

THREE DIMENSIONAL NONLINEAR INSTABILITY ANALYSIS OF ELECTROCONVECTIVE FINITE DIELECTRIC FLUIDS

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Abstract: The nonlinear electrohydrodynamic Rayleigh-Taylor instability analysis for the interface between two finite fluid layers with interfacial transfer of heat and mass is studied for two and three dimensions cases. Multiple scales perturbation method is applied to get general linear dispersion relation and a nonlinear Ginzburg-Landau equation, representing the behaviour of the system. The stability of the system is analyzed theoretically and graphically. In linear case, the stability criterion is independent of heat and mass transfer coefficient, and both the dimensions and fluid depths have destabilizing effects, while both the electric field and the surface tension have stabilizing influences. In the nonlinear case, in the two-dimensional disturbances, for small electric field values E_0 , stability holds for only small wavenumbers range which decreases by increasing E_0 , while for high values of E_0 , stability exists also at high wavenumbers range which increases by increasing E_0 . The surface tension T and the fluid depths d_1 and d_2 have destabilizing effects, while the heat and mass transference coefficient α has a dual role on the stability of the system, i.e stabilizing as well as destabilizing. In the three - dimensional disturbances, we found that the stability regions decreases under the effects of all the previous physical parameters.

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1. Introduction

The Rayleigh-Taylor instability (RTI) is a fundamental fluid dynamical phenomenon that occurs at the interface between a dense fluid supported by a lighter fluid under gravity. Because of its importance for geophysical, astrophysical, industrial processes and controlled fusion, different characteristics of fluids have been included in the Rayleigh-Taylor instability through theoretical investigations [1, 2]. The effects of surface tension, variable density and viscosity on the RTI have been well documented in the Chandrasekhar's treatise [3]. The study of nonlinear RTI has studied by many researchers [4, 5]. Nayfeh [6] considered the nonlinear stability of a surface wave propagation over a plane interface between two fluids. He obtained nonlinear Schrödinger equation representing the disturbance. The stability of the system could be deduced by the sign of the coefficients of this equation.

The problem of interfacial stability of two streaming fluids separated by an interface is usually discussed without mass and heat transfer effects. But there are situations, like film boiling or pool boiling of fluids, in which the effect of heat and mass transfer over the interface plays an important role in stability discussions. The technique of mass and heat transfer over an interface is of great importance in many environmental and industrial processes. These encompass the design of many types of contracting equipment, e.g. evaporators, boilers, gas-absorbers, condensers, pipelines, chemical reactors and nuclear reactors. Hsieh and his partners [7-9] presented a mathematical establishment for a liquid vapor interface with mass and heat transfer and obtained the stability conditions for RTI for inviscid vapor and liquid.

Nonlinear treatment for our topic is very important according to the nature of the flows are nonlinear. Hsieh [10] studied the nonlinear RTI with mass and heat transfer of inviscid fluids and remarked that in the nonlinear case, the stability range increases if there are a strong heat and mass transfer. Lee [11,12] studied the nonlinear analysis of RTI of inviscid fluids with mass and heat transfer and observed that stronger heat flux enlarges the region of stability. Awasthi and Agrawal [13] investigated the nonlinear capillary instability of cylindrical interface with mass and heat transfer between vapor and liquid phases using viscous potential flow theory, They found that the heat and mass transfer has

stabilizing effect for both axisymmetric and asymmetric disturbances in the nonlinear analysis.

The topic of electrohydrodynamics (EHD) has a great area of importance in various physical situations. A review on EHD with distinguished reference to many progress in this field was done by Melcher [14]. In the linear EHD stability theory, it is known that the normal electric field has a destabilizing effect, while the tangential one always has a stabilizing influence [15,16]. Problems on nonlinear EHD stability have been studied by many authors in recent years. El-Sayed and Callebaut [17-19] investigated the problem of the interfacial capillary-gravity waves of two dielectric fluids of uniform thickness in the presence of both horizontal and vertical electric fields to the interface between the two fluids, in the absence or presence of surface charges at their interfaces. They also obtained the envelope solutions in terms of the Jacobian elliptic functions of the steady state in (2+1) dimensions and these solutions may exist depending on the relative sign of the dispersive and nonlinear terms. For recent development of the topic, see the works of Papageorgiou and Petropoulos [20], Shankar and Sharma [21], Griffiths [22], Tomar et al [23], Uguz and Aubry [24], Ersoy and Uguz [25], and Jalaal et al [26].

On the other hand, A recent review on the subject of EHD instability problems of the interface between two fluids in both plane and cylindrical geometries in the presence of mass and heat transfer shows that the analysis of these studies was confined within the framework of dielectric or magnetic fluids, e.g. Mohamed et al. [27], Elhefnawy et al.[28], El-Sayed et al. [29,30], and Moatimid [31, 32]. Also, viscous potential flow theory have been used extensively in the study of the effect of different parameters on the stability of the fluids. For example, see Asthana and Agrawal [33], Awasthi and Agrawal [34], Maher and Moatimid [35], Moatimid and Hassan [36], and Awasthi [37]

Studies on nonlinear EHD instability analysis of two dielectric fluids in two and three dimensions have received great interest in recent years. A great number of published papers are dealing with this subject, for recent developments of this topic see the investigations of Young et al.[38], Abo Eldahab [39], Choudhury and Das [40], Moatimid and Hassan [41], El-sayed and co-workers [42-45].

In this paper, we study the general problem of nonlinear EHD instability analysis of two finite thickness layers of dielectric fluids with interfacial transfer of mass and heat in three-dimensions. This problem, to the best of our knowledge, has not been treated yet, Therefore, in this article, we shall first formulate the general interfacial problem for two dielectric bounded fluids in the presence of mass and heat transfer, and a uniform horizontal electric field. The linear and nonlinear analyses, using multiple scales method, are carried out. The stability

conditions are obtained for both the linear and nonlinear cases and discussed analytically and numerically. novel results are listed in a concluding remarks section with the main findings of different physical parameters including in the analysis.

2. Mathematical Formulation

Let us examine the three-dimensional finite amplitude capillary -gravity stokes wave propagating on the interface $z = 0$ that represent the balance case. The perturbed interface $z = \zeta(x, y, t)$ separates two incompressible dielectric fluids with uniform thickness. The lower fluid (with subscript 1) which occupies the layer $-d_1 < z < \zeta(x, y, t)$, has density ρ_1 , and dielectric constant ε_1 , while the upper fluid (with subscript 2), which occupies the layer $\zeta(x, y, t) < z < d_2$, has density ρ_2 , dielectric constant ε_2 . The temperatures at $z = 0$, $z = -d_1$ and $z = d_2$ are T_0 , T_1 and T_2 , respectively. The two fluids are affected by a constant horizontal electric field E_0 acts along positive x -direction. The surface tension forces T are considered between the two fluids. The system undergoes a uniform gravitational field g acting along negative z -direction. The interface is defined as

$$S(x, y, z, t) = z - \zeta(x, y, t) = 0 \quad (1)$$

the outer normal unit vector is expressed as

$$\mathbf{n} = \frac{\nabla S}{|\nabla S|} = [1 + \zeta_x^2 + \zeta_y^2]^{-\frac{1}{2}} (-\zeta_x, -\zeta_y, 1) \quad (2)$$

The fluids motion is assumed to be irrotational, thus there exist velocity potentials $\phi_j(x, y, z, t)$ such that

$$\mathbf{v}_j = \nabla \phi_j, \quad j = 1, 2. \quad (3)$$

where $\mathbf{v}_j$ is the fluid velocity. For incompressible fluids, the velocity potentials ϕ_j , ($j = 1, 2$), satisfy Laplace's equation [19]. i.e.

$$\nabla^2 \phi_1 = 0, \quad \text{for } -d_1 < z < \zeta(x, y, t) \quad (4)$$

$$\nabla^2 \phi_2 = 0, \quad \text{for } \zeta(x, y, t) < z < d_2 \quad (5)$$

In EHD, a valid quasi-static approximation is generally assumed. Then the electrical equations

$$\nabla \times \mathbf{E} = \mathbf{0}, \quad \text{and} \quad \nabla \cdot (\varepsilon \mathbf{E}) = 0. \quad (6)$$

Thus, the electric fields can be written about an electrostatic potentials $\psi_j(x, y, z, t)$ such that

$$\mathbf{E}_j = E_0 \mathbf{e}_x - \nabla \psi_j, \quad j = 1, 2. \quad (7)$$

Therefore, from equations (6) and (7), the electrostatic potentials also satisfy Laplace's equation, i.e.

$$\nabla^2 \psi_1 = 0, \quad \text{for} \quad -d_1 < z < \zeta(x, y, t) \quad (8)$$

$$\nabla^2 \psi_2 = 0, \quad \text{for} \quad \zeta(x, y, t) < z < d_2 \quad (9)$$

3. Boundary conditions.

The velocity potential ϕ_j and electrostatic potential ψ_j ($j = 1, 2$) solutions should satisfy the suitable boundary conditions [9-10, 46]

- (i) At the rigid lower and upper boundaries $z = -d_1$ and $z = d_2$,
The normal fluid velocities vanish, and then we have

$$\frac{\partial \phi_j}{\partial z} = 0 \quad \text{on} \quad z = (-1)^j d_j, \quad j = 1, 2, \quad (10)$$

The normal electric fields components also vanish, i.e.

$$\frac{\partial \psi_j}{\partial z} = 0 \quad \text{on} \quad z = (-1)^j d_j, \quad j = 1, 2, \quad (11)$$

- (ii) the various boundary conditions at the free interface $z = \zeta(x, y, t)$, are
The conservation of mass across the interface yields

$$\zeta_x \|\rho \phi_x\| + \zeta_y \|\rho \phi_y\| - \|\rho \phi_z\| + \zeta_t \|\rho\| = 0 \quad (12)$$

where $\|*\| = *_2 - *_1$ is the jump over the interface.

The tangential electric field component is assumed to be continuous cross the interface, and then we have

$$\|\psi_x\| + \zeta_x \|\psi_z\| = 0, \quad (13)$$

$$\|\psi_y\| + \zeta_y \|\psi_z\| = 0, \quad (14)$$

$$\zeta_x \|\psi_y\| - \zeta_y \|\psi_x\| = 0, \quad (15)$$

The normal electric displacement is assumed to be continuous at the interface, and then we get

$$\zeta_x \|\varepsilon \psi_x\| + \zeta_y \|\varepsilon \psi_y\| - \|\varepsilon \psi_z\| - \zeta_x E_0 \|\varepsilon\| = 0, \quad (16)$$

The conservation of energy at the interface

$$\rho_1(\phi_{1z} - \zeta_t - \zeta_x \phi_{1x} - \zeta_y \phi_{1y}) = \alpha(\zeta + \alpha_2^2 \zeta^2 + \alpha_3 \zeta^3) \quad (17)$$

where the coefficients of mass and heat transfer α , α_2 and α_3 are given by

$$\alpha = \frac{G}{L} \left(\frac{1}{d_1} + \frac{1}{d_2} \right), \quad \alpha_2 = \frac{1}{d_2} - \frac{1}{d_1}, \quad \text{and} \quad \alpha_3 = \frac{1}{d_1^2} - \frac{1}{d_1 d_2} + \frac{1}{d_2^2}. \quad (18)$$

where L is the latent heat of transformation from one fluid to another, $G = \frac{K_2(T_0 - T_2)}{d_2} = \frac{K_1(T_1 - T_0)}{d_1}$ is the equilibrium heat flux and K_1 and K_2 represent the heat conductivities of the two fluids,

The conservation of momentum balance gives

$$\begin{aligned} & \alpha(\zeta + \alpha_2^2 \zeta^2 + \alpha_3 \zeta^3) \left\{ \|\phi_z\| - \zeta_x \|\phi_x\| - \zeta_y \|\phi_y\| \right\} [1 + \zeta_x^2 + \zeta_y^2]^{-1} \\ & - T \left\{ \frac{1}{2} \zeta_{xx} (2 - 3\zeta_x^2 - \zeta_y^2) - 2\zeta_x \zeta_y \zeta_{xy} + \frac{1}{2} \zeta_{yy} (2 - 3\zeta_y^2 - \zeta_x^2) \right\} \\ & - \frac{1}{2} \|\rho(\nabla \phi)^2\| - \|\rho \phi_t\| - g\zeta \|\rho\| + \frac{1}{2} \|\varepsilon(\psi_x^2 + \psi_y^2 - \psi_z^2)\| \\ & + 2\zeta_x \|\varepsilon \psi_x \psi_z\| + 2\zeta_y \|\varepsilon \psi_y \psi_z\| - E_0^2 \zeta_x^2 \|\varepsilon\| - E_0 \|\varepsilon \psi_x\| \\ & + 2E_0 \zeta_x \left\{ \zeta_x \|\varepsilon \psi_x\| + \zeta_y \|\varepsilon \psi_y\| - \|\varepsilon \psi_z\| \right\} = 0 \quad \text{at} \quad z = \zeta(x, y, t) \end{aligned} \quad (19)$$

4. Perturbation scheme and linear theory

The multiple scales analysis [47] was performed to investigate the nonlinear interactions of small but finite amplitude waves. We first expand ζ , ψ_j and ϕ_j ($j = 1, 2$) in the following series

$$\zeta(x, y, t) = \sum_{n=1}^3 \epsilon^n \zeta_n(x_0, x_1, x_2, y_0, y_1, y_2, t_0, t_1, t_2) + o(\epsilon^4) \quad (20)$$

and

$$\begin{pmatrix} \phi_j \\ \psi_j \end{pmatrix}(x, y, z, t) = \sum_{n=1}^3 \epsilon^n \begin{pmatrix} \phi_{jn} \\ \psi_{jn} \end{pmatrix}(x_0, x_1, x_2, y_0, y_1, y_2, z, t_0, t_1, t_2) + o(\epsilon^4) \quad (21)$$

where ϵ is a small dimensionless parameter showing the strength of the nonlinearity. The scales $x_n = \epsilon^n x$, $y_n = \epsilon^n y$, and $t_n = \epsilon^n t$ are supposed to satisfy the following expansions

$$\frac{\partial}{\partial \beta} = \sum_{n=1}^3 \epsilon^n \frac{\partial}{\partial \beta_n} + o(\epsilon^4) \quad (22)$$

where β represent t , x , and y . Here, the short scales x_0 , y_0 represent the wavelength while the long scales x_j , y_j , ($j = 1, 2$) denote to the spatial modulations of the phase and the amplitude. The fast scale t_0 represents the wave frequency, whereas t_1 and t_2 stand respectively for the slow temporal scales of the phase and the amplitude. Following the usual procedure [42], we get the following quasi-monochromatic wave solution to the first order perturbation problem

$$\zeta_1 = Ae^{i\theta} + c.c., \quad (23)$$

$$\phi_{j1} = \pm \frac{(\alpha - i\omega\rho_j) \cosh K(z \pm d_j)}{K\rho_j \cosh Kd_j} Ae^{i\theta} + c.c. + p_{j1}, \quad (24)$$

$$\psi_{j1} = \frac{ikE_0(\varepsilon_2 - \varepsilon_1)}{K(\varepsilon_1\sigma_1 + \varepsilon_2\sigma_2)} \frac{\cosh K(z \pm d_j)}{\cosh Kd_j} Ae^{i\theta} + c.c., \quad (25)$$

where $\sigma_j = \tanh Kd_j$ ($j = 1, 2$), $\theta = (kx_0 + ly_0 - \omega t_0)$ is the carrier wave phase, $K = (k^2 + l^2)^{1/2}$, k and l represent the wavenumbers elements in x - and y -directions respectively, while ω , A , i , $c.c.$ denote for the angular frequency, the complex amplitude of the surface elevation, the imaginary unit, and the complex conjugate of the previous terms respectively. While the real functions p_{11} and p_{21} represent independent quantities of the lowest scales x_0 , y_0 and t_0 . These additional functions are of slow scales x_j , y_j , t_j , ($j = 1, 2$). Now, to obtain a nontrivial starting solution, the frequency ω and the wavenumber K must satisfy the dispersion equation

$$F(\omega, K) = a_0\omega^2 + ia_1\omega + a_2 = 0, \quad (26)$$

where

$$\begin{aligned} a_0 &= \left(\frac{\rho_1}{\sigma_1} + \frac{\rho_2}{\sigma_2} \right) \\ a_1 &= \alpha \left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2} \right) \\ a_2 &= -\frac{k^2 E_0^2 (\varepsilon_2 - \varepsilon_1)^2}{(\varepsilon_1 \sigma_1 + \varepsilon_2 \sigma_2)} + gK(\rho_2 - \rho_1) - TK^3 \end{aligned} \quad (27)$$

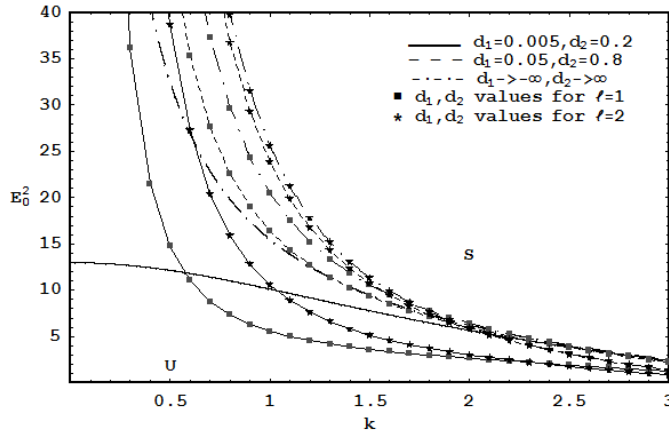


Figure 1: Variation of E_0^2 given by equation (30), with k for various values of d_1 and d_2 , if $l = 0$, $\rho_1 = 0.000542 \text{ gr/cm}^3$, $\rho_2 = 0.97708 \text{ gr/cm}^3$, $\varepsilon_1 = 1.00785 \text{ farad/cm}$, $\varepsilon_2 = 61.03 \text{ farad/cm}$, $g = 980 \text{ cm/sec}^2$, $T = 58.5 \text{ dyn/cm}$, when $d_1 = 0.005 \text{ cm}$, $d_2 = 0.2 \text{ cm}$ (solid), $d_1 = 0.05 \text{ cm}$, $d_2 = 0.8 \text{ cm}$ (dashed) and $d_1 \rightarrow -\infty$, $d_2 \rightarrow \infty$ (dashed dotted). The curves with symbols $\blacksquare$ and $\star$ represent the cases when $l = 1$ and $l = 2$, respectively.

The linear dispersion relation (26) was originally obtained by Mohamed et al. [27] for the two dimensional case, Qingfu [48] for the case of three dimensions without heat and mass transfer ($\alpha = 0$), the electric field ($E_0 = 0$) and the surface tension ($T = 0$). Using Routh-Hurwitz stability criterion [49] to equation (26), since a_0 is always positive, we obtain the stability conditions (especially, when the imaginary part of ω is negative) as

$$a_1 > 0 \quad \text{and} \quad a_2 \leq 0, \quad (28)$$

From conditions (28), we note that since the coefficient α is always positive, hence the first condition $a_1 > 0$ is trivially satisfied, while the second one $a_2 \leq 0$ tends to

$$E_0^2 \geq E_c^2, \quad (29)$$

where

$$E_c^2 = \frac{K [g(\rho_2 - \rho_1) - K^2 T] [\varepsilon_1 \sigma_1 + \varepsilon_2 \sigma_2]}{k^2 (\varepsilon_2 - \varepsilon_1)^2}. \quad (30)$$

Hence, our system is unstable or stable depending on if the electric field E_0^2 is smaller or larger than the critical electric field E_c^2 separating unstable from

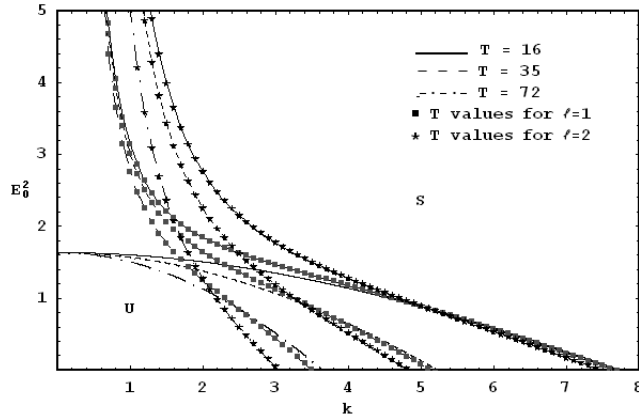


Figure 2: Variation of E_0^2 given by equation (30), with k for various values of the surface tension T , if $l = 0$, $\rho_1 = 0.000542$ gr/cm³, $\rho_2 = 0.97708$ gr/cm³, $\varepsilon_1 = 1.00785$ farad/cm, $\varepsilon_2 = 61.03$ farad/cm, $d_1 = 0.05$ cm, $d_2 = 0.1$ cm, $g = 980$ cm/sec², when $T = 16$ dyn/cm (solid), $T = 35$ dyn/cm (dashed) and $T = 72$ dyn/cm (dashed dotted). The curves with symbols ■ and ★ represent the cases when $l = 1$ and $l = 2$, respectively.

stable disturbances. From the stability conditions (28), it is clear that the influence of both horizontal electric field and surface tension are stabilizing, while the mass and heat transfer coefficient (α) has no effect on the stability condition (29). These results are in agreement with the same that are obtained by Mohamed et al[27]. Now, to detect the effects of different parameters including in the analysis on the linear stability of our considered system, we will draw (using Mathematica 7.0) the transition curves described by equation (30) in the $E_0^2 - k$ plane when $E_0^2 = E_C^2$. These transition curves that are presented in Figures 1-2, separate the unstable "U" from stable "S" regions. The region higher the curve is stable, while the lower region is unstable.

In Figure 1 we sketch E_0^2 against k for different values of the fluid depths d_1 and d_2 in two-and three-dimensional disturbances cases ($l = 0$ and $l \geq 1$) respectively. When $l = 0$, the unstable region increases by increasing the fluid depths d_1 and d_2 , and this instability increases by increasing the dimension l (i.e. when $l \geq 1$) for small wavenumber values, but it decreases after a constant wavenumber value. Also, for any value of l , for small E_0^2 values, the system is found to be unstable, while for high E_0^2 values, the electric field has been found to have a stabilizing effect, since stability region increases by increasing the electric field E_0^2 values. Therefore, both the fluid depths and the dimension

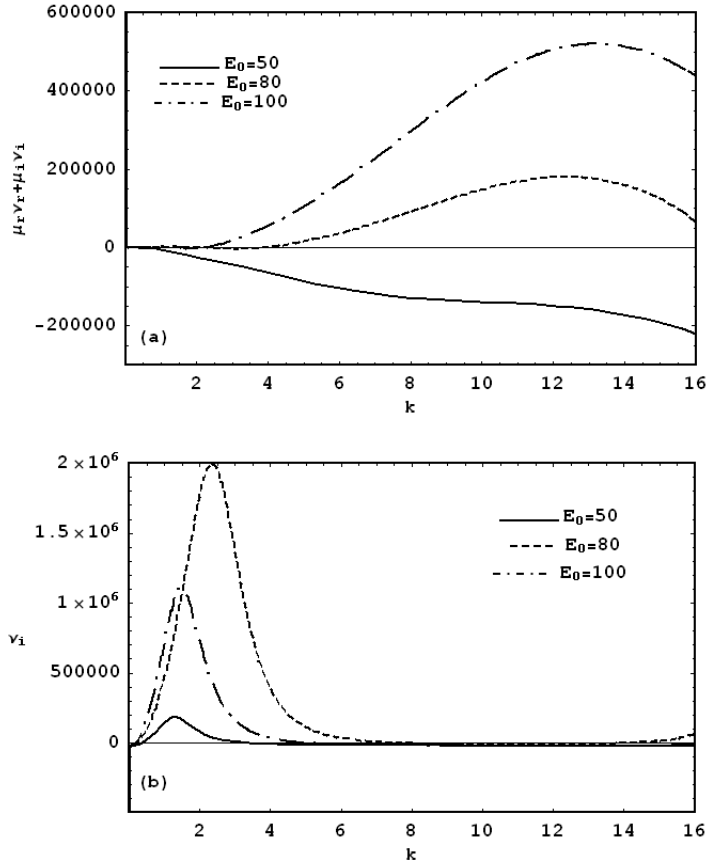


Figure 3: Variation of (a) $\mu_r \nu_r + \mu_i \nu_i$ and (b) ν_i given by equation (47) with k for various values of the electric field E_0 , for the system having $\rho_1 = 0.000542$ gr/cm³, $\rho_2 = 0.97708$ gr/cm³, $\varepsilon_1 = 1.00785$ farad/cm, $\varepsilon_2 = 61.03$ farad/cm, $d_1 = 0.05$ cm, $d_2 = 0.1$ cm, $g = 980$ cm/sec², $\alpha = 0.05$ gr/cm³sec, $l = 0$ and $T = 58.5$ dyn/cm, when $E_0 = 50$ statV/cm (solid), $E_0 = 80$ statV/cm (dashed) and $E_0 = 100$ statV/cm (dashed dotted).

have destabilizing effects, while the electric field has a stabilizing effect on the system which is unstable without the electric field effect.

In Figure 2 we study the influence of the surface tension T on the stability of the system in two-and three-dimensional disturbances cases ($l = 0$ and $l \geq 1$) respectively. When $l = 0$, the unstable region decreases by increasing the

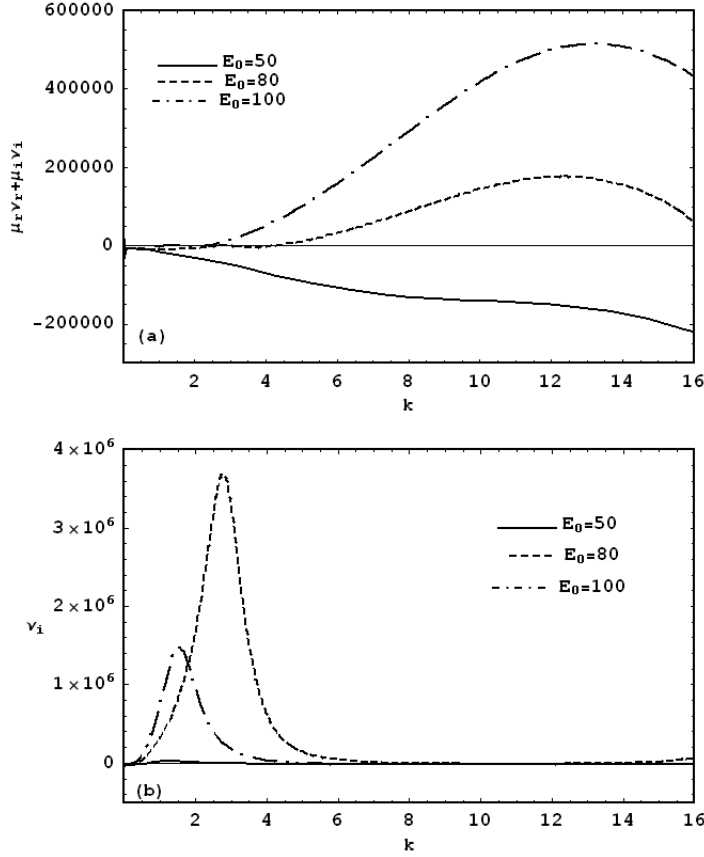


Figure 4: Variation of (a) $\mu_r \nu_r + \mu_i \nu_i$ and (b) ν_i given by equation (47) with k for various values of the electric field E_0 , for the same system considered in figure (3) but with $l = 1$.

surface tension T , while it increases by increasing the dimension ($l \geq 1$). Therefore, the surface tension T has a stabilizing effect. The effects of both dimension and electric field on the system are found to be similar to their effects observed from Figure 1, i.e. both the dimension and the horizontal electric field have destabilizing and stabilizing effects, respectively.

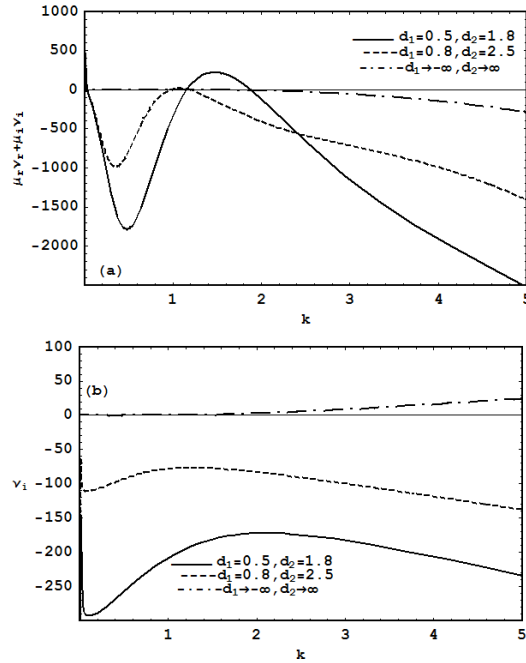


Figure 5: Variation of (a) $\mu_r \nu_r + \mu_i \nu_i$ and (b) ν_i given by equation (47) with k for various values of the fluid depths h_1 and h_2 , for the system having $\rho_1 = 0.000542$ gr/cm³, $\rho_2 = 0.97708$ gr/cm³, $\varepsilon_1 = 1.00785$ farad/cm, $\varepsilon_2 = 61.03$ farad/cm, $\alpha = 0.05$ gr/cm³sec, $g = 980$ cm/sec², $E_0 = 30$ statV/cm, $l = 0$ and $T = 58.5$ dyn/cm, when $d_1 = 0.5$ cm, $d_2 = 1.8$ cm (solid), $d_1 = 0.8$ cm, $d_2 = 2.5$ cm (dashed) and $d_1 \rightarrow -\infty$, $d_2 \rightarrow \infty$ (dashed dotted).

5. Nonlinear Ginzburg-Landau equation

Perform the same steps used in our previous article [42] to investigate the amplitude modulation for travelling waves, we obtain, after very lengthy calculations, (but they will not be included here to save space), the following nonlinear evolution equations. The solvability conditions for the second order perturbation are

$$\frac{\partial A}{\partial t_1} + v_k \frac{\partial A}{\partial x_1} + v_l \frac{\partial A}{\partial y_1} = 0, \quad (31)$$

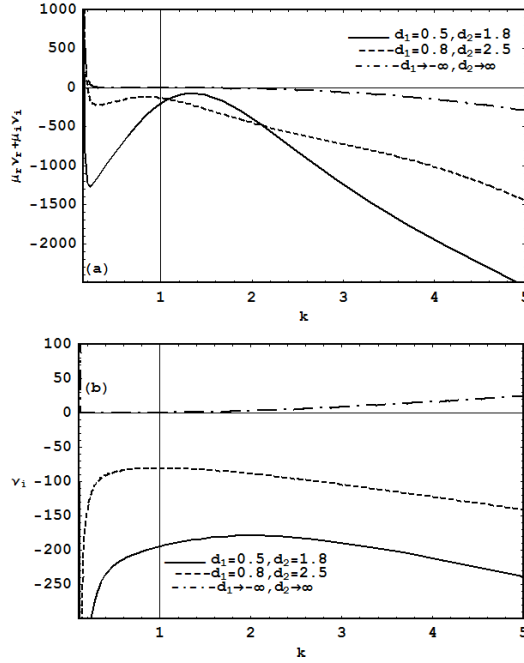


Figure 6: Variation of (a) $\mu_r \nu_r + \mu_i \nu_i$ and (b) ν_i given by equation (47) with k for various values of the fluid depths d_1 and d_2 , for the same system considered in figure (5) but with $l = 1$.

associated with its c.c., where v_k and v_l are the group velocities of the wavetrain along x and y directions, respectively, represented as

$$v_k = - \left(\frac{\partial F}{\partial k} \right) \left(\frac{\partial F}{\partial \omega} \right)^{-1} \quad \text{and} \quad v_l = - \left(\frac{\partial F}{\partial l} \right) \left(\frac{\partial F}{\partial \omega} \right)^{-1} \quad (32)$$

$$B_0 + 2\alpha_2 |A|^2 = 0, \quad (33)$$

and

$$\left(\rho_2 \frac{\partial p_{21}}{\partial t_1} - \rho_1 \frac{\partial p_{11}}{\partial t_1} \right) + g B_0 (\rho_2 - \rho_1) + Q |A|^2 = 0, \quad (34)$$

where Q is given by

$$Q = \frac{(1 - \sigma_2^2) (\rho_2^2 \omega^2 + \alpha^2)}{\rho_2 \sigma_2^2} - \frac{(1 - \sigma_1^2) (\rho_1^2 \omega^2 + \alpha^2)}{\rho_1 \sigma_1^2} - \frac{k^2 E_0^2 (\varepsilon_2 - \varepsilon_1)^2}{(\varepsilon_1 \sigma_1 + \varepsilon_2 \sigma_2)^2} [\varepsilon_2 (1 - \sigma_2^2) - \varepsilon_1 (1 - \sigma_1^2)]. \quad (35)$$

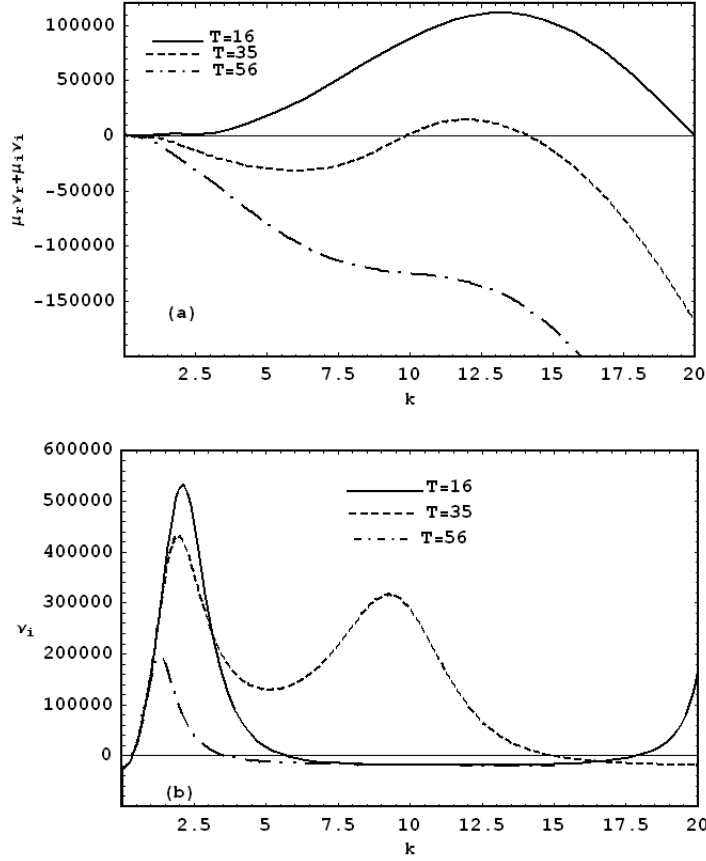


Figure 7: Variation of (a) $\mu_r \nu_r + \mu_i \nu_i$ and (b) ν_i given by equation (47) with k for various values of the surface tension T , for the system having $\rho_1 = 0.000542$ gr/cm³, $\rho_2 = 0.97708$ gr/cm³, $\varepsilon_1 = 1.00785$ farad/cm, $\varepsilon_2 = 61.03$ farad/cm, $d_1 = 0.05$ cm, $d_2 = 0.1$ cm, $g = 980$ cm/sec², $\alpha = 0.05$ gr/cm³sec, $l = 0$ and $E_0 = 50$ statV/cm, when $T = 16$ dyn/cm (solid), $T = 35$ dyn/cm (dashed) and $T = 56$ dyn/cm (dashed dotted).

where B_0 in (33) refers to the induced mean motion and the symbol $|*|$ is the modulus. The constants $p_{j2}, (j = 1, 2)$ in equation (34) are real functions of the slow scales $x_j, y_j, t_j, (j = 1, 2)$ will be determined later by considering the higher order equations. Equation (31) show the modulations on the time scale ϵ^{-1} propagate without changing its shape with the group velocity (v_k, v_l) . The solvability conditions for the third order perturbation problem are

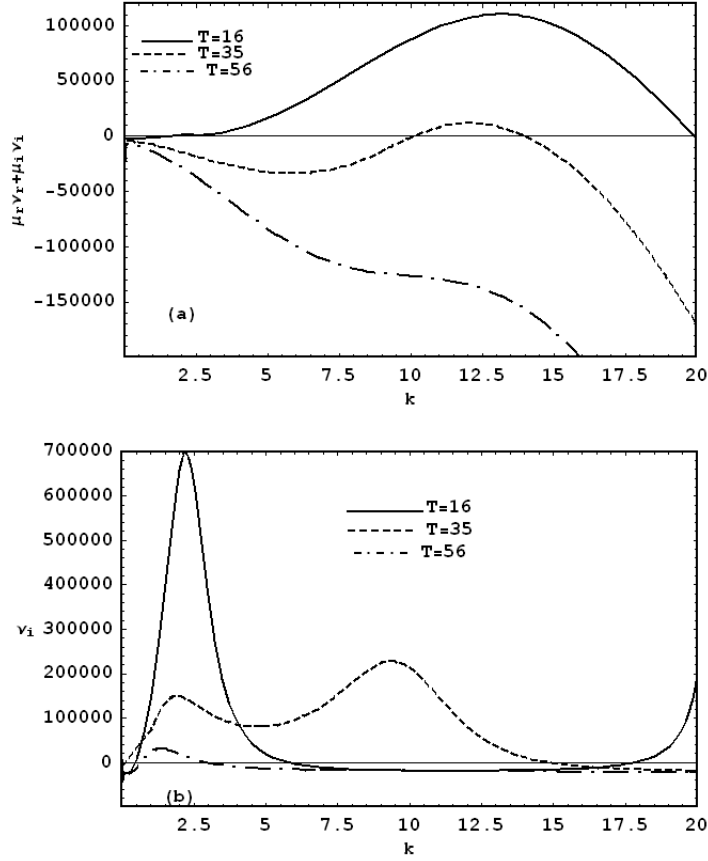


Figure 8: Variation of (a) $\mu_r \nu_r + \mu_i \nu_i$ and (b) ν_i given by equation (47) with k for various values of the surface tension T , for the same system considered in figure (7) but with $l = 1$.

$$\begin{aligned}
 & i \left\{ \frac{\partial F}{\partial \omega} \frac{\partial A}{\partial t_2} - \frac{\partial F}{\partial k} \frac{\partial A}{\partial x_2} - \frac{\partial F}{\partial l} \frac{\partial A}{\partial y_2} \right\} \\
 & + \left\{ \frac{\partial^2 F}{\partial \omega \partial k} \frac{\partial^2 A}{\partial t_1 \partial x_1} + \frac{\partial^2 F}{\partial \omega \partial l} \frac{\partial^2 A}{\partial t_1 \partial y_1} - \frac{\partial^2 F}{\partial k \partial l} \frac{\partial^2 A}{\partial x_1 \partial y_1} \right\} \\
 & - \frac{1}{2} \left\{ \frac{\partial^2 F}{\partial \omega^2} \frac{\partial^2 A}{\partial t_1^2} + \frac{\partial^2 F}{\partial k^2} \frac{\partial^2 A}{\partial x_1^2} + \frac{\partial^2 F}{\partial l^2} \frac{\partial^2 A}{\partial y_1^2} \right\} + GA^2 \bar{A} + RA = 0, \quad (36)
 \end{aligned}$$

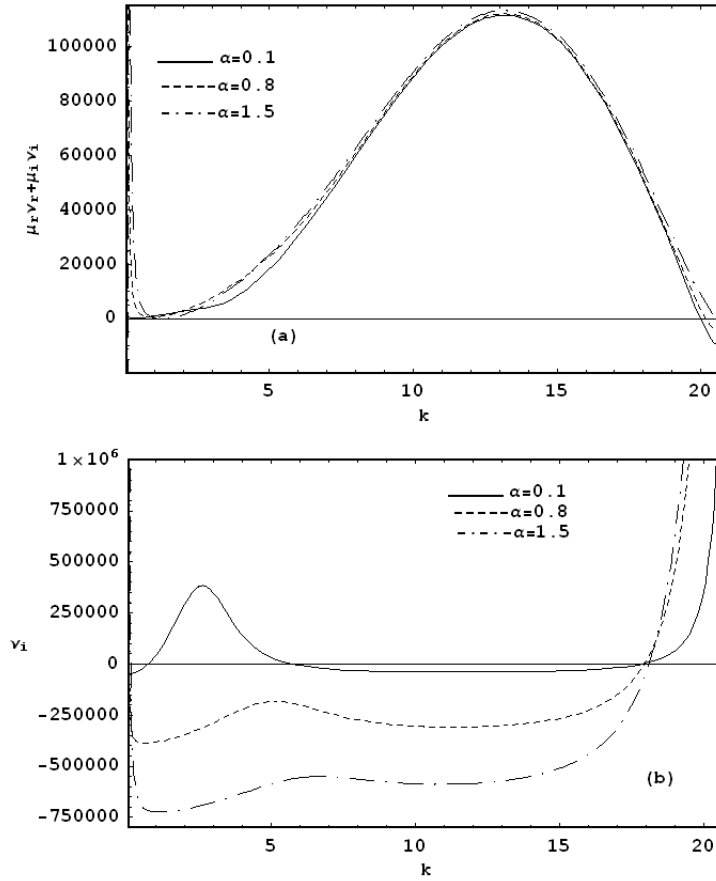


Figure 9: Variation of (a) $\mu_r \nu_r + \mu_i \nu_i$ and (b) ν_i given by equation (47) with k for various values of the heat and mass transfer coefficient α , for the system having $\rho_1 = 0.000542$ gr/cm³, $\rho_2 = 0.97708$ gr/cm³, $\varepsilon_1 = 1.00785$ farad/cm, $\varepsilon_2 = 61.03$ farad/cm, $d_1 = 0.05$ cm, $d_2 = 0.1$ cm, $g = 980$ cm/sec², $E_0 = 50$ statV/cm, $l = 0$ and $T = 16$ dyn/cm, when $\alpha = 0.1$ (solid) , $\alpha = 0.8$ (dashed) and $\alpha = 1.5$ (dashed dotted) .

where

$$G = \frac{3}{2}TK^4 + \Lambda\omega^2 \left\{ \frac{\rho_2}{\sigma_2^2} (3 - \sigma_2^2) - \frac{\rho_1}{\sigma_1^2} (3 - \sigma_1^2) \right\} \\ + \omega K \left\{ \frac{1}{\sigma_1^3} [2\omega\rho_1 (1 - 2\sigma_1^2) + i\alpha (1 - 3\sigma_1^2)] \right\}$$

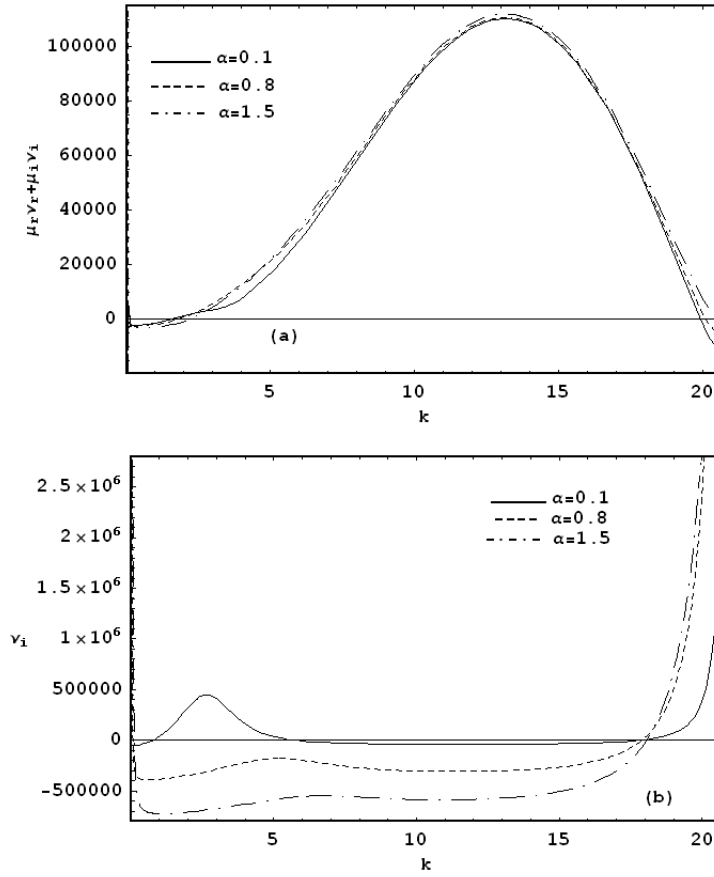


Figure 10: Variation of (a) $\mu_r \nu_r + \mu_i \nu_i$ and (b) ν_i given by equation (47) with k for various values of the heat and mass transfer coefficient α , for the same system considered in figure (9) but with $l = 1$.

$$\begin{aligned}
& + \frac{1}{\sigma_2^3} [2\omega\rho_2 (1 - 2\sigma_2^2) + i\alpha (1 - 3\sigma_2^2)] \Bigg\} + \frac{i\alpha\omega (\sigma_1 + \sigma_2)}{K \sigma_1^2 \sigma_2^2} \\
& \times \{K (\sigma_2 - \sigma_1) (\Lambda - \alpha_2) + \sigma_1 \sigma_2 (2\Lambda\alpha_2 + 3\alpha_3)\} \\
& + \frac{1}{2}\alpha^2 \left\{ \frac{(1 - \sigma_1^2)}{\rho_1 \sigma_1^3} (2K - \Lambda\sigma_1 - \alpha_2\sigma_1) \right. \\
& \left. + \frac{(1 - \sigma_2^2)}{\rho_2 \sigma_2^3} (2K + \Lambda\sigma_2 + \alpha_2\sigma_2) \right\} + \frac{\Lambda k^2 E_0^2 (\varepsilon_2 - \varepsilon_1)^2}{(\varepsilon_1 \sigma_1 + \varepsilon_2 \sigma_2)^2}
\end{aligned}$$

$$\begin{aligned}
& \times \left[\frac{\varepsilon_1 \sigma_1}{(1 + \sigma_1^2)} + \frac{\varepsilon_2 \sigma_2}{(1 + \sigma_2^2)} \right]^{-1} \left\{ \frac{\varepsilon_2^2 \sigma_2 (-3 + \sigma_2^2)}{(1 + \sigma_2^2)} - \frac{\varepsilon_1^2 \sigma_1 (-3 + \sigma_1^2)}{(1 + \sigma_1^2)} \right. \\
& + \frac{\varepsilon_1 \varepsilon_2}{(1 + \sigma_1^2)(1 + \sigma_2^2)} [\sigma_2 (3 + \sigma_1^4) - \sigma_1 (3 + \sigma_2^4) + 2\sigma_2^3 (1 - \sigma_1^2) \\
& \left. - 2\sigma_1^3 (1 - \sigma_2^2)] \right\} + \frac{2k^2 E_0^2 K (\varepsilon_2 - \varepsilon_1)^2}{(\varepsilon_1 \sigma_1 + \varepsilon_2 \sigma_2)^2} \left[\frac{\varepsilon_1 \sigma_1}{(1 + \sigma_1^2)} + \frac{\varepsilon_2 \sigma_2}{(1 + \sigma_2^2)} \right]^{-1} \\
& \times \left\{ \frac{\varepsilon_1^2 (-1 + 2\sigma_1^2)}{(1 + \sigma_1^2)} + \frac{\varepsilon_2^2 (-1 + 2\sigma_2^2)}{(1 + \sigma_2^2)} + \frac{\varepsilon_1 \varepsilon_2}{(1 + \sigma_1^2)(1 + \sigma_2^2)} \right. \\
& \left. \times [2(1 - \sigma_1^2)(1 - \sigma_2^2) + \sigma_1 \sigma_2 (3\sigma_1^2 + 3\sigma_2^2 - 2)] \right\}. \quad (37)
\end{aligned}$$

and

$$\begin{aligned}
R = & (H_1 + iH_2) B_0 - \left(\frac{2\rho_1 \omega + i\alpha}{K\sigma_1} \right) \left(k \frac{\partial p_{11}}{\partial x_1} + l \frac{\partial p_{11}}{\partial y_1} \right) \\
& - \left(\frac{2\rho_2 \omega + i\alpha}{K\sigma_2} \right) \left(k \frac{\partial p_{21}}{\partial x_1} + l \frac{\partial p_{21}}{\partial y_1} \right) \quad (38)
\end{aligned}$$

with

$$\begin{aligned}
H_1 = & \frac{\rho_2^2 \omega^2 (1 - \sigma_2^2)}{\rho_2 \sigma_2^2} - \frac{\rho_1^2 \omega^2 (1 - \sigma_1^2)}{\rho_1 \sigma_1^2} \\
& - \frac{k^2 E_0^2 (\varepsilon_2 - \varepsilon_1)^2}{(\varepsilon_1 \sigma_1 + \varepsilon_2 \sigma_2)^2} [\varepsilon_2 (1 - \sigma_2^2) - \varepsilon_1 (1 - \sigma_1^2)], \quad (39)
\end{aligned}$$

and

$$H_2 = \frac{\alpha \omega (\sigma_1 + \sigma_2) [K(\sigma_1 - \sigma_2) + 2\alpha_2 \sigma_1 \sigma_2]}{K \sigma_1^2 \sigma_2^2} \quad (40)$$

Moreover from the solvability conditions for ζ_3 we have

$$\begin{aligned}
& (\rho_2 - \rho_1) \frac{\partial B_0}{\partial t_1} - \frac{2\omega}{K} \left(\frac{\rho_1}{\sigma_1} + \frac{\rho_2}{\sigma_2} \right) \left(k \frac{\partial |A|^2}{\partial x_1} + l \frac{\partial |A|^2}{\partial y_1} \right) \\
& - \rho_1 h_1 \left(\frac{\partial^2 p_{11}}{\partial x_1^2} + \frac{\partial^2 p_{11}}{\partial y_1^2} \right) - \rho_2 h_2 \left(\frac{\partial^2 p_{21}}{\partial x_1^2} + \frac{\partial^2 p_{21}}{\partial y_1^2} \right) = 0 \quad (41)
\end{aligned}$$

Let us suppose that A , p_{j1} , ($j = 1, 2$) and B_0 depend on x_1, y_1 and t_1 only through the transformation [17, 50], $\theta_1 = K^{-1} \{(kx_1 + ly_1) - (kv_k + lv_l)t_1\}$,

then with the aid of equation (33), equations (34) and (41), after integrating with respect to θ_1 , yield

$$\begin{aligned} \frac{\partial p_{j1}}{\partial \theta_1} = & \left\{ \rho_j K (d_1 + d_2) (kv_k + lv_l) \right\}^{-1} \\ & \times \left\{ (kv_k + lv_l) \left[2\alpha_2(\rho_2 - \rho_1) (kv_k + lv_l) - 2\omega K \left(\frac{\rho_1}{\sigma_1} + \frac{\rho_2}{\sigma_2} \right) \right] \right. \\ & \left. + (-1)^j d_{3-j} K^2 [Q - 2g\alpha_2(\rho_2 - \rho_1)] \right\} |A|^2 + r_j(x_2, y_2), \quad j = 1, 2 \end{aligned} \quad (42)$$

By using the suitable initial and/or boundary conditions of our considered problem, The slow functions $r_j(x_2, y_2)$, $j = 1, 2$ in equations (42) could be determined. Moreover, we can eliminate these terms by a simple transformation [50], from the terminal results and has no effect on the stability nature. Hence $B_0, \partial p_{j1}/\partial y_1$ and $\partial p_{j1}/\partial x_1$, ($j = 1, 2$) in equation (38) will be written about A . Considering that A depends on t_2, x_2 and y_2 through $\theta_2 = K^{-1} \{ (kx_2 + ly_2) - (kv_k + lv_l) t_2 \}$ and $\tau = t_2$. Plug the expressions for $B_0, \partial p_{j1}/\partial y_1$ and $\partial p_{j1}/\partial x_1$, ($j = 1, 2$) into equations (36), (38). We finally obtain a nonlinear Gingburg-Landau equation of complex coefficients

$$i \frac{\partial A}{\partial \tau} + \mu \frac{\partial^2 A}{\partial \theta_1^2} = \nu |A|^2 A \quad (43)$$

where the complex coefficients μ and ν are defined as

$$\mu = \frac{1}{2K^2} \{ k^2 v_{kk} + 2kl v_{kl} + l^2 v_{ll} \} \quad (44)$$

and

$$\nu = -\frac{(G + R)}{(\partial F / \partial \omega)} \quad (45)$$

Equation (43) depicts the nonlinear behaviour of the capillary gravity waves on fluid layers of uniform thickness. Note that in the limiting case when $d_2 \rightarrow \infty$ and absence of heat and mass transfer (when $\alpha = 0$), then equation (43) reduces to the same nonlinear evolution equation (in dimensional form) obtained previously by El-Sayed and Callebaut [18] in studying the soliton envelope solutions of the resulting equation. It is of interest to note that the instability modulation of a nonlinear plane wave solution of eq. (43) is controlled by the two complex coefficients μ and ν .

6. Instability of progressive waves

In equation (43), the complex coefficients μ and ν take the form

$$\mu = \mu_r + i\mu_i \quad \text{and} \quad \nu = \nu_r + i\nu_i \quad (46)$$

We can use eq. (43) to investigate the stability behaviour of the system. Matkowsky and Volpert [51] and Lange and Newell [52] discuss the stability conditions of Ginzburg-Landau equation (43). Recently Pelap and Faye [53] examined Lange and Newell's conditions for instability modulation of Stokes waves. They explain that the stability criteria, together occur, for our system are

$$\mu_r\nu_r + \mu_i\nu_i > 0 \quad \text{and} \quad \nu_i < 0. \quad (47)$$

otherwise, the system is unstable. Now, we shall discuss the stability of our system numerically by sketching the transition curves $\mu_r\nu_r + \mu_i\nu_i$ and ν_i against the wavenumber k for different values of parameters E_0 , α , T , l , d_1 , and d_2 , including in the analysis.

Figures 3 a, b show the variation of (a) $\mu_r\nu_r + \mu_i\nu_i$ and (b) ν_i with k for different electric field values E_0 , for a system having: $\rho_2 = 0.97708 \text{ gr/cm}^3$, $\rho_1 = 0.000542 \text{ gr/cm}^3$, $\varepsilon_2 = 61.03 \text{ farad/cm}$, $\varepsilon_1 = 1.00785 \text{ farad/cm}$, $\alpha = 0.05 \text{ gr/cm}^3\text{sec}$, $d_1 = 0.05 \text{ cm}$, $d_2 = 0.1 \text{ cm}$, $T = 58.5 \text{ dyn/cm}$, and $g = 980 \text{ cm/s}^2$. It is found that when $l = 0$ (two-dimensional disturbances), for $E_0 \leq 50$, the system is stable only for the wavenumbers range $0 < k < 0.2$, and it is unstable afterwards. As E_0 increases, in addition to this stable region, there exists new stability regions, e.g. when $E_0 = 80$, the stability holds for the wavenumbers range $7.6 < k < 14.1$, while when $E_0 = 100$, it occurs for $5.05 < k < 19.25$. Figs.(4 a, b) show the variation of (a) $\mu_r\nu_r + \mu_i\nu_i$ and (b) ν_i with k for the same system in Figs.(3 a, b) but with $l = 1$. It is found that when $l \geq 1$ (three-dimensional disturbances), for $E_0 \leq 50$, the system is stable only for the wavenumber range $0 < k < 0.03$, and it is unstable afterwards. As E_0 increases, in addition to this stable region, there exists when ($l = 1$) new stability region, e.g. when $E_0 = 80$, the stability holds for the wavenumber range $7.75 < k < 14$, while when $E_0 = 100$, it occurs for $5.06 < k < 19.1$. Also, by increasing the dimension l (figures are excluded to save space), we found that for $l = 2$: when $E_0 = 80$, stability occurs for $8.25 < k < 13.5$, and when $E_0 = 100$, it occurs for $5.15 < k < 19$, and for $l = 3$: when $E_0 = 80$, stability holds for $9.2 < k < 12.7$, but when $E_0 = 100$, stability occurs for $5.4 < k < 18.7$.

From the preceding discussion, we conclude that in two dimensional disturbances case ($l = 0$), the first stable region is slightly decreased, while the second

stable one increases by increasing the electric field values, i.e. the applied electric field has a stabilizing effect on the considered system. In three dimensional disturbances case ($l \geq 1$), the first stable region is found to be smaller than the corresponding one when ($l = 0$), while for a fixed E_0 value, the stable region decreases by increasing l value, and for a fixed l value ($l = 1, 2, 3, \dots$), the stable region increases by increasing E_0 value. Therefore, the electric field has a stabilizing effect, while the system is destabilizing for three dimensional disturbances and this instability increases by increasing the dimension l .

Figs.(5 a, b) illustrate the variation of (a) $\mu_r\nu_r + \mu_i\nu_i$ and (b) ν_i with k for different fluid depths values d_1 and d_2 , when $E_0 = 30$. It is found that when $l = 0$ (two dimensional disturbances), for ($d_1 = 0.5, d_2 = 1.8$) the system is stable only for the wavenumber range $1.2 < k < 1.9$, otherwise, it is unstable. By increasing values of the fluid depths d_1 and d_2 , i.e. when ($d_1 = 0.8, d_2 = 2.5$), the system is stable only for a smaller wavenumber range $1 < k < 1.2$. For high fluid depths values, i.e. when $d_1 \rightarrow -\infty, d_2 \rightarrow \infty$, (the case of semi infinite fluids) the stability occurs for the wavenumber range $0.02 < k < 0.45$. Therefore, the fluid depths d_1 and d_2 have destabilizing effects on the considered system. Figs.(6 a, b) are sketched for the same system in Figs. (5 a, b) but with $l = 1$. It is found that when $l \geq 1$ (three dimensional disturbances), for ($d_1 = 0.5, d_2 = 1.8$), the system is stable only for the wavenumber range $0.1 < k < 0.2$, otherwise, it is unstable. As d_1 and d_2 increase, when ($d_1 = 0.8, d_2 = 2.5$), the stability occurs in the same range $0.1 < k < 0.2$, while when $d_1 \rightarrow -\infty, d_2 \rightarrow \infty$, it occurs for $0.15 < k < 0.35$.

From the preceding discussion, we conclude that in two-dimensional disturbances case ($l = 0$), the stable region decreases, by increasing the fluid depths values d_1 and d_2 , i.e. the fluid depths d_1 and d_2 have destabilizing effects. In three-dimensional disturbances case ($l \geq 1$), for a fixed d_1 and d_2 values, the stable region decreases by increasing l value from ($l = 0$) to ($l = 1, 2, 3, \dots$). Also, the system is destabilizing for the case of three-dimensional disturbances of two semi-infinite fluids and this instability increases by increasing the dimension l .

Figs.(7 a, b) show the variation of (a) $\mu_r\nu_r + \mu_i\nu_i$ and (b) ν_i with k for different surface tension values T , when $E_0 = 50$. It is found that when $l = 0$ (two-dimensional disturbances), for $T = 16$, the system is stable only for the wavenumber range $5.75 < k < 17.9$, otherwise, it is unstable. As T increases, the stable region disappeared, e.g. when $T \geq 35$, the system is always unstable. Figs.(8 a, b) is drawn for the same system in Figs.(7a, b) but with $l = 1$. When $l \geq 1$ (three-dimensional disturbances), for $T = 16$, the system is stable only for the wavenumber range $5.8 < k < 17.85$. As T increases, the stable region disappeared, e.g. when $T \geq 35$, the system is always unstable. Also for $l = 2$

: when $T = 16$, stability occurs only for the wavenumber range $6 < k < 17.65$, and when $T \geq 35$ the system is always unstable.

Therefore, we conclude that as l increases, and for any fixed value of T , stability region, decreases, i.e. l has a destabilizing effect, while, as T increases, for any fixed value of l , the surface tension T has a destabilizing effect.

Figs.(9 a, b) and (10 a, b) are drawn for various values of heat and mass transference coefficient α , when $l = 0$, and $l = 1$, respectively. It is seen from Figs.(9 a, b) when $l = 0$ that for small values of α (≈ 0.1), there are only one stable region occurs at the wavenumber range $5.8 < k < 17.5$, otherwise the system is unstable. By increasing α value we note that the stability range increase, for example, when $\alpha = 0.8$, stability occurs in the wavenumber range $0.1 < k < 17.6$, while for $\alpha = 1.5$, stability holds in the range $0.1 < k < 18$. Therefore, the heat and mass transference coefficient α has a stabilizing effect of two-dimensional disturbances. It is obvious also from Figs. (10 a, b), when $l = 1$ that for small values of α (≈ 0.1), the stability occurs for the wavenumber range $5.8 < k < 17.9$, while for $\alpha = 0.8$, stability occurs in the wavenumber range $1.8 < k < 17.9$, i.e. α has a stabilizing effect in this case since the stability range increases by increasing α , and also for $\alpha = 1.5$, the stability holds for the wavenumber range $2.2 < k < 17.9$, i.e. α has a destabilizing effect in this case, since the stability range decreases by increasing α . Therefore, we conclude that the system is stable for small values of the parameter α , and it is unstable for higher values of α , i.e. the heat and mass transference coefficient α has a dual role on the stability of three-dimensional disturbances, which means that the dimension l has a destabilizing effect in this case.

7. Conclusions

In conclusion, The nonlinear RTI analysis of two finite superposed fluids in three dimensions in presence of interfacial transfer of heat and mass and a constant tangential electric field is investigated. Based on multiple scales perturbation method, a general linear dispersion relation is obtained and numerically discussed. The main results in that case show that

- (1) The mass and heat transference has no effect on the considered system.
- (2) Both the fluid depths and the dimension have destabilizing effects.
- (3) Both the electric field and the surface tension have stabilizing effects.
- (4) The system in two-dimensional disturbances case is more stable than the case of three-dimensional disturbances.

While in the nonlinear study, a nonlinear Ginzburg-Landau equation with

complex coefficients is obtained , describing the disturbed system behaviour. The results for both two- and three- dimensional disturbances are summarized in the following :

(1) For small electric field values (including the case when $E_0 = 0$), stability holds for only small wave numbers range which decreases by increasing E_0 , while for high values of E_0 , stability holds also at high wavenumbers range which increases by increasing E_0 .

(2) The system is stable for bounded fluids, but this stability reduces by exceeding the fluid depths d_1 and d_2 (including two semi-infinite fluids case), i.e. d_1 and d_2 have destabilizing effects.

(3) The surface tension force T is found to has a destabilizing effect.

(4) The heat and mass transfere coefficient α has a stabilizing effect for two-dimensional disturbances case, and this result agree with the same results that obtained earlier by Hseih[10] and Mohamed et al[27], while it plays a dual role on the stability for three-dimensional disturbances case, i.e stabilizing as well as destabilizing depending on the value of α .

(5) Also, in the three-dimensional disturbances, we found that the instability regions increase (or the stability regions decreases) under the effects of all the previous physical parameters, which means that the dimension has always a destabilizing effect on the system.

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