FLOW IN A COMPOSITE POROUS CYLINDRICAL CHANNEL OF VARIABLE PERMEABILITY COVERED WITH POROUS LAYER OF UNIFORM PERMEABILITY

Vineet Kumar Verma\textsuperscript{1,\textcopyright}, Sanjeeva Kumar Singh\textsuperscript{2}
\textsuperscript{1,2}Department of Mathematics and Astronomy
University of Lucknow
Lucknow, 226007, INDIA

Abstract: In the present article we have consider the steady flow of a viscous incompressible fluid in a composite porous cylindrical channel. Inner and outer part of the cylindrical channel are of different permeability. The porous channel consist of two part. The inner porous cylinder is of variable permeability which is covered by outer porous layer of uniform permeability $k_0$. We have consider two cases of permeability variation of the inner porous cylinder; (i) linear variation, $k = k_0r$ and (ii) quadratic variation, $k = k_0r^2$. Analytical solution of the problem is obtained by using Brinkman equation. Exact expressions for the velocity, rate of volume flow, average velocity and shear stress on the impermeable boundary are obtained and exhibited graphically. Effect of permeability parameter and gap parameter on the flow characteristics has been discussed. It is found that these parameters have very strong effect on the flow.

AMS Subject Classification: 76DXX, 76SXX
Key Words: porous medium, Brinkman equation, modified Bessel function, composite channel

1. Introduction

Flow through channels partially and completely filled with a porous materials has wide range of applications in geophysics and petroleum industries. In re-
cent years considerable interest has been evidenced in the study of flow through porous channels of various shapes. Kaviany (1985) studied laminar flow through a porous channel bounded by two parallel plates maintained at constant and equal temperature. Kaviany used modified Darcy model for transfer of momentum and found excess pressure drop associated with the entrance region, decreases as $r$ increases. Pop and Cheng (1992) presented analytical study of the steady incompressible flow past a circular cylinder embedded in a constant porosity medium based on the Brinkman model. They obtained a closed form exact solution which leads to an expression for the separation parameter. Vadasz (1993) studied the problems of flow through variable permeability porous medium. The permeability of the fluid saturated porous domain varies in the vertical direction, thus affecting the imposed main flow direction is created. He found that flow solutions are possible depending on the particular variation of the permeability in the vertical direction. Govender (2006) analyzed the dean problem in heterogeneous porous media for the specific case of monotonic permeability variation in the vertical direction. He obtained analytical solution in terms of the curvature ratio, which is found to control the streamline pattern. Wang (2008) considered fully developed laminar forced convection inside a semi-circular channel filled with a Brinkman-Darcy porous medium. He found analytical solutions for flow and constant flux heat transfer are found using a mixture of Cartesian and cylindrical coordinates. Verma and Datta (2012) found analytical solution for fully developed laminar flow of a viscous incompressible fluid in an annular region between two coaxial cylindrical tubes filled with a porous medium of variable permeability when the permeability of the porous medium varies with the radial distance. Wang (2010) investigated a fully developed flow and constant flux heat transfer in super-elliptic ducts filled with a Darcy Brinkman porous medium. He used Ritz method to determine the velocity and temperature fields. Verma (2014) considered the flow of a viscous, incompressible and electrically conducting fluid in an annular pipe in the presence of radial magnetic field when viscosity is a function of radial distance. He considered linear and quadratic variation of viscosity and found exact solutions for velocity and rate of volume flow in both cases. Zaytoon et al.(2016) have studied flow through a variable permeability Brinkman porous layers with quadratic permeability function, undertaken by a Darcy porous layer of variable linear permeability function. They found that compatibility between the low order Darcy law and the Brinkman equation in the sense at interface between layers.

In the present paper we have considered steady flow of a viscous, incompressible fluid in a porous composite cylindrical channel. The inner porous
region is of variable permeability which is covered by another outer porous layer of uniform permeability $k = k_0$. We have considered two cases of permeability variation of the inner porous layer; (i) when permeability of inner porous layer is varying quadratically i.e., $k = k_0 r^2$ (ii) when permeability is varying linearly i.e., $k = k_0 r$.

![Fig. 1](image_url): Cross-sections of the porous channel.

### 2. Mathematical Formulation

In the present problem we consider steady flow of viscous, incompressible fluid flow in a composite porous cylindrical channel impermeable boundary surface. The composite channel consist of two regions. Outer annular porous cylindrical region II is of uniform permeability $k_0$ and inner porous region I is of variable permeability, as shown in Fig.(1). We assume two cases of permeability variation of inner porous region; (i) linear variation i.e., $k = k_0 r$ (ii) and quadratic variation i.e., $k = k_0 r^2$. Radius of inner region I is $a$ and thickness of outer annular region II is $(b - a); \ (b > a)$. Flow in both the regions takes place along the axis of cylindrical channel which is taken Z-axis under the common pressure gradient $\frac{\partial p^*}{\partial z^*}$. Flow in the composite porous channel is governed by the Brinkman equation (1947). For the present case Brinkman equation in cylindrical polar coordinates $(r^*, \theta, z^*)$ is given by

$$\mu_e \left( \frac{d^2 u^*}{dr^{*2}} + \frac{1}{r^*} \frac{du^*}{dr^*} \right) - \frac{\mu}{k} u^* = \frac{\partial p^*}{\partial z^*} \quad (1)$$

where $u^*$ is the fluid velocity, $\mu_e$ is the effective viscosity, $\mu$ is the fluid viscosity, $k$ is the permeability of the porous medium and $\frac{\partial p^*}{\partial z^*}$ is the constant applied pressure gradient. According to Brinkman (1947) and Chikh et al. (1995), for
high porosity cases, we can take $\mu_e = \mu$. This consideration reduces the Eq.(1) to the following form

$$\frac{d^2u^*}{dr^2} + \frac{1}{r^*} \frac{du^*}{dr^*} - \frac{u^*}{k} = \frac{1}{\mu} \frac{\partial p^*}{\partial z^*}$$

(2)

Now we introduce dimensionless variables as follows

$$r^* = \frac{r}{a} \quad \text{and} \quad u = \frac{\mu u^*}{a^2(-\partial p^*/\partial z^*)}$$

Here, the characteristic velocity being determined by $a^2\mu(-\partial p^*/\partial z^*)$. Using the above dimensionless variables in Eq.(2) and after dropping the star index for our convenience, we get governing Brinkman equation of motion as

$$\frac{d^2u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{a^2}{k} u = -1$$

(3)

### 3. Solution of the Problem

**Case I:**

When permeability of inner porous region I is $k_0r^2$ and permeability of region II is $k_0$.

Thus for inner porous cylinder of permeability $k_0r^2$, the governing equation of motion is

$$r^2 \frac{d^2u}{dr^2} + r \frac{du}{dr} - \alpha^2 u = -r^2; \quad (0 \leq r \leq 1)$$

(4)

where $a^2/k_0 = \alpha^2$ is called permeability variation parameter for region I and $u$ is the velocity in region I. General solution of Eq.(4) is

$$u(r) = A_1r^{\alpha} + A_2 \frac{r^2}{(-4 - \alpha^2)}; \quad \text{for} \ \alpha \neq 2, \quad 0 \leq r \leq 1$$

(5)

$$u(r) = A_1 \cosh(2 \log r) + A_2 \sinh(2 \log r) + \frac{1}{16} [-r^4 \sinh(2 \log r) + r^4 \cosh(2 \log r) - 4 \log r \sinh(2 \log r) - 4 \log r \cosh(2 \log r)]; \quad \text{for} \ \alpha = 2$$

For outer porous cylinder is of permeability $k_0$, the governing equation of motion is

$$r \frac{d^2v}{dr^2} + \frac{dv}{dr} - \alpha^2 rv = -r; \quad (1 \leq r \leq q = b/a)$$

(6)
Where \( v \) is the velocity in region II. General solution of Eq.(6) is

\[
v(r) = B_1 I_0(\alpha r) + B_2 K_0(\alpha r) + \frac{1}{\alpha^2}; \quad 1 \leq r \leq q
\]  

(7)

where, \( I_0 \) and \( K_0 \) are the modified Bessel functions of zeroth order of first and second kind respectively. To determine the constants \( A_1, A_2, B_1 \) and \( B_2 \), we use following boundary conditions

(i) No slip boundary condition at \( r = q \).

(ii) Continuity of velocity and tangential stress at the interface of two porous regions (\( r = 1 \)).

(iii) At the centre (\( r = 0 \)) velocity gradient is zero.

These boundary conditions mathematically can be expressed as

\[
\begin{align*}
v(r) &= 0 \quad \text{at} \quad r = q \\
u(r) &= v(r) \quad \text{at} \quad r = 1 \\
u'(r) &= v'(r) \quad \text{at} \quad r = 1 \\
u'(r) &= 0 \quad \text{at} \quad r = 0
\end{align*}
\]  

(8)

Using above boundary conditions we can find constant of integration as \( A_1, A_2, B_1 \) and \( B_2 \) as given below

\[
A_1 = \frac{1}{\Delta(\alpha - 2)} \left[-\frac{I_1(\alpha)}{\alpha^2} \{ \alpha^2 - 4 \} K_0(\alpha) + 4K_0(q\alpha) \right] + I_0(\alpha) \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \{ 2\alpha K_0(q\alpha) - (\alpha^2 - 4) K_1(\alpha) \} - 2\{ \alpha K_0(\alpha) + 2K_1(\alpha) \} I_0(q\alpha),
\]

\[
A_2 = 0,
\]

\[
B_1 = \frac{1}{\Delta} \left[ 2K_0(q\alpha) - (\alpha + 2) K_0(\alpha) - (\alpha + 2) K_1(\alpha) \right],
\]

\[
B_2 = \frac{1}{\Delta} \left[ (\alpha + 2) I_0(\alpha) - (\alpha + 2) I_1(\alpha) - 2I_0(q\alpha) \right] 
\]  

(9)

where

\[
\Delta = \alpha^2(\alpha + 2)\left[ \{ I_1(\alpha) - I_0(\alpha) \} K_0(q\alpha) + \{ K_0(\alpha) + K_1(\alpha) \} I_0(q\alpha) \right]
\]

(10)

The dimensionless velocity of the fluid at any point within the region I and II (inner and outer) when permeability of the inner and outer region is \( k_0r^2 \) and \( k_0 \), respectively, are given by Eqs.(5) and (6) on insertion of the preceding values of constants \( A_1, A_2, B_1 \) and \( B_2 \). The graphical presentation of velocity profiles for different \( \alpha \) is given in Fig.(2). In the limiting case, when \( \alpha \to 0 \) (i.e.,
when permeability of the medium is infinite in both the regions) in Eqs.(5) and (6), we obtain

$$\lim_{\alpha \to 0} u = \lim_{\alpha \to 0} v = \frac{(q^2 - r^2)}{4}$$

(11)

which is well known classical result for Hagen-poiseuille flow in a tube of radius $q$.

### 3.1. Rate of volume flow

The dimensionless rate of volume flow through cross-section of the inner region I is given by

$$Q_1 = 2\pi \int_0^1 u(r)rdr$$

(12)

Substituting $u(r)$ from Eq.(5) and after integration, we obtain

$$Q_1 = \frac{\pi [4(\alpha - 2)A_1 + 1]}{2(\alpha^2 - 4)}$$

(13)

Similarly, the dimensionless rate of volume flow through cross-section of the outer annular region II is given by

$$Q_2 = 2\pi \int_1^q v(r)r \, dr$$

(14)

Substituting $v(r)$ from Eq.(7) and after integration, we obtain

$$Q_2 = \frac{\pi}{\alpha^2} [2\alpha \{-B_1 I_1(\alpha) + B_1 q I_1(q\alpha) + B_2 (K_1(\alpha) - qK_1(q\alpha))\} + q^2 - 1]$$

(15)

where $I_1$ and $K_1$ are the modified Bessel functions of first kind of order one and $A_1$, $A_2$, $B_1$ and $B_2$ are given by Eq.(9). In the evaluation of above integrals the following identity [Ref. Abramowitz and Stegun (1970)] has been used

$$\left(\frac{1}{z} \frac{d}{dz}\right)^m \{z^\nu \mathcal{L}_\nu(z)\} = z^{\nu - m} \mathcal{L}_{\nu - m}(z)$$

with $m = 1$ and $\nu = 1$. $\mathcal{L}_\nu$ denotes $I_\nu$ and $e^{\nu \pi i}K_\nu$.

Total dimensionless rate of volume flow is given by

$$Q = Q_1 + Q_2$$
FLOW IN A COMPOSITE POROUS CYLINDRICAL CHANNEL... 327

\[ Q = \frac{\pi}{2\alpha^2(\alpha^2 - 4)} \left\{ 4(\alpha - 2)A_1 + 1 \right\} + 2(\alpha^2 - 4) \left\{ -2\alpha B_1 I_1(\alpha) + 2\alpha B_1 q I_1(q\alpha) + 2\alpha B_2 (K_1(\alpha) - qK_1(q\alpha)) + q^2 - 1 \right\} \]  

(16)

The dimensionless volume flow rate \( Q_0 \) for clear fluid flow (when permeability is infinite) can be obtained by taking limit \( \alpha \to 0 \) in Eq.(16). We get,

\[ Q_0 = \lim_{\alpha \to 0} Q = \frac{\pi q^4}{8} \]  

(17)

3.2. Average velocity

The dimensionless average velocity of the flow is defined as

\[ u_{\text{avg}} = \frac{Q}{\pi q^2} \]  

(18)

Substituting \( Q \) from the Eq.(16) into above Eq.(18), the average velocity of the flow within the composite channel is

\[ u_{\text{avg}} = \frac{1}{2q^2\alpha^2(\alpha^2 - 4)} \left\{ 4(\alpha - 2)A_1 + 1 \right\} + 2(\alpha^2 - 4) \left\{ -2\alpha B_1 I_1(\alpha) + 2\alpha B_1 q I_1(q\alpha) + 2\alpha B_2 (K_1(\alpha) - qK_1(q\alpha)) + q^2 - 1 \right\} \]  

(19)

where constants \( A_1, A_2, B_1 \) and \( B_2 \) are given by Eq.(9). For clear fluid flow average velocity of the flow is obtained by taking limit \( \alpha \to 0 \) in Eq.(19). We get

\[ \lim_{\alpha \to 0} u_{\text{avg}} = \frac{q^2}{8} \]  

(20)

which is well known average velocity for classical Hagen-Poiseuille flow.

3.3. Shearing stress on the surface of cylinders

The dimensionless shearing stress at any point within the region II is given by,

\[ \tau_{rz}(r) = -\frac{dv}{dr} \]  

(21)

Substituting \( v \) from Eq.(7) in above equation and after differentiation we get,

\[ \tau_{rz}(r) = -\alpha[B_1 I_1(\alpha r) - B_2 K_1(\alpha r)] \]  

(22)
where \( I_1 \) and \( K_1 \) are modified Bessel function of order one. Shear stress on the impermeable surface of composite channel by putting \( r = q \) in Eq.(22) and using the appropriate sign. Thus, providing

\[
\tau_{rz}(q) = -\alpha [B_1 I_1(\alpha q) - B_2 K_1(\alpha q)]
\]

(23)

where \( B_1 \) and \( B_2 \) are given by Eq.(9). Dimensionless shearing stress on the impermeable surface for clear fluid flow is obtained by taking limit \( \alpha \to 0 \) in Eq.(23). We get

\[
\lim_{\alpha \to 0} \tau_{rz}(q) = \frac{q}{2}
\]

(24)

**Case II:**

When permeability of inner region I vary linearly, according to the law \( k_0 r \), the governing Brinkman equation of motion for flow in this region is

\[
r \frac{d^2 u}{dr^2} + \frac{du}{dr} - \alpha^2 u = -r; \quad (0 \leq r \leq 1)
\]

(25)

where, \( u \) is the velocity in region I. General solutions of Eq.(25) is

\[
u(r) = C_1 I_0(2\alpha \sqrt{r}) + C_2 K_0(2\alpha \sqrt{r}) + \frac{1}{\alpha^3}(1 + r\alpha^2); \quad (0 \leq r \leq 1)
\]

(26)

For outer porous cylinder is of permeability \( k_0 \), the governing equation of motion is

\[
r \frac{d^2 v}{dr^2} + \frac{dv}{dr} - \alpha^2 rv = -r; \quad (1 \leq r \leq q = b/a)
\]

(27)

Where \( v \) is the velocity in region II. General solution of Eq.(27) is

\[
v(r) = D_1 I_0(\alpha r) + D_2 K_0(\alpha r) + \frac{1}{\alpha^2}; \quad (1 \leq r \leq q = b/a)
\]

(28)

where, \( I_0 \) and \( K_0 \) are the modified Bessel functions of zeroth order of first and second kind respectively. Using boundary conditions (8), we get constants \( C_1, C_2, D_1 \) and \( D_2 \) as given below

\[
C_1 = \frac{1}{\lambda} \left[ -I_1(\alpha)\{\alpha^2 K_0(\alpha) + K_0(q\alpha)\} - \{\alpha K_0(\alpha) + K_1(\alpha)\}I_0(q\alpha) + \alpha I_0(\alpha)\{K_0(q\alpha) - \alpha K_1(\alpha)\} \right],
\]
\[ C_2 = 0, \]
\[ D_1 = \frac{1}{\lambda} \left[ \alpha I_0(2\alpha) \{ K_0(q\alpha) - \alpha K_1(\alpha) \} - I_1(2\alpha) \{ \alpha^2 K_0(\alpha) + K_0(q\alpha) \} \right] \]
\[ D_2 = \frac{1}{\lambda} \left[ I_1(2\alpha) \{ \alpha^2 I_0(\alpha) + I_0(q\alpha) \} - \alpha I_0(2\alpha) \{ \alpha I_1(\alpha) + I_0(q\alpha) \} \right] \] (29)

where
\[ \lambda = \alpha^4 \left[ \{ I_0(2\alpha) I_1(\alpha) - I_0(\alpha) I_1(2\alpha) \} K_0(q\alpha) + \{ I_1(2\alpha) K_0(\alpha) + I_0(2\alpha) K_1(\alpha) \} I_0(q\alpha) \right] \] (30)

The dimensionless velocity of the fluid at any point within the region I and II (inner and outer) when permeability of the inner and outer region is \( k_0 r \) and \( k_0 \) respectively, is given by Eqs.(26) and (28) on insertion of the preceding values of constants \( C_1, C_2, D_1 \) and \( D_2 \). The graphical presentation of velocity profiles for different \( \alpha \) is given in Fig.(2). In the limiting case, when \( \alpha \to 0 \) (i.e., when permeability of the medium is infinite in both the regions) in Eqs.(26) and (28), we obtain
\[ \lim_{\alpha \to 0} u = \lim_{\alpha \to 0} v = \frac{(q^2 - r^2)}{4} \] (31)

**Fig. 2:** Variation of velocity with radial distance \( r \) for different \( \alpha \) for fixed \( q = 2 \).

### 3.3.1. Rate of volume flow

The dimensionless rate of volume flow through cross-section of the composite cylinder when permeability of the inner region I is \( k_0 r \) can be evaluated as in
previous case I, we get

\[ Q = \frac{\pi}{3\alpha^4} [6\alpha^2(\alpha C_1 I_1(2\alpha) - C_1 I_2(2\alpha) - \alpha D_1 I_1(\alpha) + \alpha D_1 q I_1(q\alpha)) + \alpha D_2(K_1(\alpha) - q K_1(q\alpha))) + \alpha^2(3q^2 - 1) + 3] \]

The dimensionless volume flow rate \( Q_0 \) for clear fluid flow (when permeability is infinite) can be obtained by taking limit \( \alpha \to 0 \) in Eq.(32). We get,

\[ Q_0 = \lim_{\alpha \to 0} Q = \frac{\pi q^4}{8} \]

From Fig. 3: Variation of volume flow rate with permeability parameter \( \alpha \) for different values of \( q \).

### 3.3.2. Average velocity

The dimensionless average velocity of the flow is defined as

\[ u_{avg} = \frac{Q}{\pi q^2} \]

Substituting \( Q \) from the Eq.(32) into above Eq.(34), the average velocity of the flow within the composite channel is

\[ u_{avg} = \frac{1}{3q^2\alpha^4} [6\alpha^2(\alpha C_1 I_1(2\alpha) - C_1 I_2(2\alpha) - \alpha D_1 I_1(\alpha) + \alpha D_1 q I_1(q\alpha)) + \alpha^2(3q^2 - 1) + 3] \]
\[ Q = K_\alpha r^2 \]

\[ Q = K_\alpha r \]

\[ \alpha = 4 \]

\[ \alpha = 8 \]

\[ q \]

\[ q \]

Fig. 4: Variation of volume flow rate with gap parameter \( q \) for different values of \( \alpha \).

\[ +\alpha D_2(K_1(\alpha) - qK_1(q\alpha))) + \alpha^2(3q^2 - 1) + 3 \]

(35)

where constants \( C_1, C_2, D_1 \) and \( D_2 \) are given by Eq.(29). For clear fluid flow average velocity of the flow is obtained by taking limit \( \alpha \to 0 \) in Eq.(35). We get

\[ \lim_{\alpha \to 0} u_{avg} = \frac{q^2}{8} \]

(36)

which is well known average velocity for classical Hagen-Poiseuille flow.

### 3.3.3. Shearing stress on the surface of cylinders

The dimensionless shearing stress at any point in the region II is given as

\[ \tau_{rz}(r) = -\frac{dv}{dr} \]

(37)

Substituting \( v \) from Eq.(28) in above equation and after differentiation we get,

\[ \tau_{rz}(r) = -[\alpha D_1 I_1(\alpha r) - \alpha D_2 K_1(\alpha r)] \]

(38)

where \( I_1 \) and \( K_1 \) are modified Bessel function of order one. Shear stress on the impermeable surface of the channel is obtained by putting \( r = q \) in Eq.(38) and using the appropriate sign. Thus, providing

\[ \tau_{rz}(q) = -\alpha[D_1 I_1(\alpha q) - D_2 K_1(\alpha q)] \]

(39)
where $C_1$, $D_1$ and $D_2$ are given by Eq.(29).

Dimensionless shearing stress on the surface of outer cylinder for clear fluid flow (i.e., when $\alpha = 0$) is obtained by taking limit $\alpha \to 0$ in Eq.(39). We get

$$\lim_{\alpha \to 0} \tau_{rz}(q) = \frac{q}{2}$$

(40)

Fig. 5: Variation of shear stress with permeability parameter $\alpha$ for different values of $q$.

4. Discussion

Fig.(2) represents velocity profiles of the flow in the composite channel for both the cases of permeability variation, $k_0r$ and $k_0r^2$. Velocity profiles are sketched by using Eqs.(5), (7) and Eqs.(26), (28) for permeability parameter $\alpha = 3$, 6 and 9. Figure reveals that as permeability parameter $\alpha$ increases velocity decreases. This is because increase in $\alpha$ caused decrease in the permeability of the porous region-II. We also observe that for fixed $\alpha$ velocities for permeability variation $k_0r$ and $k_0r^2$ have remarkable difference in both regions I and II. For both permeability variation $k_0r$ and $k_0r^2$ velocity increases in region-I for all $\alpha$ and decreases in region-II. In region-I velocity for permeability variation $k_0r$ is greater than that for $k_0r^2$.

Fig.(3) shows variation of rate of volume flow $Q$ with permeability parameter $\alpha$ for gap parameter $q = 1.2$ and 1.5. We found that $Q$ decreases as $\alpha$
increases. Figure also reveals that for fixed \( q \), \( Q \) is larger for linear permeability variation \( (k_0r) \) than that of quadratic permeability variation \( (k_0r^2) \).

Fig.(4) represents variation of rate of volume flow with gap parameter \( q \) for permeability parameter \( \alpha = 3 \) and 7. We observe that \( Q \) increases as \( q \) increases. Also, there is remarkable difference in the rate of volume flow \( Q \) for permeability variation \( k_0r^2 \) and \( k_0r \) when \( q \) is large. Figure also reveals that \( Q \) is larger for linear permeability variation \( (k_0r) \) than that of quadratic permeability variation \( (k_0r^2) \) for fixed \( \alpha \).

Fig.(5) shows the variation of shearing stress (skin friction) \( \tau_{rz} \) on the impermeable surface of the channel at \( r = q \) with \( \alpha \). We observe that \( \tau_{rz} \) decreases with \( \alpha \). Also \( \tau_{rz} \) is larger for permeability variation \( k_0r^2 \) than that of \( k_0r \) and this difference with increase in gap parameter \( q \).

5. Conclusion

Steady flow of viscous, incompressible fluid in a composite cylindrical channel. Inner and outer part of the cylindrical channel has been considered. Channel is consist of two part. The outer porous region-II is of uniform permeability \( k_0 \) and inner porous region-I of variable permeability \( k = k_0r^2 \) and \( k = k_0r \). Analytical solution of the flow is obtained for two cases of permeability variation of the inner region-I ; Case I- when permeability vary according to law \( k_0r^2 \) and Case II- when permeability vary according to law \( k_0r \). We use Brinkman model to analyze the flow in the porous channel. Exact expressions for velocity, volumetric flow rate, average velocity and skin friction of impermeable boundary surface of the channel has been obtained.

Effect of various parameters such as permeability parameter \( \alpha \) and gap parameter \( q \) on the flow has been examined. We found that these parameters have strong influence on the flow characteristics. The obtained results are very useful for the flow through porous channels where permeability is variable.

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