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NUMERICAL MODEL FOR NESTED SHALLOW WATER EQUATIONS

Huda Altaie[1 §], Pierre Dreyfuss[2]
[1,2]Laboratoire J.A. Dieudonné (Mathématiques) UMR CNRS 7351
Université de Nice
Sophia Antipolis, FRANCE

Abstract: A center finite difference scheme to construct two-way nested techniques for structured grids in a bounded domain for 2D shallow water model is proposed. This model has two interactive components, the coarse grid and fine grid domains. The interaction is two-way, the coarse grid domain is interpolated to provide boundary conditions for the fine grid domain and the variables on the fine grid are suitably averaged onto the coarse grid in order to drive the coarse grid model. Nested calculation with 5:1 grid ratio is presented. The formulation of the mesh nesting algorithm allows flexibility in deciding the number of mesh and the ratio of grid resolutions between adjacent meshes.

Comparison of the results of a fine grid and a coarse grid in case the nested are 1:3 and 1:5 shows the ability and accuracy of the two- way nesting technique over different periods of time and these results indicate good performance of the nesting technique.

AMS Subject Classification: 35Lxx, 35Qxx, 65Zxx, 65Pxx, 65Mxx
Key Words: 2D Shallow water model, multi-nested grid, adaptive scheme, refinement factor, nested grid, coarse grid

1. Introduction

Nesting is a fine mesh within a coarse mesh grid model represents an important way to improve the horizontal resolution in ocean models and weather models. The best resolution of the horizontal scale can be obtained without requiring

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§Correspondence author

a fine resolution grid throughout the full model domain. This allows a more accurate solution to the Ocean thus, saving computer time and memory space. A large number of researchers [9], [5], [4] has used such multiply nested grids primarily to provide a more gradual change between grid meshes and to give smoother solutions near the boundary. The nested method involves the embedding of a higher resolution grid into a lower resolution grid, which covers the full model domain.

Nesting methods allow to reduce computer resources by embedding finer resolution grid within a coarser grid as well as the variable grid spacing. We emphasize flexibilities of the nesting method compared with the other grid systems. The finer higher resolution grid can be embedded in a favorite area of the existing coarse grid when this area becomes interesting for users. In weather predictions nesting methods have mainly been used [5], [3]. Nesting for ocean models have recently been attempted [11], [7].

We intend to provide the two- way nesting as one of the options to simulate shallow water model effectively. There are two types of nesting, one-way type and two-way type. One-way nesting means that information of the coarse grid is only put on the boundary of the nested grid. On the other hand, two-way nesting means that information of the nested grid is additionally reflected on the coarse grid. Our nesting has the both one-way (passive method) and two-way(interactive nesting method) options, [3]. There are not many examples of two-way nested models. [10] developed a two-way nesting and applied it to a barotropic modon and a baroclinic vortex in a flat ocean [9], extended their methods to an ocean with topography and attempted further refinements of the grid in the vertical direction. Also, it is not clear that a conservative scheme can give a better result than a non conservative one [10], because in a nested model the coarse grid model cannot resolve the solution of the fine grid model and a conservative scheme imposes the coarse grid fluxes on the fine grid solution and may spread the erroneous coarse grid field more fast into the fine grid region.

The paper is organized as follows. In section 2, description the nesting model. In section 3, discuss some numerical examples of 2D non-linear shallow-water model using an explicit center finite difference in space and leapfrog schemes with Asselin-Roberts filter in time and compare the results when the model has the spacial refinement ratio 1: 3 and 1:5. Finally, in section 4, the main conclusions are summarized.

2. Description of Model

2.1. Governing Equations

Consider the 2D depth-averaged nonlinear shallow water equation which contains the as fallows (see [6], [8]):

$$\frac{\partial \eta}{\partial t} + \frac{\partial Hu}{\partial x} + \frac{\partial Hv}{\partial y} = 0, \tag{1}$$

$$\begin{aligned}
\frac{\partial Hu}{\partial t} + \frac{\partial Hu^2}{\partial x} + \frac{\partial Huv}{\partial y} - fHv = &- gH\frac{\partial \eta}{\partial x} + \nu\left[\frac{\partial}{\partial x}(H\frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(H\frac{\partial u}{\partial y})\right] \\
&+ \frac{\tau_u^w}{\rho_0} - \frac{\tau_u^b}{\rho_0}, \\
\frac{\partial Hv}{\partial t} + \frac{\partial Hvu}{\partial y} + \frac{\partial Hv^2}{\partial y} + fHu = &- gH\frac{\partial \eta}{\partial y} + \nu\left[\frac{\partial}{\partial x}(H\frac{\partial v}{\partial x}) + \frac{\partial}{\partial y}(H\frac{\partial v}{\partial y})\right] \\
&+ \frac{\tau_v^w}{\rho_0} - \frac{\tau_v^b}{\rho_0}.
\end{aligned}$$

Here:

- x, y are the horizontal coordinates.
- t is the time.
- $u = u(x, y, t)$ is the depth-averaged horizontal velocity in the x direction.
- $v = v(x, y, t)$ is the depth-averaged horizontal velocity in the y direction.
- $H = H(x, y, t)$ is the depth from the surface level to the bottom (water height).
- ν is the horizontal turbulent viscosity.
- g stains for the gravity acceleration.
- $f = 1.01 \times 10^{-4} rad/s$ is the Coriolis frequency at 42° of latitude.
- $\rho_0 = 1033 kg/m^3$ is the water mean density.
- η is the water level relative to rest.

- τ_u^w is the bottom stress zonal component and τ_u^b is the wind stress zonal component.

The finite difference scheme used in this model is based upon the alternating an explicit center finite difference scheme in space and leapfrog scheme with Asselin-Roberts filter in time with Dirichlet open boundary condition and linear interpolated (both spatially and temporally) and updating byfull-weighting scheme (both spatially and temporally).

2.2. General Formulation of the Nested Models

We consider the general case of a high-resolution model covering the local domain ω embedded in a coarser resolution model covering the larger domain Ω. With obvious notations the local high-resolution grid and the global coarse resolution grid are denoted respectively as ω_h and Ω_H. The corresponding state vectors are denoted respectively as x_h and x_H. We also denote as ω_H, the part of the grid Ω_H corresponding to the local domain ω, [2].

For both one-way and two-way interactions, we use the same notation F for the coarse and fine models in order to simplify the notations. It can of course be different at least at the discrete level.

In the case of one-way interaction, the coarse grid model provides boundary conditions to the high-resolution model using an interpolation operator I_H^h. Semi-discretized equations of the nested system can be written as follows:

Domain Ω_H:

$$\frac{\partial x_H}{\partial t} = F(x_H) \text{ on } \Omega_H \times [0,T],$$

$$x_H(t=0) = x_H^0.$$

Domain ω_h:

$$\frac{\partial x_h}{\partial t} = F(x_h, x_{\partial\omega}) \text{ on } \omega_h \times [0,T],$$

$$x_h(t=0) = x_h^0,$$

$$x_{\partial\omega} = I_H^h(x_H) \text{ on } \partial\omega_h \times [0,T].$$

Here $x_{\partial\omega}$ represents the information coming from the coarse grid onto $\partial\omega_h$, the boundary of the fine grid. The one-way interaction is said to be passive since there is no retroaction from the local model onto the global model. From a practical point of view this also means that both models do not have to be run simultaneously (the global model can be run first and its solution can then be used offline by the local model).

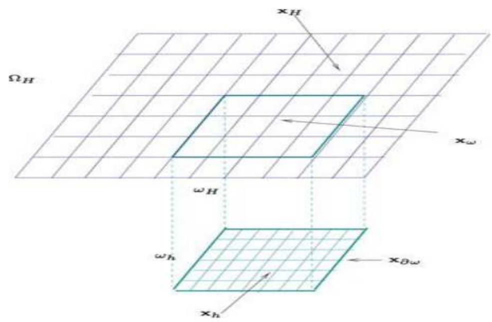


Figure 1: Notations used in the definitions of the nested models

In the case of *two-way interactions* a feedback term from the fine grid onto the coarse grid is added. The coarse solution is updated locally (in ω_H°, the interior of ω_H) by the high-resolution solution using a restriction operator G_h^H. Semi-discretized equations of the nested system can be written as follows:

Domain Ω_H:

$$\frac{\partial x_H}{\partial t} = F(x_H, x_\omega) \text{ on } \Omega_H \times [0,T],$$

$$x_H(t=0) = x_H^0,$$

$$x_\omega = G_h^H(x_h) \text{ on } \omega_H^\circ \times [0,T].$$

Domain ω_H:

$$\frac{\partial x_h}{\partial t} = F(x_h, x_{\partial\omega}) \text{ on } \omega_h \times [0,T],$$

$$x_h(t=0) = x_h^0,$$

$$x_{\partial\omega} = I_H^h(x_H) \text{ on } \partial\omega_h \times [0,T].$$

Here x_ω represents the information coming from the fine grid onto the coarse grid in ω_H°. The two-way interactions are said to be active. In that case both models must be run simultaneously since they permanently exchange information.

After discretization the problems have to be integrated in time in a specific order. The model is first integrated on the coarse grid Ω_H and then on the high-resolution ω_h grid with boundary conditions given by a spatial and temporal interpolation of the coarse values. Finally a feedback can be applied.

3. Nesting Procedure

3.1. Feedback of Boundary Data

Interpolation techniques are required for effective data transmission as data is transferred between domains of different spatial and temporal resolution. There are two main goals for an interpolation scheme to be optimum: (1) to maximize the information being transferred and (2) to minimize the generation of noise.

Interpolation techniques used in the transfer of information from the coarse domain to the nested domain are usually of a polynomial form or a linear/bilinear form.

Problems can arise with the use of polynomial techniques in areas of sharp gradients due to the formation of surplus oscillation of the interpolation variables. Therefore linear interpolation is more widely used for both spatial and temporal interpolation.

There are four main updating interpolation procedures for the transfer of information from the fine domain into the coarse domain (1) direct copy (2) basic averaging procedure (3) Shapiro and (4) fully weighted averaging procedure, [3].

(1) **Direct copy** is the most severe interpolation technique with only the nested grid point that lies directly in the region of the coarse grid point being used in the procedure.

$$\phi^c_{i,j} = \phi^n_{i,j},$$

where $\phi^c_{i,j}$ represents the coarse grid point and $\phi^n_{i,j}$ represents the nested grid point that overlays the centre of the coarse grid cell.

(2) **The average procedure** takes into account all fine grid points that are enclosed in the coarse cell, [1]. This scheme is based on the assumption that the fine grid variables over laying the one coarse grid cell have a uniform distribution of value.

$$\begin{aligned}\phi^c_{i,j} =& \frac{1}{9}(\phi^n_{i-1,j-1} + \phi^n_{i-1,j} + \phi^n_{i-1,j+1} + \phi^n_{i,j-1} + \phi^n_{i,j} + \phi^n_{i,j+1} + \phi^n_{i+1,j-1} \\ &+ \phi^n_{i+1,j} + \phi^n_{i+1,j+1}),\end{aligned}$$

$\phi^c_{i,j}$ representing the coarse point that is being updated and $\phi^n_{i,j}$ being the fine grid values in the same cell.

(3) **The Shapiro interpolation scheme** is based on the assumption that the nested grid point that lies in the central region of the coarse grid is of equal

importance to the sum of the other nested grid points enclosed in the coarse grid cell, [3].

$$\phi^c_{i,j} = \frac{1}{16}(\phi^n_{i-1,j-1} + \phi^n_{i-1,j} + \phi^n_{i-1,j+1} + \phi^n_{i,j-1} + 8\phi^n_{i,j} + \phi^n_{i,j+1} + \phi^n_{i+1,j-1} + \phi^n_{i+1,j} + \phi^n_{i+1,j+1}).$$

(4) The final interpolation scheme is the full weighted averaging method and assumes that the interpolated value used for the updating procedure should be influenced mainly by nested grid points close to the centre of the coarse grid point being updated and less by the more distant points, [3].

$$\phi^c_{i,j} = \frac{1}{20}(\phi^n_{i-1,j-1} + 2\phi^n_{i-1,j} + \phi^n_{i-1,j+1} + 2\phi^n_{i,j-1} + 8\phi^n_{i,j} + 2\phi^n_{i,j+1} + \phi^n_{i+1,j-1} + 2\phi^n_{i+1,j} + \phi^n_{i+1,j+1})$$

3.2. Algorithms of Two-Way Nesting Grid

3.2.1. Algorithm of two-way nesting grid when the space refinement factor is 1:3 and time refinement factor is 1:2

Case 1. Suppose all flux values in the finer and the coarser region are known at time level $t = n\Delta t$ and we need to solve the finer and the coarser region values at the next time step $t = (n+2)\Delta t$, Since the coarser grid region and the finer grid region adopt different grid sizes, the time step sizes for each region are different due to the requirement of stability. Assume that the time step of the finer region is one half of the coarser region.

1. Get the solution of the free surface elevation at $t = (n + 1)\Delta t$ in the coarser region by solving continuity equation.
2. Get the free surface elevation at $t = (n + 1/2)\Delta t$ in the finer region by solving continuity equation.
3. Get the flux values at $t = (n + 1)\Delta t$ in the finer grid region by solving momentum equations.
4. Get the free surface elevation at $t = (n + 3/2)$ in the finer region by solving continuity equation.
5. To transfer all the information from the finer grid region at $t = (n+1)\Delta t$ to the coarser region by using updating full-weighting scheme.
6. Get the flux values at $t = (n + 2)\Delta t$ in the coarser region by solving momentum equation.
7. Get the flux values at $t = (n + 2)\Delta t$ in the finer region by solving momentum equation.

8. Transfer the informationat $t = (n + 2)\Delta t$, from the finer grid region to the coarser region.

Case 2. Assume that all model variables at time t are known and the time step of the finer region is one half of that of the coarser region.

1. Get The solution for the free surface elevation at $t = (n + 1/2)\Delta t$ in the coarser region using continuity equation.

2. Solve the continuity equation in the finer region, we need to have the flux information along the connected boundary at $t = n\Delta t$. So the flux values in the coarser grids at the connected boundary are linearly interpolated and then those interpolated values are set to the fluxes in the finer at the boundary.

3. Solve the free surface elevation at $t = (n+1/4)\Delta t$ in the finer grid region by solving continuity equation.

4. Solve the flux values at $t = (n+1/2)\Delta t$ in the finer grid region by solving momentum equations.

5. Solve the free surface elevation at $t = (n+3/4)\Delta t$ in the finer grid region by solving continuity equation.

6. Transfer all the information at $t = (n+1/2)\Delta t$ from the finer grid region to the coarser region with update schemes by using full-weighting method.

7. Get the flux values at $t = (n + 1)\Delta t$ in the finer region by solving momentum equations.

9. Get the flux values at $t = (n + 1)\Delta t$ in the coarser region by solving momentum equations.

8. Transefer all the information of the flux values at $t = (n+1)\Delta t$ from the finer to coarser region.

3.2.2. Algorithm of two-way nesting grid when the space and time refinement factor is 1:3

Suppose all flux values in the finer region and the coarser region, are known at time level $t = n\Delta t$ and we need to solve the finer and the coarser region values at the next time step $t = (n + 2)\Delta t$.

1. Get the free surface elevation at $t = (n + 1)\Delta t$ in the coarser region by solving continuity equation.

2. To solve the continuity equation in the finer region , we need to have the flux information along the connected boundary at $t = n\Delta t$. So the flux values in the coarser grids at the connected boundary are linearly interpolated and then those interpolated values are set to the fluxes in the finer at the boundary.

3. Get the free surface elevation at $t = (n + 1/3)\Delta t$ in the finer region by solving continuity equation.

4. Get the flux values at $t = (n + 2/3)\Delta t$ in the finer grid region by using momentum equations.

5. Get the free surface elevation at $t = (n + 1)\Delta t$ in the finer grid region by using continuity equation.

6. Get the the flux values at $t = (n + 4/3)\Delta t$ in the finer grid region by solving momentum equations.

7. Get the free surface elevation at $t = (n + 5/3)\Delta t$ in the finer grid region by solving continuity equation.

8. Transfer the information at $t = (n + 1)\Delta t$ from the finer grid region to the coarser region using updating scheme (copy grid).

9. Solve the flux values at $t = (n + 2)\Delta t$ in the finer region by using momentum equations.

10. Solve the flux values at $t = (n + 2)\Delta t$ in the coarser region using momentum equations.

11. Transfer all the information at $t = (n + 2)\Delta t$ from the finer grid region to the coarser region.

4. Model Calculations

In this section, we apply multiple nested for 2D depth- averaged non-linear shallow water equations in structured grids by using an explicit finite difference in space and leapfrog with Asselin-Roberts filter schemes in time with initial condition $\eta = u = v = 0$ and Dirichlet boundary condition, linear interpolation and for update interpolation scheme, we use full-weighting scheme and discusses some examples when the space refinement ratio 1:5 and the time refinement ratio 1:2. To verify the performance the nesting technique, compare the results in this case with the results in case 1:3. The results show the performance of nesting technique.

Example 1. when the space refinement is 1:5 and the temporal refinement is 1:2.

In this example, we find the relative error l2 for the 2D depth-averaged nonlinear shallow watere quations in Region 1 (coarse grids) which contains one fine grid (child embedded or separable to parent) located in Region 2 (fine grid) which contain another fine grid located in Region 3 at difference times t=1000 , . . ., 5000 hr when the coarse grid length 5×5, the time step in fine grid is one half time in coarse grids at each level and time steps in coarse grid =0. 010 (the information about the coarse and fine grid are given in table (1):

Information	Grid 01	Grid 21	Grid 31
Number of grids	100×100	100×100	100×100
length grid size	5	1	. 20
parent grid	non	grid 01	grid 21
grid size ratio	non	5	5
time step in sec	0. 010	0. 005	0. 0025
SWEs	non-linear	non-linear	non-linear
Latitude (North-south)	1-100	61-80	61-80
longitude Eest-west)	1-100	61-80	11-30
CFL condition	—	0. 7	0. 6

Table 1: The information on the set up of the different grids for the 2D non-linear shallow water equations are given below

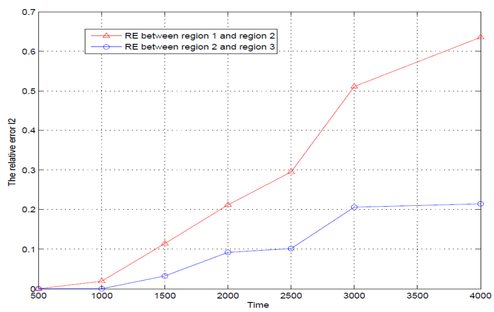


Figure 2: Compare relative errorl2 between Region1, Region2 and Region2-Region3 with the space refinement ratio 1:3

The following figure compare relative errorl2 between(coarse grid in level 1 and fine grid in level 2) and the relative errorl2 between (level 2 and level 3) in case 1:3 and case1:5. We can show the results when the space refinement ratio 1:5 are better than the case 1:3.

Example 2. When the space refinement ratio is 1:5 and the temporal refinement ratio is 1:2

In this example, we find the relative errorl2 for 2d depth-averaged linear shallow water equations in Region 1 (coarse grids) which contain one fine grid located in Region 2 (fine grid) which contain another fine grid located in Region 3 for difference times t=1000 , . . . , 5000 hr when the coarse grid length 5×5

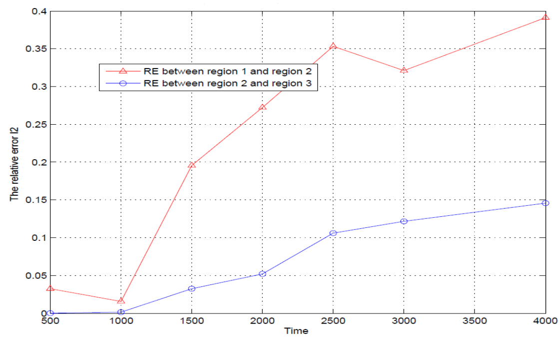


Figure 3: Compare relative errorl2 between Region1, Region2 and Region2-Region3 with the space refinement ratio 1:5

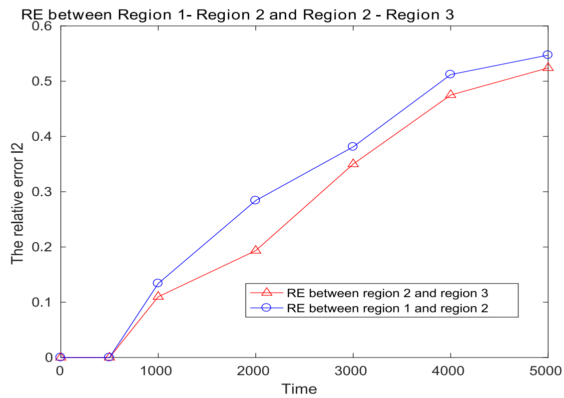


Figure 4: tcompare between the relative error in (level 1 -level 2) and (level 2 -level 3) in case 1:5.

, the time step in fine grid is one half time in coarse grids at each level, time steps in coarse grid =0. 010 (the information about the coarse and fine grid the same previous examples):

The first figure compare between the relative errorl2 (in level 1 -level 2) and (level 2 -level 3) in case 1:5 and the second figure compare between the relative error in (level 1 -level 2) and (level 2 -level 3) in case 1:3.

Example 3. When the ratio refinement factor in both space and time is equal 1:5.

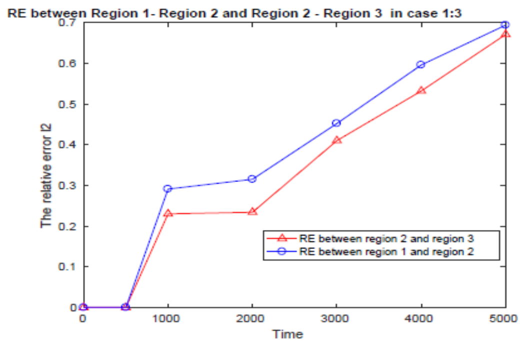


Figure 5: tcompare between the relative error in (level 1 -level 2) and (level 2 -level 3) in case 1:3.

In this example , we use 2D depth-averaged linear shallow water equations with non-rotated f=0, wind stress and bottom stress=0. If we take different values of time t= 20, 30, 40,. . ., 100 days for finding the relative error between carse grid and fine grid when nx=300 , ny=300 , dx=3 , dy=3 in coarse grid and nx=200 , ny=200 , dx=0. 6 , dy=0. 6 fine grid in , the space and time refinement ratio factor is 1:5, the time step in coarse grid is 0. 05 by using the Dirichlet open boundary condition

The following figuers compare the absolute error and relative error l2 in one-way and two- way nesting for the 2D depth-averaged linear shallow water equations.

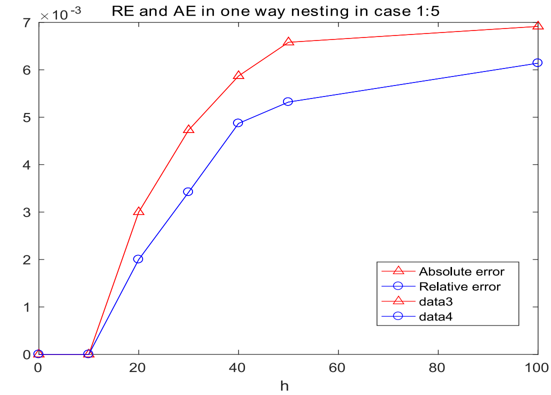
When compare the results with the results in case 1:3 , we obtained good results when use the space refinement ratio 1:5 because the large ratio gives very well connected boundary conditions.

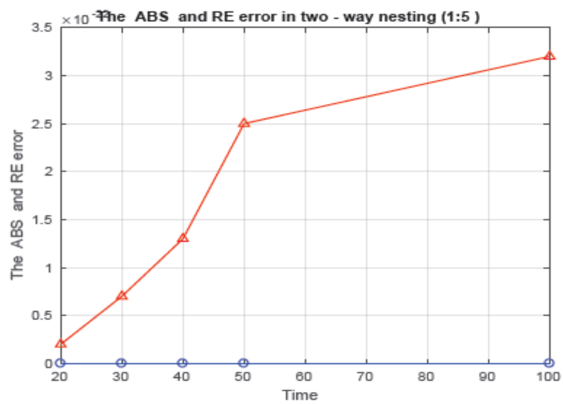
The following figuers show the ABS error and RE error in one-way and two-way nesting in case 1:3

Example 4: when the space refinement ratio is 1:5 and temporal refinement ratio is 1:2

In this example , we find the absolute errorl2 for 2D depth- averaged linear shallow water equation with non-rotated f=0, wind stress and bottom stress =0 in case fine grid contain again one fine grid in another level at difference times t= 10, 20, . . . , 300 days by using the Dirichlet open boundary condition when nx=ny=200 , dx=dy=3 , time step=0. 5 in coarse grid and nx=ny=200, dx=dy=0. 6, time step=0. 25 in fine grid.

The following figure compare the absolute error between two way





nesting in (coarse grid - fine grid level 1) and (fine - fine) grid in level. When compare these results with the results in case 1:3, we obtained very well results when use the space refinement ratio 1:5.

The following figure compare the absolute error between (coarse grid - fine grid in level 1) and (fine - fine) grid in level 2 in case the ratio 1:3

Example 5. When the space refinement ratio is 1:5 (mesh refinement factor) and temporal refinement ratio is 1:2.

In this example, we find the relative errorl2 in one- way nesting and two way nesting for 2D depth-averaged nonlinear shallow water equations with non-rotated f=0 , $\nu = 0$ wind and bottom stress =0 , if we take different values of time t= 100, 200 hr, . . when nx=ny=200, dx=dy=1 , dt=0. 01 , the interpolation techniques used in the transfer of information from the coarse domain to the nested domain are usually of a polynomial form or a linear form

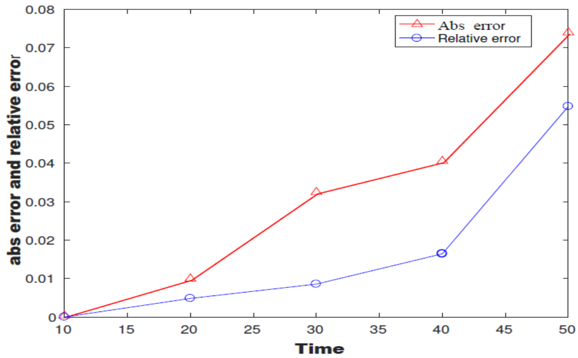


Figure 6: figure represent ABS error and RE error in one-way nesting in case 1:3

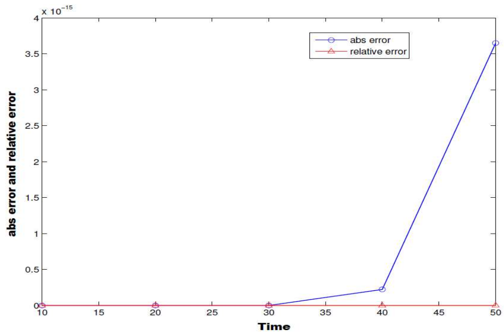


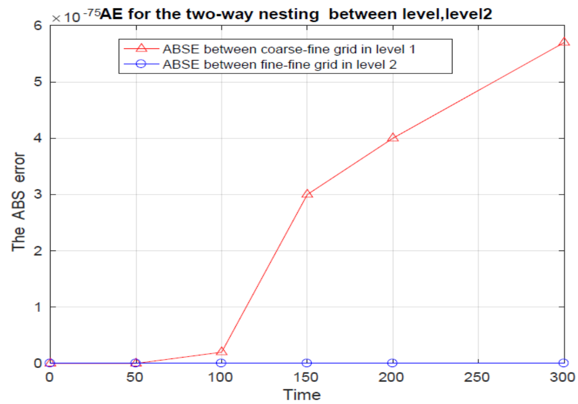
Figure 7: figure represent ABS error and RE error in two-way nesting in case 1:3

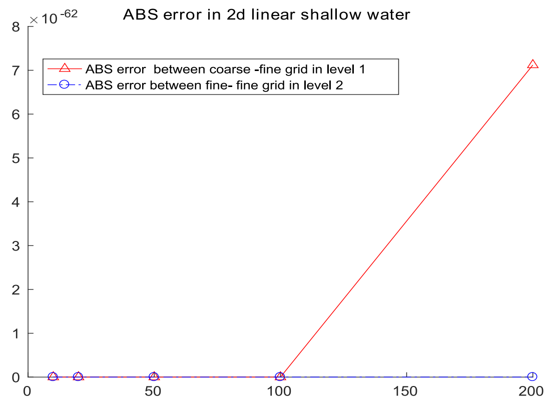
and for updating interpolation used full-weighting procedures.

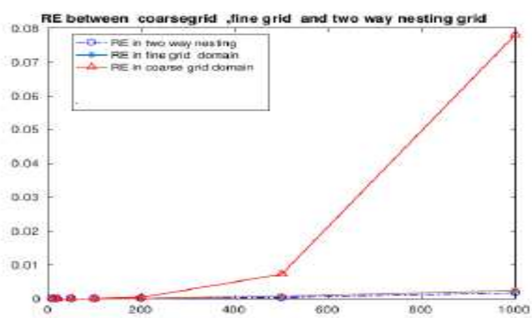
The following figure shows the relative errorl2 of free surface in one-way and two- way nesting grids with space ratio 1:5 . The resuls in this case is very good compair to the results in case 1:3.

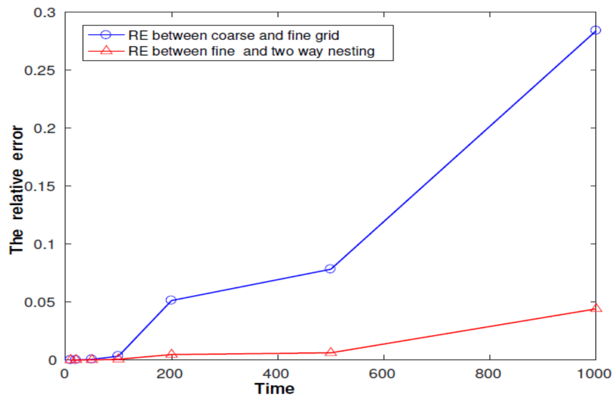
Example 6. Compare the results when both the space and temporal refinement ratio are 1:3 and 1:5

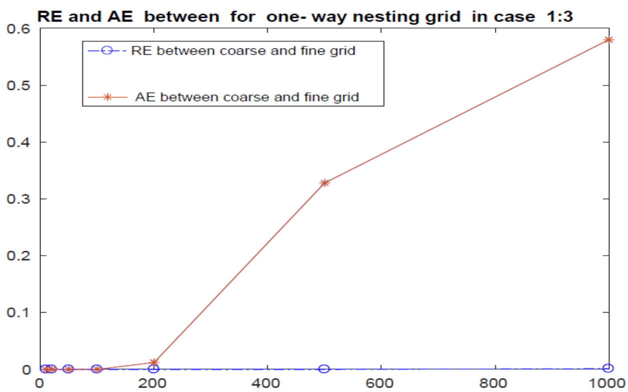
In this example, we find relative errorl2 in one- way nesting and two way nesting for 2D depth-averged nonlinear shallow water equations (with non-rotated f=0 , $\nu = 0$, wind and bottom stress =0) at different values of time t= 100, 200, . . . , 1000 hr when nx=ny=300, dx=dy=1 , dt=0. 01 with the initial condition u=v=0 .


AE for the two-way nesting between level,level2
ABSE between coarse-fine grid in level 1
ABSE between fine-fine grid in level 2
The ABS error
Time


ABS error in 2d linear shallow water
ABS error between coarse -fine grid in level 1
ABS error between fine- fine grid in level 2


RE between coarsegrid ,fine grid and two way nesting grid
RE in two way nesting
RE in fine grid domain
RE in coarse grid domain

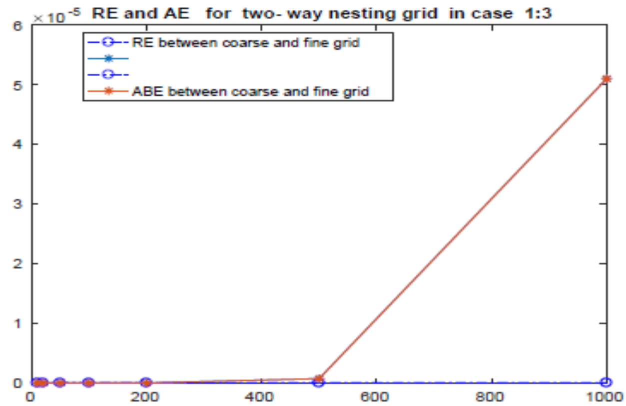


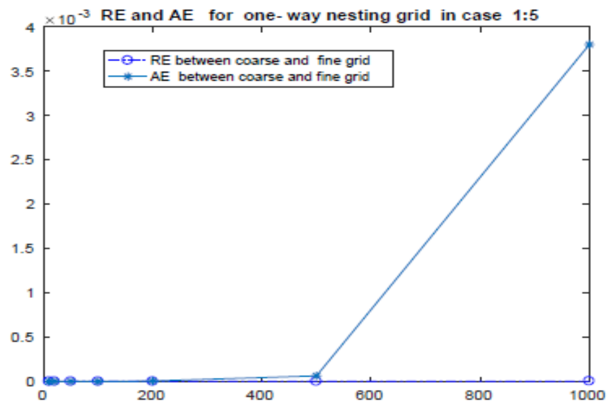


The following figures show the relative error and absolut error in one-way and two- way nesting grids in two-cases 1:3 and 1:5.

Example 7. In this example , we use system of the 2D depth-averged nonlinear shallow water equations with(wind stress=0 and f=0). A sequence of snapshots for elevation of the free surface , u velocity in x-momentum and v velocity in y-momentum at time 1000, 2000 hr, . . . 4000 hr in coarse grid by using nx=150 , ny=150 , dx=10 , dy=10 (grid length) and the time steps in fine grid is one half time step in coarse grid t=0. 25 and the space and temporal refinement ratio is 1:3.

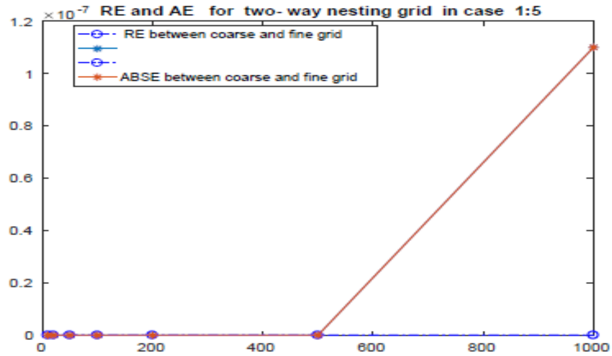
The following figuer shows the relative errorl2 for free surface , u- velocity and v- velocity between coarse grid and fine grid.





5. Summary and Conclusion

In this paper, a new technique of a two-way nested grid was described and a new approach is presented to treat this problem. The nesting procedure has been tested with data under different conditions. The approach introduced in the paper presented the possibility of increasing accuracy and efficiency of the modeling results within a two-way nesting grid model. To verify the nested multiple grid models, several numerical examples were presented and it was shown that two-way nesting techniques perform very well when the refinement factor 1:3 and 1:5. In particular, two-way nesting ensures dynamical consistency



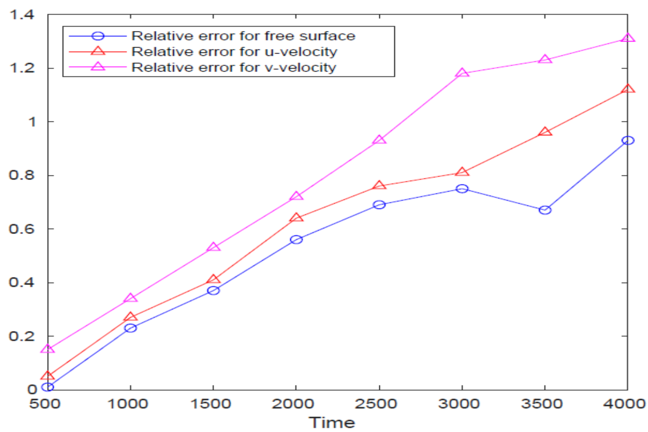


Figure 8: Relative errorl2 for free surface , u-velocity and v- velocity

between the coarse grid and fine grid occurs frequently. Also, when use the refinement factor 1:5 the connecting boundary conditions are very good.

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