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A NOTE ON THE "MEAN VALUE" SOFTWARE RELIABILITY MODEL

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Abstract: In this paper we study the Hausdorff approximation of the Heaviside step function $h_{t_0}(t)$ by sigmoidal curve based on the "mean value" software reliability model and find an expression for the error of the best approximation.

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Key Words: software reliability model, Gompertz curve, Hausdorff approximation, Heaviside step function

1. Introduction

Ohishi, Okamura and Dohi [7] formulate Gompertz software reliability model based on the following *deterministic curve model*:

$$M(t) = \omega a^{b^t} \tag{1}$$

where $a, b \in (0, 1)$.

Satoh [17] and Satoh and Yamada [18] introduced a discrete Gompertz curve by discretization of the differential equations for the Gompertz curve and applied the discrete Gompertz curve to predict the number of detected software faults.

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Yamada [23] constructed a model with the following mean value function

$$M(t) = \omega \left(a^{b^t} - a \right). \tag{2}$$

Some software reliability models, can be found in [6]–[12].

In this note we study the Hausdorff approximation of the Heaviside step function $h_{t_0}(t)$ by sigmoidal curve based on the "mean value" software reliability model and find an expression for the error of the best approximation.

2. The "mean value" software reliability model

We consider the following "mean value" software reliability model – (MVSRM):

$$N(t) = \omega \left(a^{b^t} - a \right), \;\; t_0 = \frac{1}{\ln b} \ln \left(\frac{\ln \left(\frac{1+2a}{a} \right)}{\ln a} \right) \tag{3}$$

$$N(t_0) = \frac{1}{2}, \;\; \omega = 1. \tag{4}$$

Definition 1. [19] The Hausdorff distance (the H–distance) $\rho(f, g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f, g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t, x)|| = \max\{|t|, |x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A - B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

The one–sided Hausdorff distance d between the Heaviside step function

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0, 1 - a], & \text{if} \quad t = t_0 \\ 1 - a, & \text{if} \quad t > t_0 \end{cases} \tag{5}$$

and the sigmoid ((3)–(4)) satisfies the relation

$$N(t_0 + d; \theta, \lambda) = 1 - a - d. \tag{6}$$

The following theorem gives upper and lower bounds for d

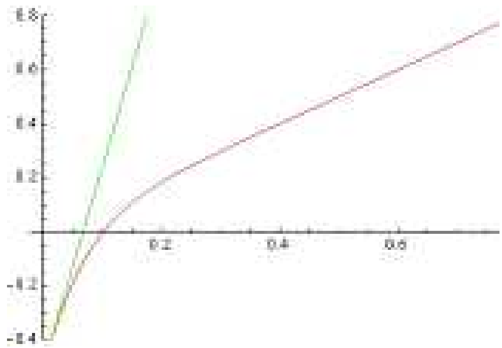

Figure 1: The functions $F(d)$ and $G(d)$ for $a = 0.000001$, $b = 0.00000001$.

Theorem. Let

$$p = -1 + a^{b^{t_0}}$$

$$q = 1 + a^{b^{t_0}} b^{t_0} \ln a \ln b.$$

For the one–sided Hausdorff distance d between h_{t_0} and the curve ((3)–(4)) the following inequalities hold for:

$$\frac{2q}{-p} > e^2; \quad a < 0.5$$

$$d_l = \frac{1}{2\frac{q}{-p}} < d < \frac{\ln(2\frac{q}{-p})}{2\frac{q}{-p}} = d_r. \tag{7}$$

Proof. Let us examine the functions:

$$F(d) = N(t_0 + d) - 1 + a + d. \tag{8}$$

$$G(d) = p + qd. \tag{9}$$

From Taylor expansion we obtain $G(d) - F(d) = O(d^2)$.

Hence $G(d)$ approximates $F(d)$ with $d \to 0$ as $O(d^2)$ (see Fig. 1).

In addition $G'(d) > 0$.

Further, for $\frac{2q}{-p} > e^2$; $a < 0.5$ we have $G(d_l) < 0$ and $G(d_r) > 0$.

This completes the proof of the theorem.

The model ((3)–(4)) for $a = 0.000001$, $b = 0.00000001$, $t_0 = 0.162443$ is visualized on Fig. 3.

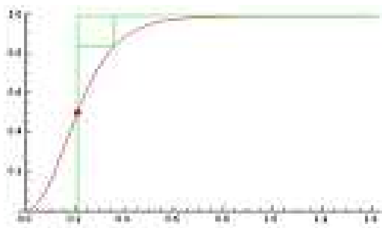

Figure 2: The model ((3)–(4)) for $a = 0.01$, $b = 0.001$, $t_0 = 0.208752$; H–distance $d = 0.152434$.

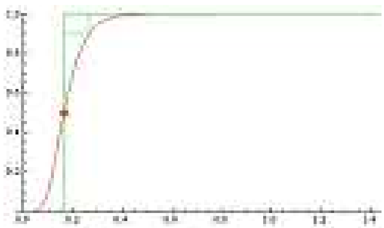

Figure 3: The model ((3)–(4)) for $a = 0.000001$, $b = 0.00000001$, $t_0 = 0.162443$; H–distance $d = 0.101446$; $d_l = 0.033856$; $d_r = 0.114625$.

The model ((3)–(4)) for $a = 0.01$, $b = 0.001$, $t_0 = 0.208752$ is visualized on Fig. 2.

Remark 1. We propose a software module (see Fig. 4) within the programming environment *CAS Mathematica* for the analysis of the considered family of "mean value" functions.

Remark 2. In many cases it is appropriate to use the following model (called by us for brevity) "*inverted deterministic software model*" [9]:

$$M(t) = a^{b^{\frac{k}{t}}}. \tag{10}$$

Numerical example. This example is based on the data reported by Musa [9]. For the first 12 hours of testing, the number of failures each hour is given in Table 1.

Approximate solution by model (10) for $k = 1.23565$, $a = 118.71$ and $b = 0.671538$ is visualized on Fig. 5.

```
Clear[b]
Clear[a]
Manipulate[Dynamic@Show[Plot[f[t], {t, 0, 1.2}, LabelStyle →
    Directive[Blue, Bold], PlotLabel → a^{b^{t}} - a],
   PlotRange → {Automatic, {0, 1 - a}}], {{b, 0.00000001}, 0.0000001, 0.1,
  Appearance → "Open"}, {{a, 0.00000001}, 0.0000001, 0.1,
  Appearance → "Open"}, Initialization :→ (f[t_] := a^{b^{t}} - a)]
```

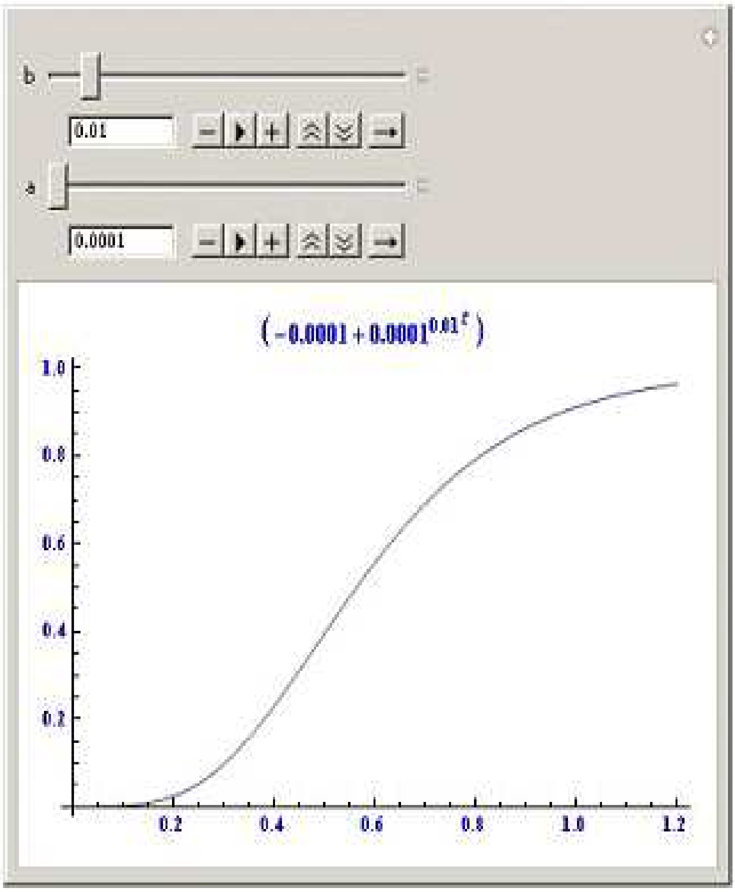

Figure 4: An example of the usage of dynamical and graphical representation for the function $N(t)$ for given a and b.

$Hour$	$Number\,of\,failures$	$Cumulative\,failures$
1	27	27
2	16	43
3	11	54
4	10	64
5	11	75
6	7	82
7	2	84
8	5	89
9	3	92
10	1	93
11	4	97
12	7	104

Table 1: Cumulative failures

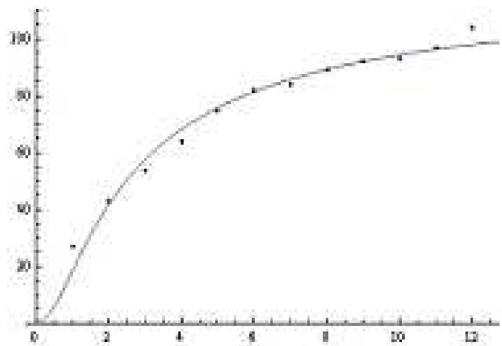

Figure 5: Approximate solution.

Acknowledgments

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