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ON MORE TOPOLOGICAL INDICES OF JAHANGIR GRAPHS

Ashaq Ali[1], Imran Hashim[2],
Waqas Nazeer[3], Shin Min Kang[4 §]

[1,2]Department of Mathematics and Statistics
The University of Lahore
Lahore, 54100, PAKISTAN

[3]Division of Science and Technology
University of Education
Lahore, 54000, PAKISTAN

[4]Department of Mathematics and RINS
Gyeongsang National University
Jinju, 52828, KOREA

Abstract: A topological index of graph G is a numerical parameter related to G which characterizes its topology and is preserved under isomorphism of graphs. Properties of the chemical compounds and topological indices are correlated. In this report, we compute newly defined topological indices, namely, AG_1 index, SK index, SK_1 index and SK_2 index of Jahangir graphs. We also compute sum connectivity index and modified Randić index of underling graph.

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[§]Correspondence author

1. Introduction

Cheminformatics is an emerging field in which quantitative structure-activity and Structure-property relationships predict the biological activities and properties of nano-material see [11, 29]. In these studies, some physcio-chemical properties and topological indices are used to predict bioactivity of the chemical compounds see [13].

The branch of chemistry which deals with the chemical structures with the help of mathematical tools is called mathematical chemistry. Chemical graph theory is that branch of mathematical chemistry which applies graph theory to mathematical modeling of chemical phenomena. In chemical graph theory a molecular graph is a simple graph (having no loops and multiple edges) in which atoms and chemical bonds between them are represented by vertices and edges respectively. A graph with vertex set $V(G)$ and edge set $E(G)$ is connected, if there exist a connection between any pair of vertices in G. The degree of a vertex is the number of vertices which are connected to that fixed vertex by the edges. The number of vertices of G, adjacent to a given vertex v, is the "degree" of this vertex, and will be denoted by d_v. The concept of degree in graph theory is closely related to the concept of valence in chemistry. One can see [29] for details on basics of graph theory.

The topological index of a molecule structure can be considered as a non-empirical numerical quantity which quantifies the molecular structure and its branching pattern in many ways. In this point of view, the topological index can be regarded as a score function which maps each molecular structure to a real number and is used as a descriptor of the molecule under testing. Topological indices gives a good predictions of variety of physico-chemical properties of chemical compounds containing boiling point, heat of evaporation, heat of formation, chromatographic retention times, surface tension, vapor pressure and partition coefficients could be rationalized by the assumption that Wiener index is roughly proportional to the van der Waals surface area of the respective molecule. For details about topological indices, we refer [1–8, 15–22].

The first Zagreb index and the second Zagreb index, introduced by Gutman and Trinajstic [14] and are defined as:

$$M_1(G) = \sum_{v \in V(G)} (d_v)^2$$

and

$$M_2(G) = \sum_{uv \in E(G)} d_u d_v.$$

Sum connectivity index is defined as

$$SC(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_u + d_v}}$$

and modified Randić index is defined as

$$mR(G) = \sum_{uv\in E(G)} \frac{1}{\max\{d_u, d_v\}}.$$

Shigehalli and Kanabur [27] introduced following new degree-based topological indices:

$$AG_1(G) = \sum_{uv\in E(G)} \frac{d_v + d_u}{2\sqrt{d_v d_u}},$$

$$SK(G) = \sum_{uv\in E(G)} \frac{d_v + d_u}{2},$$

$$SK_1(G) = \sum_{uv\in E(G)} \frac{d_v d_u}{2},$$

$$SK_2(G) = \sum_{vu\in E(G)} \left(\frac{d_v + d_u}{2}\right)^2.$$

The Jahangir graph $J_{n,m}$ is a graph on $nm + 1$ vertices and $m(n + 1)$ edges $\forall n \geq 2$ and $m \geq 3$. $J_{n,m}$ consist of a cycle $C_{n,m}$ with one additional vertex which is adjacent to m vertices of $C_{n,m}$ at distance n to each other, Following figure 1 shows some particular cases of $J_{n,m}$.

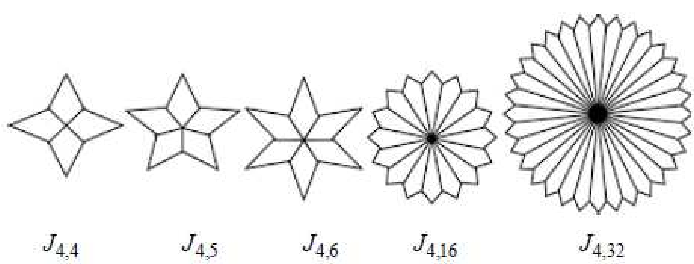

Figure 1. The graphs of $J_{4,4}, J_{4,5}, J_{4,6}, J_{4,16}, J_{4,32}$

The Figure $J_{2,8}$ is carved on Jahangir's tomb. It is situated at 5 kilometer north-west of Lahore Pakistan. In [23], authors computed the pebbling number of Jahangir graph $J_{2,m}$ for $m \geq 8$. Authors in [24], computed domination number in $J_{2,m}$ and Ramsey number for $J_{3,m}$ in [10]. Authors computed Weiner index and Hosoya polynomial of $J_{2,m}$ in [12] and $J_{4,m}$ in [28]. All these results are partial and need to be generalized for all values of m and n. In this paper, we aim to compute some new degree-based topological indices of Jahangir graph .

2. Main Results

In this part, we give our main computational results.

Theorem 2.1. *Let $J_{n,m}$ be the Jahangir graph with defining parameters m and n. Then*

(1) $SC(J_{n,m}) = \frac{m(n-2)}{2} + \frac{2m}{\sqrt{5}} + \frac{m}{\sqrt{3+m}}$.

(2) $mR(J_{n,m}) = \begin{cases} \frac{mn}{2} - \frac{m}{3} + 1; & \text{if } m > 3 \\ \frac{mn}{2}; & \text{if } m \leq 3. \end{cases}$

Proof. Clearly, we have $|V(J_{n,m})| = 8n+2$ and $|E(J_{n,m})| = 10n+1$. Based on the degree of end vertices, we can divide the edge set into following three partitions.

$E_1(J_{n,m}) = \{e = uv \in E(J_{n,m}) : d_u = d_v = 2\}$,
$E_2(J_{n,m}) = \{e = uv \in E(J_{n,m}) : d_u = 2, d_v = 3\}$,
$E_3(J_{n,m}) = \{e = uv \in E(J_{n,m}) : d_u = 3, d_v = m\}$.

In addition, $E_1(J_{n,m}) = m(n-2)$, $E_2(J_{n,m}) = 2m$, $E_3(J_{n,m}) = m$.

Now

$$\begin{aligned} SC(J_{n,m}) &= \sum_{uv \in E(J_{n,m})} \frac{1}{\sqrt{d_u + d_v}} \\ &= \sum_{uv \in E_1(J_{n,m})} \frac{1}{\sqrt{d_u + d_v}} + \sum_{uv \in E_2(J_{n,m})} \frac{1}{\sqrt{d_u + d_v}} \\ &\quad + \sum_{uv \in E_3(J_{n,m})} \frac{1}{\sqrt{d_u + d_v}} \\ &= |E_1(J_{n,m})| \frac{1}{\sqrt{2+2}} + |E_2(J_{n,m})| \frac{1}{\sqrt{2+3}} + |E_3 J_{n,m})| \frac{1}{\sqrt{3+m}} \\ &= \frac{m(n-2)}{2} + \frac{2m}{\sqrt{5}} + \frac{m}{\sqrt{3+m}}. \end{aligned}$$

$$
\begin{aligned}
mR(J_{n,m}) &= \sum_{uv\in E(J_{n,m})} \frac{1}{\max\{d_u, d_v\}} \\
&= \sum_{uv\in E_1(J_{n,m})} \frac{1}{\max\{d_u, d_v\}} + \sum_{uv\in E_2(J_{n,m})} \frac{1}{\max\{d_u, d_v\}} \\
&\quad + \sum_{uv\in E_3(J_{n,m})} \frac{1}{\max\{d_u, d_v\}} \\
&= \begin{cases} \frac{mn}{2} - \frac{m}{3} + 1; & \text{if } m > 3 \\ \frac{mn}{2}; & \text{if } m \leq 3. \end{cases}
\end{aligned}
$$

□

Theorem 2.2. *Let $J_{n,m}$ be the Jahangir graph with defining parameters m and n. Then*

(1) $AG_1(J_{n,m}) = m(n-2) + \frac{5m}{\sqrt{6}} + (3+m)\frac{\sqrt{m}}{2\sqrt{3}}$.

(2) $SK(J_n, m) = 2mn + \frac{5m}{2} + \frac{m^2}{2}$.

(3) $SK_1(J_n, m) = 2mn + 2m + \frac{3m^2}{2}$.

(4) $SK_2(J_n, m) = 4mn + \frac{27m}{4} + \frac{m^3}{4} + \frac{3m^2}{2}$.

Proof. Let $J_{n,m}$ be Jahangir graph with defining parameters of m and n. Then

$$
\begin{aligned}
AG_1(J_{n,m}) &= \sum_{uv\in E_(J_{n,m})} \frac{d_u + d_v}{2\sqrt{d_u d_v}} \\
&= \sum_{uv\in E_1(J_{n,m})} \frac{d_u + d_v}{2\sqrt{d_u d_v}} + \sum_{uv\in E_2(J_{n,m})} \frac{d_u + d_v}{2\sqrt{d_u d_v}} \\
&\quad + \sum_{uv\in E_3(J_{n,m})} \frac{d_u + d_v}{2\sqrt{d_u d_v}} \\
&= |E_1(J_{n,m})|\frac{2+2}{2\sqrt{4}} + |E_2(J_{n,m})|\frac{2+3}{2\sqrt{6}} + |E_3(J_{n,m})|\frac{3+m}{2\sqrt{3m}} \\
&= m(n-2) + \frac{5m}{\sqrt{6}} + (3+m)\frac{\sqrt{m}}{2\sqrt{3}}.
\end{aligned}
$$

$$\begin{aligned} SK(J_{n,m}) &= \sum_{uv\in E(J_{n,m})} \frac{d_u+d_v}{2} \\ &= \sum_{uv\in E_1(J_{n,m})} \frac{d_u+d_v}{2} + \sum_{uv\in E_2(J_{n,m})} \frac{d_u+d_v}{2} \\ &\quad + \sum_{uv\in E_3(J_{n,m})} \frac{d_u+d_v}{2} \\ &= |E_1(J_{n,m})|\frac{2+2}{2} + |E_2(J_{n,m})|\frac{2+3}{2} + |E_3(J_{n,m})|\frac{3+m}{2} \\ &= 2mn + \frac{5m}{2} + \frac{m^2}{2}. \end{aligned}$$

$$\begin{aligned} SK_1(J_{n,m}) &= \sum_{uv\in E(J_{n,m})} \frac{d_u.d_v}{2} \\ &= \sum_{uv\in E_1(J_{n,m})} \frac{d_u d_v}{2} + \sum_{uv\in E_2(J_{n,m})} \frac{d_u d_v}{2} + \sum_{uv\in E_3(J_{n,m})} \frac{d_u d_v}{2} \\ &= |E_1(J_{n,m})|\frac{4}{2} + |E_2(J_{n,m})|\frac{6}{2} + |E_3(J_{n,m})|\frac{3m}{2} \\ &= 2mn + 2m + \frac{3m^2}{2}. \end{aligned}$$

$$\begin{aligned} SK_2(J_{n,m}) &= \sum_{uv\in E(J_{n,m})} \left(\frac{d_u+d_v}{2}\right)^2 \\ &= \sum_{uv\in E_1(J_{n,m})} \left(\frac{d_u+d_v}{2}\right)^2 + \sum_{uv\in E_2(J_{n,m})} \left(\frac{d_u+d_v}{2}\right)^2 \\ &\quad + \sum_{uv\in E_3(J_{n,m})} \left(\frac{d_u+d_v}{2}\right)^2 \\ &= |E_1(J_{n,m})|\left(\frac{2+2}{2}\right)^2 + |E_2(J_{n,m})|\left(\frac{2+3}{2}\right)^2 \\ &\quad + |E_3(J_{n,m})|\left(\frac{3+m}{2}\right)^2 \\ &= 4mn + \frac{27m}{4} + \frac{m^3}{4} + \frac{3m^2}{2}. \end{aligned}$$

□

3. Conclusions and Discussion

In this article we computed Arithmetic-Geometric index (AG_1 index), SK index, SK_1 index and SK_2 index, sum connectivity index and modified Randić index of Jahangir graph. Our results play a vital role in determining properties of this graph and its uses in industry, electronics, and pharmacy [9, 25, 26].

References

[1] M.S. Ahmad, W. Nazeer, S.M. Kang, C.Y. Jung, Some computational aspects for the line graph of Banana tree graph, *Global J. Pure Appl. Math.*, **13** (2016), 2601-2627.

[2] M.S. Ahmad, W. Nazeer, S.M. Kang, C.Y. Jung, M-polynomials and degree based topological indices for the line graph of firecracker graph, *Global J. Pure Appl. Math.*, **13** (2016), 2749-2776.

[3] M. Ajmal, S.M. Kang, W. Nazeer, M. Munir, C.Y. Jung, Some topological invariants of the Möbius ladders, *Global J. Pure Appl. Math.*, **12** (2016), 5317-5327.

[4] M. Ajmal, W. Nazeer, W. Khalid, S.M. Kang, Zagreb polynomials and multiple Zagreb indices for the line graphs of Banana tree graph, firecracker graph and subdivision graphs, *Global J. Pure Appl. Math.*, **13** (2016), 2659-2672.

[5] M. Ajmal, W. Nazeer, W. Khalid, S.M. Kang, Forgotten polynomial and forgotten index for the line graphs of Banana tree graph, firecracker graph and subdivision graphs, *Global J. Pure Appl. Math.*, **13** (2016), 2673-2682.

[6] M. Ajmal, W. Nazeer, M. Munir, S.M. Kang, C.Y. Jung: The M-polynomials and topological indices of generalized prism network, *Int. J. Math. Anal.*, **11** (2017), 293-303. **doi:** 10.12988/ijma.2017.7118

[7] M. Ajmal, W. Nazeer, M. Munir, S.M. Kang, C.Y. Jung, The M-polynomials and topological indices of toroidal polyhex network, *Int. J. Math. Anal.*, **11** (2017), 305-315. **doi:** 10.12988/ijma.2017.7119

[8] M. Ajmal, W. Nazeer, M. Munir, S.M. Kang, Y.C. Kwun, Some algebraic polynomials and topological indices of generalized prism and toroidal polyhex networks, *Symmetry*, **9** (2017), Article ID 5, 12 pages. **doi:**10.3390/sym9010005

[9] A. Ali, H. Iqbal, W. Nazeer, S.M. Kang, On topological indices of line graph of Firecracker graph, *Int. J. Pure Appl. Math.*, **116** (2017), 1035-1042. **doi:** 10.12732/ijpam.v116i4.18

[10] K. Ali, E.T. Baskoro, and I. Tomescu, On the Ramsey number of Paths and Jahangir graph $J_{3,m}$, The 3rd International Conference on 21st Century Mathematics 2007, Government College University, Lahore, Pakistan, 2007.

[11] E. Deutsch, S. Klavžar, S, M-polynomial and degree-based topological indices, *Iranian J. Math. Chem.*, **6** (2015), 93-102. **doi:** 10.22052/ijmc.2015.10106

[12] M.R. Farahani, The Wiener index and Hosoya polynomial of a class of Jahangir graphs $J_{3,m}$, *Fundam. J. Math. Math. Sci.*, **3** (2015), 91-96.

[13] I. Gutman, Degree-based topological indices, *Croat. Chem. Acta*, **86** (2013), 351-361. **doi:** 10.5562/cca2294

[14] I. Gutman, N. Trinajstić, Graph theory and molecular orbitals. Total π-electron energy of alternant hydrocarbons, *Chem. Phys. Lett.*, **17** (1972), 535-538. **doi:** 10.1016/0009-2614(72)85099-1

[15] M.K. Jamil, A. Javed, W. Nazeer, M.R. Farahani, Y. Gao, Four vertex-degree-based topological indices of $VC_5C_7[p;q]$ nanotubes, *Commun. Math. Appl.*, **9** (2017), 99-105.

[16] M. Munir, W. Nazeer, S.M. Kang, M.I. Qureshi, A.R. Nizami, Y.C. Kwun, Some invariants of Jahangir graphs, *Symmetry*, **9**(2017), Article 17, 15 pages. **doi:** 10.3390/sym9010017

[17] M. Munir, W. Nazeer, A. Nizami, S. Rafique, S.M. Kang, M-polynomials and topological indices of titania nanotubes, *Symmetry* **8** (2016), Article ID 117, 9 pages. **doi:** 10.3390/sym8110117

[18] M. Munir, W. Nazeer, S. Rafique, A.R. Nizami, S.M. Kang, Some computational aspects of Boron triangular nanotubes, *Symmetry*, **9** (2017), Article ID 6, 11 pages. **doi:** 10.3390/sym9010006

[19] M. Munir, W. Nazeer, S. Rafique, S.M. Kang, M-polynomial and related topological indices of nanostar dendrimers, *Symmetry*, **8** (2016), Article ID 97, 12 pages. **doi:** 10.3390/sym8090097

[20] M. Munir, W. Nazeer, S. Rafique, S.M. Kang, M-polynomial and degree-based topological indices of polyhex nanotubes, *Symmetry*, **8** (2016), Article ID 149, 8 pages. **doi:** 10.3390/sym8120149

[21] M. Munir, W. Nazeer, Z. Shahzadi, S.M. Kang, Some invariants of circulant graphs, *Symmetry*, **8** (2016), Article 134, 8 pages. **doi:** 10.3390/sym8110134

[22] Y.C. Kwun, M. Munir, W. Nazeer, S. Rafique, S.M. Kang, M-polynomials and topological indices of V-phenylenic nanotubes and nanotori, *Scientific Reports* **7** (2017), Article ID 8756, 9 pages. **doi:** 10.1038/s41598-017-08309-y

[23] A. Lourdusamy, S.S. Jeyaseelan, T. Mathivanan, On pebbling Jahangir graph, *Gen. Math. Notes*, **5** (2011), 42-49.

[24] D.A. Mojdeh and A.N. Ghameshlou, Domination in Jahangir Graph $J_{2,m}$, *Int. J. Contemp. Math. Sciences*, **2** (2007), 1193-1199. **doi:** 10.12988/ijcms.2007.07122

[25] H.M. Rehman, R. Sardar, A.Raza, Computing topological indices of hex board and its line graph, *Open J. Math. Sci.*, **1** (2017), 62-71.

[26] M.S. Sardar, S. Zafar, M.R. Farahani, The generalized zagres index of capra-designed planar benzenoid series $Ca_k(C_6)$, *Open J. Math. Sci.*, **1** (2017), 44-51.

[27] V.S. Shigehalli, R. Kanabur, Computation of new degree-based topological indices of graphene, **2016** (2016), Article ID 4341919, 6 pages. **doi:** 10.1155/2016/4341919

[28] S. Wang, M.R. Farahani, M.R.R. Kanna, M.K. Jamil, R.P. Kumar, The Wiener index and the Hosoya polynomial of the Jahangir graphs, *Appl. Comput. Math.*, **5** (2016), 138-141. **doi:** 10.11648/j.acm.20160503.17

[29] D.B. West, *An Introduction to Graph Theory*, Prentice-Hall, Inc., New Jersey, 1996.