beta \in \Gamma$ and $\lambda_1 \in X$.

Let Z be a non-empty set. Then a fuzzy subset ψ of Z is a mapping $\psi : Z \to [0, 1]$. If ψ is a fuzzy subset of Z, then the complement of ψ denoted by ψ^c or ψ' is a fuzzy subset in X given by $\psi'(x_1) = 1 - \psi(x_1)$ for all $x_1 \in Z$. A fuzzy subset ϕ of X is called a *fuzzy field* of X(denoted by (ϕ, X)), if it satisfies the following four conditions for all $x_1, x_2 \in X$: $(i)\phi(x_1 + x_2) \geq \phi(x_1) \wedge \phi(x_2)$, $(ii)\phi(-x_1) \geq \phi(x_1)$,$(iii)\phi(x_1x_2) \geq \phi(x_1) \wedge \phi(x_2)$, $(iv)\phi(x_1^{-1}) \geq \phi(x_1)$ for any $x_1(\neq 0) \in X$.

3. Fuzzy Γ-Near-Algebra

In this section, we define and discuss some properties of the notion of a fuzzy Γ-near-algebra over a fuzzy field.

Definition 1. Let N be a Γ-near-algebra over a field X. Let (ϕ, X) be a fuzzy field. A fuzzy subset ψ of N is called a *fuzzy* Γ*-near-algebra* of N over a fuzzy field (ϕ, X) if it satisfies the following four conditions:
(i) $\psi(x_1 + x_2) \geq \min(\psi(x_1), \psi(x_2))$,
(ii) $\psi(\lambda_1 x_1) \geq \min(\phi(\lambda_1), \psi(x_1))$,
(iii) $\psi(x_1 \alpha x_2) \geq \min(\psi(x_1), \psi(x_2))$,
(iv) $\phi(1) \geq \psi(x_1)$ for every $x_1, x_2 \in Y, \lambda_1 \in X, \alpha \in \Gamma$.
A fuzzy Γ-near-algebra ψ of N is denoted by (ψ, N).

Theorem 1. *If (ψ, N) is a fuzzy Γ-near-algebra over a fuzzy field (ϕ, X), then $\phi(0) \geq \psi(x_1)$ for every $x_1 \in N$.*

Theorem 2. *If (ψ, N) is a fuzzy Γ-near-algebra over a fuzzy field (ϕ, X), then $\psi(0) \geq \psi(x_1)$ for every $x_1 \in N$.*

Proof. $\psi(0) = \psi(x_1 - x_1) = \psi(1x_1 - 1x_1) \geq \min(\min(\phi(1), \psi(x_1)),$
$\min(\phi(-1), \psi(x_1))) \geq \min(\min(\psi(x_1), \psi(x_1)), \min(\phi(1), \psi(x_1))) \geq$
$\min(\psi(x_1), \psi(x_1)) = \psi(x_1)$. □

Example 1. Let $X = \{0, 1\}$ be a set with two binary operations additional modulo 2 and multiplication modulo 2. And let $N = \{0, a, b, c\}$ be a set with a binary operation " + " as follows:

$+$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Define a scalar multiplication on N by $0 \cdot x_1 = 0$ and $1 \cdot x_1 = x_1$ for every $x_1 \in N$ and $0, 1 \in X$. For any $x_1, x_2 \in X$, we have $x_1 - x_2 \in X$ and particularly for $x_2 \neq 0$, $x_1 x_2^{-1} \in X$, which provides X is a field.

By routine calculation, it shows that N is a linear space over the field X. Let $\Gamma = \{\alpha, \beta\}$ be a non-empty set. Define a mapping $N \times \Gamma \times N \to N$ by the following tables:

α	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	0	0	0
c	0	0	0	0

β	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	b	b	b
c	0	c	c	c

Then N is a Γ-near-algebra over a field X. Let ϕ be a fuzzy subset of X defined by

$$\phi(x_1) = \begin{cases} 0.9 & \text{if } x_1 = 0, \\ 0.8 & \textit{otherwise.} \end{cases}$$

It is clear that (ϕ, X) is a fuzzy field. Let ψ be a fuzzy subset of N defined by

$$\psi(x_1) = \begin{cases} 0.6 & \text{if } x_1 = 0, \\ 0.4 & \text{otherwise.} \end{cases}$$

It provides that (ψ, N) is a fuzzy Γ-near-algebra over a fuzzy field (ϕ, X).

Theorem 3. *Let (ϕ, X) be a fuzzy field, N be a Γ-near-algebra over a field X. Let ψ be a fuzzy subset of N. Then (ψ, N) is a fuzzy Γ-near-algebra over a fuzzy field (ϕ, X) if and only if the following three conditions hold:*
(i)$\psi(\lambda_1 x_1 + \lambda_2 x_2) \geq \min(\min(\phi(\lambda_1), \psi(x_1)), \min(\phi(\lambda_2), \psi(x_2)))$,
(ii)$\psi(x_1 \alpha x_2) \geq \min(\psi(x_1), \psi(x_2))$ *and*
(iii)$\phi(1) \geq \psi(x_1)$ *for every* $\lambda_1, \lambda_2 \in X, \alpha \in \Gamma$ *and* $x_1, x_2 \in N$.

Proof. Suppose that (ψ, N) is a fuzzy Γ-near-algebra over a fuzzy field (ϕ, X). Then (i) for any $\lambda_1, \lambda_2 \in X, \alpha \in \Gamma$ and $x_1, x_2 \in N$ we have $\psi(\lambda_1 x_1 + \lambda_2 x_2) \geq \min(\psi(\lambda_1 x_1), \psi(\lambda_2 x_2)) \geq \min(\min(\phi(\lambda_1), \psi(x_1)), \min(\phi(\lambda_2), \psi(x_2)))$. Clearly (ii) and (iii) holds directly from the definition of a fuzzy Γ-near-algebra.

Conversely, suppose that the three conditions of the hypothesis hold. Then

$$\begin{aligned}
\psi(x_1 + x_2) &= \psi(1x_1 + 1x_2) \\
&\geq \min(\min(\phi(1), \psi(x_1)), \min(\phi(1), \psi(x_2))) \\
&\geq \min(\min(\psi(x_1), \psi(x_1)), \min(\psi(x_2), \psi(x_2))) \\
&\geq \min(\psi(x_1), \psi(x_2)), \\
\psi(\lambda_1 x_1) &= \psi(\lambda_1 x_1 + 0x_1) \\
&\geq \min(\min(\phi(\lambda_1), \psi(x_1)), \min(\phi(0), \psi(x_1))) \\
&\geq \min(\min(\phi(\lambda_1), \psi(x_1)), \min(\psi(x_1), \psi(x_1))) \\
&= \phi(\lambda_1) \wedge \psi(x_1)
\end{aligned}$$

for every $x_1, x_2 \in N$ and $\lambda_1 \in X$. Remaining two conditions of the definition of a fuzzy Γ-near-algebra holds directly, because of the hypothesis. Hence (ψ, N) is a fuzzy Γ-near-algebra over a fuzzy field (ϕ, X). □

Theorem 4. *Let N and N' be two Γ-near-algebras over a field X. Let $f : N \to N'$ be an onto Γ-near-algebra homomorphism. If (ψ, N) is a fuzzy Γ-near-algebra over a fuzzy field (ϕ, X), then $(f(\psi), N')$ is a fuzzy Γ-near-algebra over a fuzzy field (ϕ, X).*

Proof. (i) Let $u, v \in N'$. Then there exists $x_1, x_2 \in N$ such that $u = f(x_1), v = f(x_2)$. Now

$$\begin{aligned} f(\psi)(u+v) &= \sup\{\psi(x_3) : x_3 \in N, f(x_3) = u+v\} \\ &\geq \sup\{\psi(x_1+x_2) : x_1, x_2 \in N, f(x_1) = u, f(x_2) = v\} \\ &\geq \sup\{\min(\psi(x_1), \psi(x_2)) : x_1, x_2 \in N, f(x_1) = u, f(x_2) = v\} \\ &= \min\{\sup(\psi(x_1) : x_1 \in N, f(x_1) = u), \\ &\qquad\qquad \sup(\psi(x_2) : x_2 \in N, f(x_2) = v)\} \\ &= \min(f(\psi)(u), f(\psi)(v)). \\ (ii) f(\psi)(\lambda_1 u) &= \sup\{\psi(x_3) : x_3 \in N, x_3 \in f^{-1}(\lambda_1 u)\} \\ &\geq \sup\{\psi(\lambda_1 x_3) : x_3 \in N, f(x_3) = u\} \\ &\geq \sup\{\min(\phi(\lambda_1), \psi(x_3)) : x_3 \in N, f(x_3) = u\} \\ &= \min(\phi(\lambda_1), \sup\{\psi(x_3) : x_3 \in N, f(x_3) = u\}) \\ &= \min(\phi(\lambda_1), f(\psi)(u)) \end{aligned}$$

for all $u \in N', \lambda_1 \in X$.
(iii) Let $u, v \in N', \alpha \in \Gamma$. If $f^{-1}(u) = \emptyset$ or $f^{-1}(v) = \emptyset$, then $f(\psi)(u\alpha v) \geq 0 = \min(f(\psi)(u), f(\psi)(v))$. If $f^{-1}(u) \neq \emptyset$ and $f^{-1}(v) \neq \emptyset$, then $f^{-1}(u\alpha v) \neq \emptyset$. Now

$$\begin{aligned} f(\psi)(u\alpha v) &= \sup\{\psi(x_3) : x_3 \in N, \alpha \in \Gamma, x_3 \in f^{-1}(u\alpha v)\} \\ &\geq \sup\{\psi(x_1 \alpha x_2) : x_1, x_2 \in N, f(x_1) = u, f(x_2) = v\} \\ &\geq \sup\{\min(\psi(x_1), \psi(x_2)) : x_1, x_2 \in N, f(x_1) = u, f(x_2) = v\} \\ &= \min\{\sup(\psi(x_1) : x_1 \in N, f(x_1) = u), \\ &\qquad\qquad \sup(\psi(x_2) : x_2 \in N, f(x_2) = v)\} \\ &= \min(f(\psi)(u), f(\psi)(v)). \end{aligned}$$

(iv) Since $\phi(1) \geq \psi(x_1)$ for every $x_1 \in N$, then for all $u \in N'$, $\phi(1) \geq \sup\{\psi(x_3) : x_3 \in N, x_3 \in f^{-1}(u)\} = f(\psi)(u)$. Thus $\phi(1) \geq f(\psi)(u)$ for every $u \in N'$.

Hence $(f(\psi), N')$ is a fuzzy Γ-near-algebra over a fuzzy field (ϕ, X). □

Theorem 5. *Let N and N' be two Γ-near-algebras over a field X. Let $f : N \to N'$ be an onto Γ-near-algebra homomorphism. If (ψ, N') is a fuzzy Γ-near-algebra over a fuzzy field (ϕ, X), then $(f^{-1}(\psi), N)$ is a fuzzy Γ-near-algebra over a fuzzy field (ϕ, X).*

Proof. Let $x_1, x_2 \in N, \alpha \in \Gamma$ and $\lambda_1, \lambda_2 \in X$. Then we have

$$\begin{aligned} f^{-1}(\psi)(\lambda_1 x_1 + \lambda_2 x_2) &= \psi(f(\lambda_1 x_1 + \lambda_2 x_2)) \\ &= \psi(\lambda_1 f(x_1) + \lambda_2 f(x_2)) \\ &\geq \min(\psi(\lambda_1 f(x_1)), \psi(\lambda_2 f(x_2))) \\ &\geq \min(\min(\phi(\lambda_1), \psi(f(x_1))), \min(\phi(\lambda_2), \psi(f(x_2)))) \\ &= \min(\min(\phi(\lambda_1), f^{-1}(\psi)(x_1)), \min(\phi(\lambda_2), f^{-1}(\psi)(x_2))) \\ f^{-1}(\psi)(x_1 \alpha x_2) &= \psi(f(x_1 \alpha x_2)) \\ &= \psi(f(x_1) \alpha f(x_2)) \\ &\geq \min(\psi(f(x_1)), \psi(f(x_2))) \\ &= \min(f^{-1}(\psi)(x_1), f^{-1}(\psi)(x_2)). \end{aligned}$$

Now for every $x_3 \in N'$, we have $\phi(1) \geq \psi(x_3)$ This implies that $\phi(1) \geq \psi(f(x_1)) = f^{-1}(\psi)(x_1)$ for every $x_1 \in N$. Hence $(f^{-1}(\psi), N)$ is a fuzzy Γ-near-algebra over the fuzzy field (ϕ, X). □

Theorem 6. *Intersection of a collection of fuzzy Γ-near-algebras is a fuzzy Γ-near-algebra.*

Proof. Let $\{\psi_i\}_{i \in \Lambda}$ be a family of fuzzy Γ-near-algebras of a Γ-near-algebra N over a fuzzy field (ϕ, X). Let $\psi(x_1) = \bigcap_{i \in \Lambda} \psi_i(x_1) = \inf_{i \in \Lambda} \psi_i(x_1)$ for every $x_1 \in N$. Then for every $x_1, x_2 \in N, \alpha \in \Gamma$ and $\lambda_1, \lambda_2 \in X$ we have

$$\begin{aligned} \psi(\lambda_1 x_1 + \lambda_2 x_2) &= \inf_{i \in \Lambda} \psi_i(\lambda_1 x_1 + \lambda_2 x_2) \\ &\geq \inf_{i \in \Lambda} [\min(\min(\phi(\lambda_1), \psi_i(x_1)), \min(\phi(\lambda_2), \psi_i(x_2)))] \\ &\geq \min(\min(\phi(\lambda_1), \inf_{i \in \Lambda} \psi_i(x_1)), \min(\phi(\lambda_2), \inf_{i \in \Lambda} \psi_i(x_2))) \\ &= \min(\min(\phi(\lambda_1), \psi(x_1)), \min(\phi(\lambda_2), \psi(x_1))), \\ \psi(x_1 \alpha x_2) &= \inf_{i \in \Lambda} \psi_i(x_1 \alpha x_2) \\ &\geq \inf_{i \in \Lambda} (\min(\psi_i(x_1), \psi_i(x_2))) \\ &\geq \min(\inf_{i \in \Lambda} \psi_i(x_1), \inf_{i \in \Lambda} \psi_i(x_2)) \\ &= \min(\psi(x_1), \psi(x_2)). \end{aligned}$$

Since each ψ_i is a fuzzy Γ-near-algebra, we get $\phi(1) \geq \psi_i(x_1)$. Thus $\phi(1) \geq \inf_{i \in \Lambda} \psi_i(x_1) = \psi(x_1)$ for every $x_1 \in N$. Hence (ψ, N) is a fuzzy Γ-near-algebra over a fuzzy field (ϕ, X). □

Theorem 7. *Let N be a Γ-near-algebra over a field X. Let (ϕ, X) be a fuzzy field and ψ be a fuzzy subset of N. Then the following statements are equivalent: for all $x_1, x_2 \in N$ and $\alpha \in \Gamma$*

(i) $\psi(x_1\alpha x_2) = \min(\psi(x_1), \psi(x_2))$
(ii) $\psi'(x_1\alpha x_2) = \max(\psi'(x_1), \psi'(x_2))$.

Proof. For definiteness suppose that $\psi(x_1) \leq \psi(x_2)$. Then $\psi(x_1\alpha x_2) = \psi(x_1)$. So $\psi'(x_1\alpha x_2) = 1 - \psi(x_1\alpha x_2) = 1 - \psi(x_1) = \psi'(x_1)$ and $\psi'(x_1) = 1 - \psi(x_1) \geq 1 - \psi(x_2) = \psi'(x_2)$. Thus $\psi'(x_1\alpha x_2) = \psi'(x_1) \geq \psi'(x_2)$. Hence $\psi'(x_1\alpha x_2) = \max(\psi'(x_1), \psi'(x_2))$. Conversely, for definiteness assume that $\psi'(x_1) \geq \psi'(x_2)$. Then $\psi'(x_1\alpha x_2) = \psi'(x_1)$. This gives that $1 - \psi(x_1\alpha x_2) = 1 - \psi(x_1)$. $\psi'(x_1) \geq \psi'(x_2)$ implies that $1 - \psi(x_1) \geq 1 - \psi(x_2)$, and hence $\psi(x_1) \leq \psi(x_2)$. Thus $\psi(x_1\alpha x_2) = \psi(x_1)$ and $\psi(x_1) \leq \psi(x_2)$.

□

4. Fuzzy Ideals of a Γ-Near-Algebra

In this section, the notion of fuzzy ideal in a Γ-near-algebra is initiated and obtain fundamental results to this notion.

Definition 2. Let (ψ, N) be a fuzzy Γ-near-algebra over a fuzzy field (ϕ, X). Then ψ is called a fuzzy ideal of N if $\psi(x_1\alpha x_2) \geq \psi(x_1)$ and $\psi(x_2\alpha(x_1 + i) - x_2\alpha x_1) \geq \psi(i)$ for every $x_1, x_2, i \in N, \alpha \in \Gamma$.
ψ is a *fuzzy right ideal* of N if $\psi(x_1\alpha x_2) \geq \psi(x_1)$ for every $x_1, x_2 \in N, \alpha \in \Gamma$.
ψ is a *fuzzy left ideal* of N if $\psi(x_2\alpha(x_1+i) - x_2\alpha x_1) \geq \psi(i)$ for every $x_1, x_2, i \in N, \alpha \in \Gamma$.

Example 2. From the example of fuzzy Γ-near-algebra over a fuzzy field, it is clear that (ψ, N) is also a fuzzy ideal over the fuzzy field (ϕ, X).

Theorem 8. *ψ be a fuzzy ideal of a Γ-near-algebra N if and only if $\psi_t = \{x_1 \in N : \psi(x_1) \geq t, t \in [0, 1], \}$ is an ideal of N.*

Theorem 9. *Let (ψ, N) be a fuzzy Γ-near-algebra over a fuzzy field (ϕ, X). If ψ is a fuzzy ideal of N, then $\psi(0) \geq \psi(x_1)$ for every $x_1 \in N$.*

Proof. $\psi(0) = \psi(x_1 - x_1) = \psi(1x_1 - 1x_1) \geq \min(\min(\phi(1), \psi(x_1)),$
$\min(\phi(-1), \psi(x_1))) \geq \min(\min(\psi(x_1), \psi(x_1)), \min(\phi(1), \psi(x_1))) \geq$
$\min(\psi(x_1), \psi(x_1)) = \psi(x_1)$. □

Theorem 10. *Let ψ be a fuzzy ideal of a zero symmetric Γ-near-algebra N over a fuzzy field (ϕ, X). Let $\phi(\lambda_1) \geq \psi(0)$ for every $\lambda_1 \in X$. Then the set $\psi_* = \{x_1 \in N : \psi(x_1) = \psi(0)\}$ is an ideal of N.*

Proof. Let $x_1, x_2 \in \psi_*$. Then $\psi(x_1) = \psi(0) = \psi(x_2)$. Since ψ is a fuzzy ideal in N, we have $\psi(x_1 - x_2) \geq \min(\psi(x_1), \psi(x_2)) = \min(\psi(0), \psi(0)) = \psi(0)$. But $\psi(0) \geq \psi(x_1 - x_2)$. Therefore $\psi(x_1 - x_2) = \psi(0)$. Thus $x_1 - x_2 \in \psi_*$. For every $\lambda_1 \in X$ we have $\psi(\lambda_1 x_1) \geq \min(\phi(\lambda_1), \psi(x_1)) \geq \min(\phi(\lambda_1), \psi(0)) = \psi(0)$. But $\psi(0) \geq \psi(\lambda_1 x_1)$ for every $\lambda_1 \in X, x_1 \in N$. Therefore $\psi(\lambda_1 x_1) = \psi(0)$ and $\lambda_1 x_1 \in N$. Thus $\lambda_1 x_1 \in \psi_*$. Hence ψ_* is a linear subspace of N.

Now let $x_1, x_2 \in N, \alpha \in \Gamma, i \in \psi_*$. Then $\psi(i) = \psi(0)$. Now $\psi(x_2\alpha(x_1 + i) - x_2\alpha x_1) \geq \psi(i) = \psi(0)$. But $\psi(0) \geq \psi(x_2\alpha(x_1 + i) - x_2\alpha x_1)$ for every $x_1, x_2 \in N, \alpha \in \Gamma, i \in \psi_*$. Therefore $\psi(x_2\alpha(x_1 + i) - x_2\alpha x_1) = \psi(0)$. Thus $x_2\alpha(x_1 + i) - x_2\alpha x_1 \in \psi_*$. Now $\psi(i\alpha x_1) \geq \psi(i) = \psi(0)$. On the other hand $\psi(0) \geq \psi(i\alpha x_1)$. Therefore $\psi(i\alpha x_1) = \psi(0)$. Thus $i\alpha x_1 \in \psi_*$. Hence ψ_* is an ideal of N. □

Theorem 11. *Intersection of a collection of fuzzy ideals of a Γ-near-algebra N over a fuzzy field (ϕ, X) is a fuzzy ideal of N.*

Proof. Let $\{\psi_i\}_{i\in\Lambda}$ be the family of fuzzy ideals of a Γ-near-algebra N over a fuzzy field (ϕ, X). Let $\psi(x_1) = \bigcap_{i\in\Lambda} \psi_i(x_1) = \inf_{i\in\Lambda} \psi_i(x_1)$. For any $x_1, x_2 \in N, \alpha \in \Gamma$ and $\lambda_1, \lambda_2 \in X$ we have

$$\begin{aligned} \psi(\lambda_1 x_1 + \lambda_2 x_2) &= \inf_{i\in\Lambda} \psi_i(\lambda_1 x_1 + \lambda_2 x_2) \\ &\geq \inf_{i\in\Lambda}[\min(\min(\phi(\lambda_1), \psi_i(x_1)), \min(\phi(\lambda_2), \psi_i(x_2)))] \\ &\geq \min(\min(\phi(\lambda_1), \inf_{i\in\Lambda} \psi_i(x_1)), \min(\phi(\lambda_2), \inf_{i\in\Lambda} \psi_i(x_2))) \\ &= \min(\min(\phi(\lambda_1), \psi(x_1)), \min(\phi(\lambda_2), \psi(x_1))). \end{aligned}$$

Since each ψ_i is a fuzzy ideal, we get $\phi(1) \geq \psi_i(x_1) \geq \inf_{i\in\Lambda} \psi_i(x_1) = \psi(x_1)$ for every $x_1 \in N$ and $i \in \Lambda$. Now

$$\begin{aligned} \psi(x_1\alpha x_2) &= \inf_{i\in\Lambda} \psi_i(x_1\alpha x_2) \\ &\geq \inf_{i\in\Lambda}(\psi_i(x_1)) \\ &= \psi(x_1). \end{aligned}$$

Thus ψ is a fuzzy right ideal of N. Let $x_1, x_2, j \in N$. Then

$$\begin{aligned} \psi(x_2\alpha(x_1 + j) - x_2\alpha x_1) &= \inf_{i\in\Lambda} \psi_i(x_2\alpha(x_1 + j) - x_2\alpha x_1) \\ &\geq \inf_{i\in\Lambda}(\psi_i(j)) \\ &= \psi(j). \end{aligned}$$

Thus ψ is a fuzzy left ideal of N. Hence ψ is a fuzzy ideal of the Γ-near-algebra N. □

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