

**ON THE EXTENDED CHEN'S AND PHAM'S SOFTWARE
RELIABILITY MODELS. SOME APPLICATIONS**

Nikolay Pavlov¹, Anton Iliev^{2 §},
Asen Rahnev³, Nikolay Kyurkchiev⁴

^{1,2,3,4}Faculty of Mathematics and Informatics
University of Plovdiv Paisii Hilendarski
24, Tzar Asen Str., 4000 Plovdiv, BULGARIA

Abstract: The Hausdorff approximation of the impulse function $\sigma^{**}(t)$ by sigmoidal functions based on the extended Chen's and Pham's cumulative functions are studied and an expression for the error of the best approximation is found. The received results are of independent significance in the study of issues related to neural networks and impulse technics. Using programming environment Mathematica we give results of many numerical examples which confirm the theory presented here. We give also real examples with data provided in [4] using extended Chen's software reliability model and extended Pham's deterministic software reliability model. Dataset included [5] Year 2000 compatibility modifications, operating system upgrade, and signaling message processing. Some direct comparisons are made.

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Key Words: three-parameters Chen's cumulative function (3Ccdf), four parameters Pham's cumulative function (4Pcdf), impulse function $\sigma^{**}(t)$, Hausdorff approximation, upper and lower bounds

1. Introduction

An important role within the hierarchical models in the procedure for quantifying the quality of software products is played by the so-called computational

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[§]Correspondence author

method based on the theoretical and empirical dependencies (usually at an early stage in their development), statistical data accumulated during tests, exploitation and the accompaniment of the program product.

An important measure of reliability assessment (completeness, accuracy and consistency) is the so-called metric (asymptotic metrics).

There are two main approaches to testing: structured and functional.

Many studies have been devoted to this overarching theme. We will only note that, depending on the test data selected and the expected results, testing is divided into: deterministic testing and stochastic testing.

Detailed description of all elements in the area of debugging theory may be found in the following books [1]–[3].

For some degradation models with applications to reliability and survival analysis, see [6].

In the book [7], we pay particular attention to both deterministic approaches and probability models for debugging theories.

A Hausdorff metric was chosen to evaluate the test data which are fitted to the sigmoid models proposed in this book.

Some of the existing cumulative distributions (Gompertz–Makeham, Yamada-exponential, Yamada–Rayleigh, Yamada–Weibull, transmuted inverse exponential, transmuted Log-Logistic, Kumaraswamy–Dagum and Kumaraswamy Quasi Lindley) are considered in the light of modern debugging and test theories.

Some software reliability models, can be found in [8]–[32].

In a number of cases, these results have independent significance in the study of issues related to neural networks and impulse technics (see, for instance [33]–[39], [40]–[52]).

In this note we study the Hausdorff approximation of the impulse function $\sigma^{**}(t)$ by sigmoidal functions based on the extended Chen's [53] and Pham's cumulative functions.

We propose a software modules (intellectual properties) within the programming environment CAS Mathematica for the analysis.

The models have been tested with real-world data.

2. Preliminaries

The typical example of impulse function from antenna feeder technique has the following shape (see, Fig. 1):

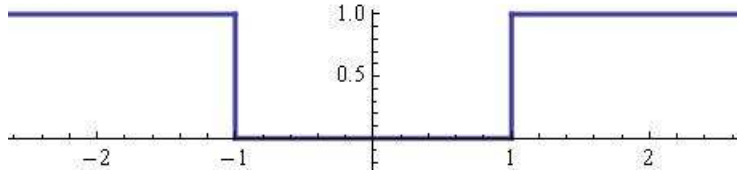


Figure 1: The signal of $\sigma^{**}(t)$ – type.

$$\sigma^{**}(t) = \begin{cases} 1, & t \in [-\infty, -1) \cup (1, +\infty) \\ 0, & t \in [-1, 1]. \end{cases} \tag{1}$$

Many probability distributions haven been introduced to analyze real datasets with bathtub failure rates.

Definition 1. The extended Chen [53] software reliability model is given as follows (see, Chaubey and Zhang [57]):

$$M(t) = \omega \left(1 - e^{\lambda(1-e^{t^\beta})} \right)^\alpha \tag{2}$$

where $\lambda > 0, \alpha > 0, \beta > 0$.

For other extensions of the Chen distribution, see [54] – [57].

Definition 2. [5] The extended deterministic Pham’s software reliability model is given as follows:

$$M_1(t) = N * \left(1 - \frac{\beta}{\beta + (at)^b} \right)^\alpha \tag{3}$$

where $a > 0, b > 0, \alpha > 0, \beta > 0$.

Definition 3. [58] The Hausdorff distance (the H–distance) $\rho(f, g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f, g) = \max\left\{ \sup_{A \in F(f)} \inf_{B \in F(g)} \|A - B\|, \sup_{B \in F(g)} \inf_{A \in F(f)} \|A - B\| \right\},$$

wherein $\|\cdot\|$ is any norm in \mathbb{R}^2 , e. g. the maximum norm $\|(t, x)\| = \max\{|t|, |x|\}$; hence the distance between the points $A = (t_A, x_A), B = (t_B, x_B)$ in \mathbb{R}^2 is $\|A - B\| = \max(|t_A - t_B|, |x_A - x_B|)$.

3. Main Results

3.1. A Note on the Extended Chen’s Function

We consider the following sigmoid:

$$M^*(t) = \left(1 - e^{\lambda(1-e^{t^\beta})}\right)^\alpha, \tag{4}$$

with $\omega = 1$, β - is an even number and

$$t_0 = \left(\ln\left(1 - \frac{1}{\lambda} \ln\left(1 - 2^{-\frac{1}{\alpha}}\right)\right)\right)^{\frac{1}{\beta}}; \quad M^*(t_0) = \frac{1}{2}. \tag{5}$$

The one-sided Hausdorff distance d between the function $\sigma^{**}(t)$ and the sigmoid ((4)–(5)) satisfies the relation

$$M^*(t_0 - d) = d. \tag{6}$$

The following theorem gives upper and lower bounds for d

Theorem 1. Let

$$p = \frac{1}{2},$$

$$q = -1 - \left(1 - \frac{1}{\lambda} \ln\left(1 - 2^{-\frac{1}{\alpha}}\right)\right) \left(1 - 2^{-\frac{1}{\alpha}}\right) \left(\frac{1}{2}\right)^{\frac{\alpha-1}{\alpha}} \\ \times \left(\ln\left(1 - \frac{1}{\lambda} \ln\left(1 - 2^{-\frac{1}{\alpha}}\right)\right)\right)^{\frac{\beta-1}{\beta}} \alpha\beta\lambda.$$

For the one-sided Hausdorff distance d between $\sigma^{**}(t)$ and the sigmoid ((4)–(5)) the following inequalities hold for:

$$-2.1q > e^{1.05} \\ d_l = \frac{1}{-2.1q} < d < \frac{\ln(-2.1q)}{-2.1q} = d_r. \tag{7}$$

Proof. Let us examine the function:

$$F(d) = M^*(t_0 - d) - d. \tag{8}$$

We see that $q < 0$. From $F'(d) < 0$ we conclude that function F is decreasing.

Consider the function

$$G(d) = p + qd. \tag{9}$$

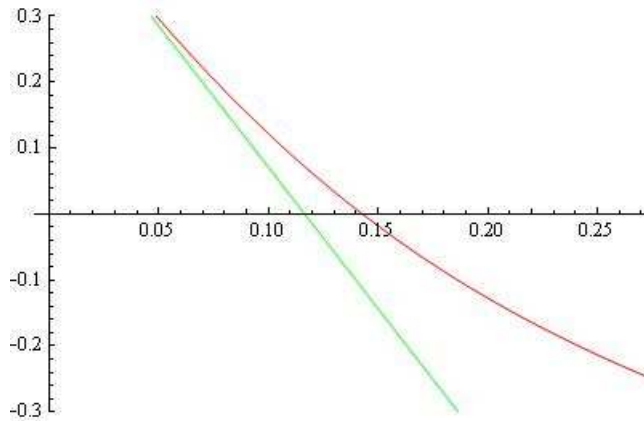


Figure 2: The functions $F(d)$ and $G(d)$.

From Taylor expansion we obtain $G(d) - F(d) = O(d^2)$.
 Hence $G(d)$ approximates $F(d)$ with $d \rightarrow 0$ as $O(d^2)$ (see Fig. 2).
 In addition $G'(d) < 0$.
 Further, for $-2.1q > e^{1.05}$ we have $G(d_l) > 0$ and $G(d_r) < 0$.
 This completes the proof of the theorem.

3.2. Numerical Examples

1. Hausdorff approximation of the impulse function $\sigma^{**}(t)$ by sigmoidal function based on the extended Chen's [53] cumulative function.

The model ((4)–(5)) for $\beta = 6$, $\alpha = 0.95$, $\lambda = 10$, $t_0 = 0.631995$ is visualized on Fig. 3.

From the nonlinear equation (6) and inequalities (7) we have: $d = 0.142801$, $d_l = 0.110996$ and $d_r = 0.243998$.

2. Application in the field of antenna–feeder technique.

After the substitution $t = kr \cos \theta + a$, where

- $k = \frac{2\pi}{l}$, l is the wave length;
- a is the phase difference;
- θ is the azimuthal angle;
- r is the distance between the emitters ($r = \frac{l}{2}$ is fixed),

the function $M^*(t)$ for $\lambda > 0$, $\alpha > 0$, $\beta > 0$ (or emitting chart of antenna factor) can be written in the form:

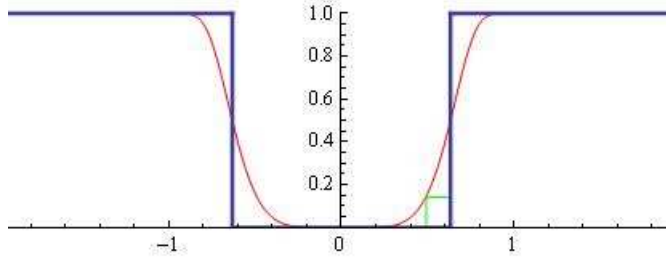


Figure 3: The model ((4)–(5)) for $\beta = 6$, $\alpha = 0.95$, $\lambda = 10$, $t_0 = 0.631995$; H-distance $d = 0.142801$, $d_l = 0.110996$, $d_r = 0.243998$.

$$M^*(\theta) = \left(1 - e^{\lambda(1 - e^{(\pi \cos \theta + \alpha)^\beta})} \right)^\alpha. \tag{10}$$

Typical emitting chart is visualized on Fig. 4.

Of course, the question of the practical realization of the activation functions which are generated as emitting charts remains open.

The mathematical apparatus proposed in the article can be successfully used for imitation and simulation of such charts.

3. Application in the field of debugging and test theory.

We give real examples with data provided in [4].

The operating time of the software is 167,900 days. 115 failures are detected for these days which contain 71 unique failures.

Table 1 shows the failures data which are united for each of the 13 months.

Dataset included [5] Year 2000 compatibility modifications, operating system upgrade, and signaling message processing.

The fitted model

$$M(t) = \omega \left(1 - e^{\lambda(1 - e^{t^\beta})} \right)^\alpha$$

based on the data of Table 1 for the estimated parameters:

$$\omega = 110; \quad \lambda = 0.026895; \quad \beta = 0.816531; \quad \alpha = 0.457081$$

is plotted on Fig. 5.

In conclusion, we will note that the determination of compulsory in area of the Software Reliability Theory components, such as confidence intervals and confidence bounds, should also be accompanied by a serious analysis of the value of the best Hausdorff approximation - the subject of study in the present paper.

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phi1[theta_] := (1 - Exp[lambda * (1 - Exp[(Pi * Cos[theta] + a) ^ beta])]) ^ alpha
Manipulate[PolarPlot[(1 - Exp[lambda * (1 - Exp[(Pi * Cos[theta] + a) ^ beta])]) ^ alpha ,
  {theta, -2 Pi, 2 Pi}], {beta, 2, 10., Appearance -> "Open"}, {alpha, 0.01, 10., Appearance -> "Open"}
, {lambda, 0.01, 30, Appearance -> "Open"}, {a, 0.01, 2 * Pi, Appearance -> "Open"}]

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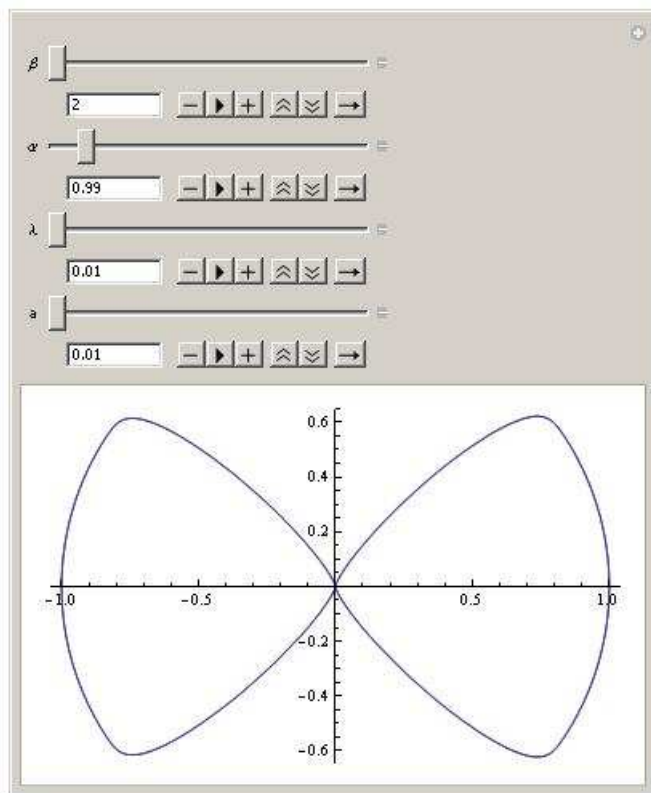


Figure 4: Typical emitting chart ($M^*(\theta)$) for $\beta = 2$; $a = 0.01$; $\alpha = 0.99$; $\lambda = 0.01$.

We hope that the results will be useful for specialists in this scientific area.

3.3. A Note on the Extended Deterministic Pham's Software Reliability Model

In this Section we study the Hausdorff approximation of the shifted Heaviside function

Month Index	System Days (Days)	System Days (Cumulative)	Failures	Cumulative Failures
1	961	961	7	7
2	4170	5131	3	10
3	8789	13,920	14	24
4	11,858	25,778	8	32
5	13,110	38,888	11	43
6	14,198	53,086	8	51
7	14,265	67,351	7	58
8	15,175	82,526	19	77
9	15,376	97,902	17	94
10	15,704	113,606	6	100
11	18,182	131,788	11	111
12	17,760	149,548	4	115
13	18,352	167,900	0	115

Table 1: Field failure data [4].

$$h_{t_0}(t) = \begin{cases} 0, & \text{if } t < t_0, \\ [0, 1], & \text{if } t = t_0 \\ 1, & \text{if } t > t_0 \end{cases} \tag{11}$$

by sigmoidal function based on the extended Pham’s cumulative function

$$M_1^*(t) = N * \left(1 - \frac{\beta}{\beta + (at)^b} \right)^\alpha \tag{12}$$

with $N = 1, b = \beta$, without loosing of generality and

$$t_0 = \frac{1}{a} \left(\beta \left(\frac{1}{1 - (\frac{1}{2})^{\frac{1}{\alpha}}} - 1 \right) \right)^{\frac{1}{\beta}} ; M_1^*(t_0) = \frac{1}{2}. \tag{13}$$

The one-sided Hausdorff distance d between the Heaviside step function $h_{t_0}(t)$ and the sigmoid $M_1^*(t)$ satisfies the relation

$$M_1^*(t_0 + d_1) = 1 - d_1. \tag{14}$$

The following theorem gives upper and lower bounds for d_1

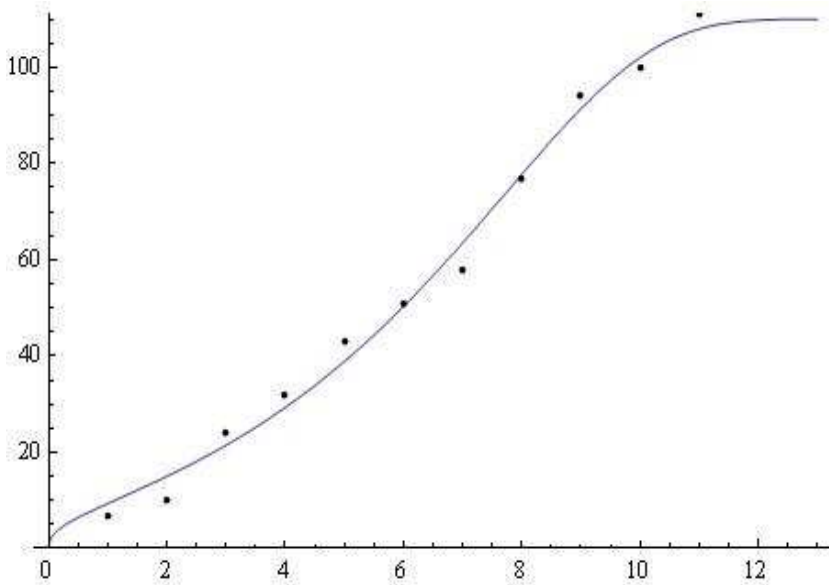


Figure 5: An example of the usage of dynamical and graphical representation for the function $M(t)$.

Theorem 2. Let

$$p_1 = -\frac{1}{2},$$

$$q_1 = 1 + \left(\beta \left(\frac{1}{1 - \left(\frac{1}{2}\right)^{\frac{1}{\alpha}}} - 1 \right) \right)^{\frac{\beta-1}{\beta}} \left(\frac{1}{2} \right)^{\frac{\alpha-1}{\alpha}} \left(1 - \left(\frac{1}{2} \right)^{\frac{1}{\alpha}} \right)^2 a\alpha.$$

For the one-sided Hausdorff distance d_1 between h_{t_0} and the sigmoid $M_1^*(t)$ the following inequalities hold for:

$$2.1q_1 > e^{1.05}$$

$$d_{l_1} = \frac{1}{2.1q_1} < d_1 < \frac{\ln(2.1q_1)}{2.1q_1} = d_{r_1}. \tag{15}$$

Proof. Let us examine the functions:

$$F_1(d_1) = M_1^*(t_0 + d_1) - 1 + d_1. \tag{16}$$

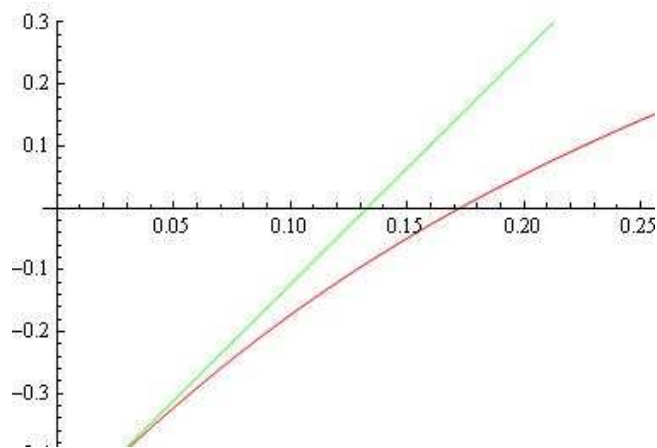


Figure 6: The functions $F_1(d_1)$ and $G_1(d_1)$.

$$G_1(d_1) = p_1 + q_1 d_1. \quad (17)$$

From Taylor expansion we obtain $G_1(d_1) - F_1(d_1) = O(d_1^2)$.

Hence $G_1(d_1)$ approximates $F_1(d_1)$ with $d_1 \rightarrow 0$ as $O(d_1^2)$ (see Fig. 6).

In addition $G'(d_1) > 0$.

Further, for $2.1q_1 > e^{1.05}$ we have $G_1(d_{l_1}) < 0$ and $G(d_{r_1}) > 0$.

This completes the proof of the theorem.

For the parameters of the function $M_1^*(t)$ at fixed t_0 we get $\alpha = 0.95$; $a = 3.9$; $\beta = 4$ and from the nonlinear equation (14) and inequalities (15) we have: $d_1 = 0.172737$, $d_{l_1} = 0.126554$ and $d_{r_1} = 0.261598$ (see, Fig. 7).

For other software reliability models, see [59]–[65]. A new class activation functions with application in the theory of impulse technics is being discussed in [52].

Acknowledgments

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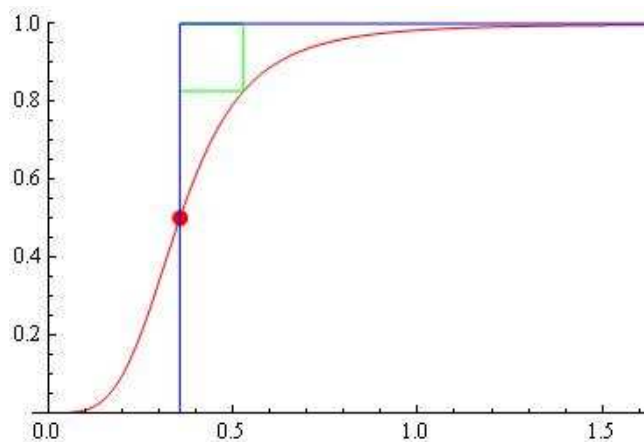


Figure 7: The function $M_1^*(t)$ for $\alpha = 0.95$; $a = 3.9$; $\beta = 4$; H-distance $d_1 = 0.172737$, $d_{l_1} = 0.126554$, $d_{r_1} = 0.261598$.

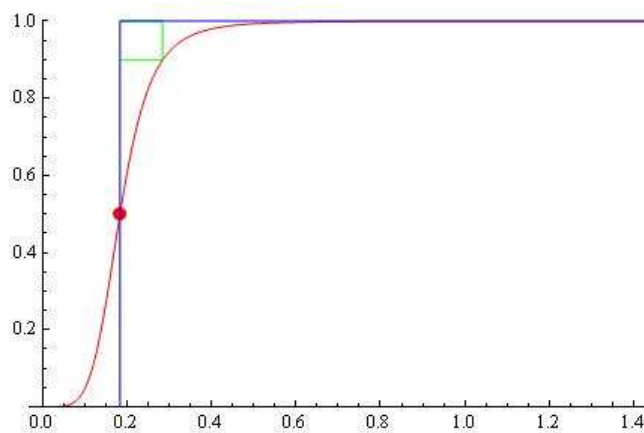


Figure 8: The function $M_1^*(t)$ for $\alpha = 0.99$; $a = 7.5$; $\beta = 5$; $t_0 = 0.183451$; H-distance $d_1 = 0.100876$, $d_{l_1} = 0.061107$, $d_{r_1} = 0.170802$.

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