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EFFECT OF THE OBLATENESS OF THE INFINITESIMAL ON EXISTENCE AND STABILITY OF OUT OF PLANE EQUILIBRIUM POINTS IN SPATIAL ELLIPTIC RESTRICTED THREE BODY PROBLEM

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Abstract: In this paper, the existence and stability of the out of plane equilibrium points $L_{6,7}$ is studied analytically and numerically for the elliptical restricted three body problem, where both the primaries and infinitesimal are oblate spheroid. Also the primaries are assumed to be radiating. It was found that the out of plane equilibrium points and its stability are affected by the oblateness of the infinitesimal. We have explored the existence of out of plane equilibrium points around the binary systems: Luyten-726 and Sirius. Also the linear stability of the system is studied analytically and graphically explored.

AMS Subject Classification: 70F07, 70E50
Key Words: elliptic restricted three body problem, out of plane equilibrium points, binary systems, oblate spheriod

1. Introduction

The investigation of existence of five planar equilibrium points and their stability for restricted three body problem has been the area of research interest for many researchers. Radzievskii[7] formulated and discussed the problem for the

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three specific bodies: the Sun, a planet and a dust particle and was the first to find that an allowance for direct solar radiation pressure results in a change in the positions of the libration points and to the appearance of new libration points in comparison with the classical problem.

Simmons etal.[10] extended the work in the circular restricted problem of three bodies taking both the gravitating bodies to be radiating. They obtained a complete solution of the problem of existence and linear stability of the equilibrium points for all values of radiation pressure of both luminous bodies and all values of mass ratios. It was found that there exists nine libration points, three collinear, two triangular and four out of the plane when the radiation pressure of the smaller mass is very high. They have shown that L_8 and L_9 are always linearly unstable, whereas L_6 and L_7 are stable for small range of radiation pressure provided that both large masses are luminous. Ragos and Zagouras[9] have studied the region of stability of these equilibrium points extensively by studying periodic solution of the system and presented the out of plane points as the saddle point of zero velocity surfaces [8]. Das etal[2] presented a new approach by discussing the existence and stability of out of plane equilibrium points, taking the radiation parameter β_2 not as an independent parameter but as the parameter dependent upon β_1, mass ratio and luminosity of the binary components, while discussing the same problem, they also studied the effects of P-R drag on the system. Abouelmagd and Mostafa [1] found the location of out of plane equilibrium points and the forbidden movement region around these points in the spatial case of non-isotropic variation of mass in restricted three body problem. Singh and Vincent [15] studied the location and stability of out of plane equilibrium points for restricted four body problem. They [16] extended their work to encompass the effect of oblateness on the location and stability of out of plane equilibrium points for the same problem.

The restricted three body problem when the oblateness of the infinitesimal is considered is another interesting extension in this feild and has been studied by some authors [3, 4]. Singh and Haruna [11] investigated the problem considering all the three participating bodies as oblate spheroid and reported the presence of five collinear equilibrium points. Also, they examined the stability of all the planar equilibrium points.

The classical Restricted Three-Body Problem has also been generalized to include additional effects observed when the primaries follows not the circular but elliptical path. If the system performs motion about circular orbit, then it is circular restricted three body problem, where as the problem becomes elliptical restricted three body problem, when the eccentricity of the orbit is also considered. The elliptical restricted three body problem while generalizing

Binary system	$M_1(M_\odot)$	$M_2(M_\odot)$	$a(AU)$	e
Luyten-726	0.101	0.99	1.95	0.62
Sirius	2.15	1.05	7.5	0.59

Table 1: Some relevant data of the binary systems.

the classical restricted three body problem still retain some exceptional and significant properties of circular model.[14] studied the dynamics of the planar ERTBP considering the oblateness of all three participating bodies and applied the model to binary pulsars.

The present paper is a discussion of the location and stability of out of plane equilibrium points when all the three participating bodies are oblate spheroids and both the primaries are luminous. The problem is discussed analytically and numerically around the two binary systems: Luyten-726-8(AB) and Sirus B. The data relevant to the paper about the two binary systems [5, 17] is shown in Table 1. The paper is organized as follows: Section 1 gives the introduction. The equations of motion are described in section 2. In section 3 the out of plane equilibrium points are obtained. In section 4 the analysis of linear stability is done. Numerical exploration and subsequent graphical presentation of the problem is given in section 5. Finally, conclusions are drawn in section 6.

2. Equation of motion

The differential equations of motion of the infinitesimal mass in the elliptical restricted three body problem under the effect of radiation pressure and oblateness of both the primaries and the oblateness of infinitesimal in a barycentric, pulsating, rotating and non dimensional coordinates system, [6] are given in the following equations:

$$\begin{aligned} x'' - 2y' &= \phi(e,f)\frac{\partial\Omega}{\partial x}, \\ y'' + 2x' &= \phi(e,f)\frac{\partial\Omega}{\partial y}, \\ z'' &= \phi(e,f)\frac{\partial\Omega}{\partial z}, \end{aligned} \tag{1}$$

where ”′” denotes the differentiation with respect to true anomaly f and

$$\Omega = \frac{1}{2}\left(x^2+y^2-z^2 e\cos f\right) + \frac{1}{n^2}\left\{(1-\mu)q_1\left(\frac{A_1}{2r_1^3} - \frac{3A_1 z^2}{2r_1^5} + \frac{1}{r_1}\right)\right.$$

$$+\mu q_2\left(\frac{A_2}{2r_2^3}-\frac{3A_2z^2}{2r_2^5}+\frac{1}{r_2}\right)+\frac{(1-\mu)A_3}{2r_1^3}\left(1-\frac{3z^2}{r_1^2}\right)+\frac{\mu A_3}{2r_2^3}\left(1-\frac{3z^2}{r_2^2}\right)\Bigg\}; \tag{2}$$

$$\phi=\frac{1}{1+e\cos f}, \tag{3}$$

Here

$$\begin{aligned}
n^2 =& \frac{1}{a^3}\left(1+\frac{3e^2}{2}+\frac{3A_1}{2}+\frac{3A_2}{2}\right),\\
r_1 =& \sqrt{(x+\mu)^2+y^2+z^2},\\
&\text{and}\\
r_2 =& \sqrt{(x+\mu-1)^2+y^2+z^2}.
\end{aligned} \tag{4}$$

Also, $\mu=\frac{m_2}{m_1+m_2}$, where m_1 and m_2 are masses of bigger and smaller primaries respectively positioned at $(x_i,0,0)$, $i=1,2$. A_1, A_2 and A_3 denote the oblateness coefficients of the two primaries and infinitesimal respectively such that $0<A_i<<1$, $(i=1,2,3)$. Furthermore the radiation factor of the primaries are denoted by $q_i=(1-\beta_i)$ where $i=1,2$ and e is the eccentricity of the orbit of either primary around the other.

3. Out-of-plane equilibrium points

The positions of the out-of-plane equilibrium points can be found from the equations of motion by putting all velocity and acceleration components equal to zero and solving the resulting system, i. e. $\Omega_x=0$, $\Omega_y=0$, and $\Omega_z=0$. The second equation $\Omega_y=0$ is satisfied for $y=0$. The other two equations are given as follows:

$$\begin{aligned}
\frac{1}{1+e\cos f}\Bigg[x-\frac{1}{n^2}\Bigg\{&\frac{(1-\mu)(x+\mu)}{r_1^3}\left(q_1+\frac{3\alpha_1}{2r_1^2}-\frac{15\alpha_1z^2}{2r_1^4}\right)\\
&+\frac{\mu(x+\mu-1)}{r_2^3}\left(q_2+\frac{3\alpha_2}{2r_2^2}-\frac{15\alpha_2z^2}{2r_2^4}\right)\Bigg\}\Bigg]=0;
\end{aligned} \tag{5}$$

$$\begin{aligned}
\frac{1}{1+e\cos f}\Bigg[-ze\cos f-\frac{z}{n^2}\Bigg\{&\frac{(1-\mu)}{r_1^3}\left(q_1+\frac{9\alpha_1}{2r_1^2}-\frac{15\alpha_1z^2}{2r_1^4}\right)\\
&+\frac{\mu}{r_2^3}\left(q_2+\frac{9\alpha_2}{2r_2^2}-\frac{15\alpha_2z^2}{2r_2^4}\right)\Bigg\}\Bigg]=0;
\end{aligned} \tag{6}$$

where,

$$\alpha_1 = A_1 q_1 + A_3,$$
$$\alpha_2 = A_2 q_2 + A_3.$$

Rearranging the terms of equation (6), we obtain the value of z as given below:

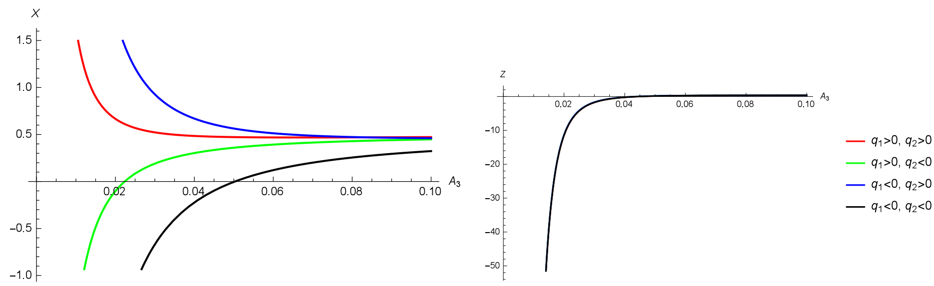
$$z^2 = \frac{2}{15} \cdot \frac{n^2 e \cos f + \frac{(1-\mu)}{r_1^3}\left(q_1 + \frac{9\alpha_1}{2r_1^2}\right) + \frac{\mu}{r_2^3}\left(q_2 + \frac{9\alpha_2}{2r_2^2}\right)}{\frac{(1-\mu)\alpha_1}{r_1^7} + \frac{\mu\alpha_2}{r_2^7}}. \tag{7}$$

The out of plane equilibrium points are denoted as L_6 and L_7 and given as $(x^*, 0, \pm z^*)$. To obtain the coordinates of L_6 and L_7, we take the initial approximation for these points as $(1-\mu, 0, \sqrt{3A_2})$ and use Software Mathematica. For both equations (5) and (7), r_1 and r_2 on the right hand side are replaced by the value of the distances according to the initial approximation that is $r_1 = r_{10} = \sqrt{1 + 3A_2}$ and $r_2 = r_{20} = \sqrt{3A_2}$, where $x_0 = 1 - \mu$, $y_0 = 0$ and $z_0 = \sqrt{3A_2}$, however the values x^* and z^* are assumed to be nonzero. Thus, the coordinates of the out of plane equilibrium points have been approximated in the form of power series as given below:

$$\begin{aligned}
x^* =& 1 - \mu + \frac{\left(2 + 3A_1 + 3e^2 + 3A_2\right) e \cos f}{2a^3} \\
&+ \frac{3\sqrt{3}A_2^{5/2}(1-\mu)\left(2 + 3A_1 + 3e^2\right) e \cos f}{2a^3 A_3 \mu} \\
&+ \frac{3\sqrt{3}A_2^{5/2}(1-\mu)\left(3a^3 A_1 q_1 + 3a^3 A_3 + 2a^3 q_1 - 3A_1 - 3e^2 - 2\right)}{2a^3 A_3 \mu}; \\
y^* =& 0; \\
z^* =& \frac{3\sqrt{A_2}}{\sqrt{5}} + \frac{A_2^{3/2} q_2}{\sqrt{5}A_3} - \frac{7A_2^{5/2} q_2^2}{6\left(\sqrt{5}A_3^2\right)}.
\end{aligned} \tag{8}$$

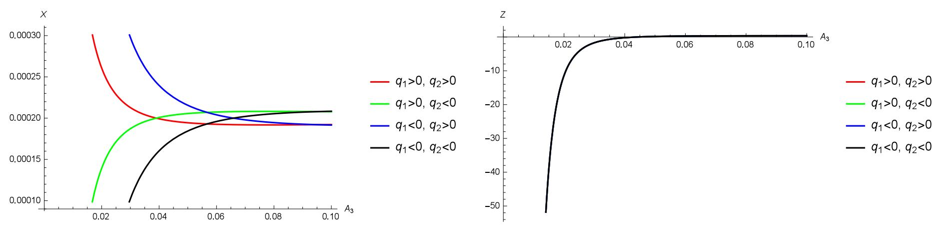
The coordinates of the out of plane equilibrium points thus obtained are found to be dependent on the true anomaly. Therefore, applying the method of averaging, the out of plane equilibrium points independent of true anomaly are obtained as:

$$\begin{aligned}
x^{*(a)} =& (1-\mu)\left(1 + \frac{3\sqrt{3}A_2^{5/2}\left((3\alpha_1 + 2) q_1 - 2n^2\right)}{2\mu\alpha_2 q_2}\right); \\
y^{*(a)} =& 0;
\end{aligned} \tag{9}$$



(a) x -coordinate (b) z -coordinate

Figure 1: The variation in the coordinates of the out of plane equilibrium point L_6 with respect to oblateness of the infinitesimal "A_3"for Luyten-726, taking $A_1 = 0.01,\ A_2 = 0.06$.



(a) x -coordinate (b) z -coordinate

Figure 2: The variation in the coordinates of the out of plane equilibrium point L_6 with respect to oblateness of the infinitesimal "A_3"for Luyten-726, taking $A_1 = 0.01,\ A_2 = 0.06$.

$$z^{*(a)} = \sqrt{3A_2}\left(\frac{3}{\sqrt{15}} + \frac{A_2}{\sqrt{15}\alpha_2} - \frac{A_2^2}{6\sqrt{15}\alpha_2^2} + \frac{3A_2^{5/2}\left(9\alpha_1+2\right)(1-\mu)q_1}{2\sqrt{5}\mu\alpha_2 q_2}\right).$$

4. Stability of out of plane equilibrium points

To study the stability of the equilibrium point denoted by (a_0, b_0, c_0) of an infinitesimal body, we displace it to the position (x, y, z) with a small displacement (u, v, w) from the point, such that $x = a_0 + u$, $y = b_0 + v$ and $z = c_0 + w$. Substituting these values in (1), we obtain the following linearized system of equations:

$$\begin{aligned} u''-2v' &= \phi\left((\Omega_{xx})^0 u+(\Omega_{xy})^0 v+(\Omega_{xz})^0 w\right), \\ v''+2u' &= \phi\left((\Omega_{yx})^0 u+(\Omega^0_{yy})^0 v+(\Omega_{yz})^0 w\right), \\ w'' &= \phi\left((\Omega_{zx})^0 u+(\Omega^0_{zy})^0 v+(\Omega_{zz})^0 w\right). \end{aligned} \quad (10)$$

here, the superscript 0 indicates that the derivatives are to be evaluated at the equilibrium points (a_0, b_0, c_0). Now the second order partial derivatives of Ω with respect to x, y and z are given as follows:

$$\begin{aligned} \Omega_{xx} = 1 - \frac{1}{n^2}\Big\{ & (1-\mu)(\mu+x)\left(-\frac{15\alpha_1(\mu+x)}{2r_1^7}+\frac{105\alpha_1 z^2(\mu+x)}{2r_1^9}-\frac{3q_1(\mu+x)}{r_1^5}\right) \\ & +\mu(\mu+x-1)\left(-\frac{15\alpha_2(\mu+x-1)}{2r_2^7}+\frac{105\alpha_2 z^2(\mu+x-1)}{2r_2^9}-\frac{3q_2(\mu+x-1)}{r_2^5}\right) \\ & +(1-\mu)\left(-\frac{15\alpha_1 z^2}{2r_1^7}+\frac{3\alpha_1}{2r_1^5}+\frac{q_1}{r_1^3}\right)+\mu\left(-\frac{15\alpha_2 z^2}{2r_2^7}+\frac{3\alpha_2}{2r_2^5}+\frac{q_2}{r_2^3}\right)\Big\}; \\ \Omega_{xy} = \Omega_{yx} = -\frac{1}{n^2}\Big\{ & (1-\mu)(\mu+x)\left(-\frac{15\alpha_1 y}{2r_1^7}+\frac{105\alpha_1 z^2 y}{2r_1^9}-\frac{3q_1 y}{r_1^5}\right) \\ & +\mu(\mu+x-1)\left(-\frac{15\alpha_2 y}{2r_2^7}+\frac{105\alpha_2 z^2 y}{2r_2^9}-\frac{3q_2 y}{r_2^5}\right)\Big\}; \\ \Omega_{xz} = \Omega_{zx} = -\frac{1}{n^2}\Big\{ & (1-\mu)(\mu+x)\left(-\frac{15\alpha_1 z}{2r_1^7}+\frac{105\alpha_1 z^2 z}{2r_1^9}-\frac{3zq_1}{r_1^5}\right) \\ & +\mu(\mu+x-1)\left(-\frac{15\alpha_2 z}{2r_2^7}+\frac{105\alpha_2 z^3}{2r_2^9}-\frac{3q_2 z}{r_2^5}\right)\Big\}; \\ \Omega_{yy} = 1 - \frac{1}{n^2}\Big\{ & (1-\mu)y^2\left(-\frac{15\alpha_1}{2r_1^7}+\frac{105\alpha_1 z^2}{2r_1^9}-\frac{3q_1}{r_1^5}\right)+\mu y^2\left(-\frac{15\alpha_2}{2r_2^7}+\frac{105\alpha_2 z^2}{2r_2^9}-\frac{3q_2}{r_2^5}\right) \\ & +(1-\mu)\left(-\frac{15\alpha_1 z^2}{2r_1^7}+\frac{3\alpha_1}{2r_1^5}+\frac{q_1}{r_1^3}\right)+\mu\left(-\frac{15\alpha_2 z^2}{2r_2^7}+\frac{3\alpha_2}{2r_2^5}+\frac{q_2}{r_2^3}\right)\Big\}; \end{aligned}$$

$$\begin{aligned} \Omega_{yz} = \Omega_{zy} = -\frac{1}{n^2}\Big\{ & (1-\mu)z(\mu+x)\left(-\frac{15\alpha_1}{2r_1^7}+\frac{105\alpha_1 z^2}{2r_1^9}-\frac{3q_1}{r_1^5}\right) \\ & +\mu(\mu+x-1)z\left(-\frac{15\alpha_2}{2r_2^7}+\frac{105\alpha_2 z^2}{2r_2^9}-\frac{3q_2}{r_2^5}\right)\Big\}; \\ \Omega_{zz} = 1 - \frac{1}{n^2}\Big\{ & (1-\mu)z^2\left(-\frac{75\alpha_1}{2r_1^7}+\frac{105\alpha_1 z^2}{2r_1^9}-\frac{3q_1}{r_1^5}\right) \\ & +\mu z^2\left(-\frac{75\alpha_2}{2r_2^7}+\frac{105\alpha_2 z^2}{2r_2^9}-\frac{3q_2}{r_2^5}\right) \\ & +(1-\mu)\left(-\frac{15\alpha_1 z^2}{2r_1^7}+\frac{9\alpha_1}{2r_1^5}+\frac{q_1}{r_1^3}\right)+\mu\left(-\frac{15\alpha_2 z^2}{2r_2^7}+\frac{9\alpha_2}{2r_2^5}+\frac{q_2}{r_2^3}\right)\Big\}; \end{aligned}$$

In order to investigate the stability of the out of plane equilibrium points, we introduce new variables as follows:

$$p_X = \frac{du}{\mathrm{df}},\ p_Y = \frac{dv}{\mathrm{df}},\ p_Z = \frac{dw}{\mathrm{df}}.$$

Assuming these values the differential equations take the form:

$$\begin{aligned}
\frac{du}{df} &= P_{14}p_X + P_{15}p_Y + P_{16}p_Z + P_{11}u + P_{12}v + P_{13}w,\\
\frac{dv}{df} &= P_{24}p_X + P_{25}p_Y + P_{26}p_Z + P_{21}u + P_{22}v + P_{23}w,\\
\frac{dw}{df} &= P_{34}p_X + P_{35}p_Y + P_{36}p_Z + P_{31}u + P_{32}v + P_{33}w, \qquad (11)\\
\frac{dp_X}{df} &= P_{44}p_X + P_{45}p_Y + P_{46}p_Z + P_{41}u + P_{42}v + P_{43}w,\\
\frac{dp_Y}{df} &= P_{54}p_X + P_{55}p_Y + P_{56}p_Z + P_{51}u + P_{52}v + P_{53}w,\\
\frac{dp_Z}{df} &= P_{64}p_X + P_{65}p_Y + P_{66}p_Z + P_{61}u + P_{62}v + P_{63}w;
\end{aligned}$$

(12)

where, the coefficients of the differential equations are $P_{14} = P_{25} = P_{36} = 1$; $P_{45} = -P_{54} = 2$; $P_{41} = \phi\Omega^0_{\mathrm{xx}}$; $P_{42} = \phi\Omega^0_{\mathrm{xy}}$; $P_{43} = \phi\Omega^0_{\mathrm{xz}}$; $P_{51} = \phi\Omega^0_{\mathrm{yx}}$; $P_{52} = \phi\Omega^0_{\mathrm{yy}}$; $P_{53} = \phi\Omega^0_{\mathrm{yz}}$; $P_{61} = \phi\Omega^0_{\mathrm{zx}}$; $P_{62} = \phi\Omega^0_{\mathrm{zy}}$; $P_{63} = \phi\Omega^0_{\mathrm{zz}}$; and the values of all other coefficients are equal to zero.

The coefficients $P_{ij}, 1 \le i, j \le 6$ are periodic functions of f of period 2π. Considering the averaged system, where the averaged coefficients are given by $P^{(0)}_{ij} = \frac{1}{2\pi}\int_0^{2\pi} P_{ij}df$.

Then characteristic equation for these averaged differential equation is given as:

$$\lambda^6 + \kappa_0\lambda^4 + \kappa_1\lambda^2 + \kappa_2 = 0; \qquad (13)$$

where,

$$\begin{aligned}
\kappa_0 &= 4 - P^{(0)}_{41} - P^{(0)}_{52} - P^{(0)}_{63}\\
\kappa_1 &= P^{(0)}_{41}P^{(0)}_{52} + P^{(0)}_{41}P^{(0)}_{63} + P^{(0)}_{52}P^{(0)}_{63} - P^{(0)2}_{42} - P^{(0)2}_{43} - P^{(0)2}_{53} - 4P^{(0)}_{63}; \qquad (14)\\
\kappa_2 &= P^{(0)}_{52}P^{(0)2}_{43} + P^{(0)2}_{42}P^{(0)}_{63} + P^{(0)}_{41}P^{(0)2}_{53} - 2P^{(0)}_{42}P^{(0)}_{53}P^{(0)}_{43} - P^{(0)}_{41}P^{(0)}_{52}P^{(0)}_{63}
\end{aligned}$$

Assuming $\lambda^2 = \rho$, the following cubic equation is obtained from equation (13):

$$\rho^3 + \kappa_0\rho^2 + \kappa_1\rho + \kappa_2 = 0. \tag{15}$$

The stability of the system will hold if the roots of equation (14) are purely imaginary, that is if $\rho_i < 0$, $i = 1, 2, 3$. This condition leads us to the following inequalities:

$$\kappa_0 > 0, \quad \kappa_1 > 0, \quad \kappa_2 > 0 \; and \; \Delta < 0; \tag{16}$$

where,

$$\Delta = \frac{(2\kappa_0^3 - 9\kappa_0\kappa_1 + 27\kappa_2)^2 + 4(3\kappa_1 - \kappa_0^2)^3}{27}. \tag{17}$$

Now, $\kappa_0 > 0$, gives the condition $3 - \frac{1}{\sqrt{1-e^2}} > 0$, that is $e < 0.866$. Further more, expanding the terms of the above inequalities, we get the following conditions:

$$0 < \kappa_1 \leq \frac{1}{3}\left(3 - \frac{1}{\sqrt{1-e^2}}\right)^2 \leq \frac{4}{3}; \tag{18}$$

and

$$\begin{aligned} 0 < &\kappa_2 < \delta_1, \; for \; \kappa_1 \leq 1; \\ \delta_1 < &\kappa_2 < \delta_2; \; for \; \kappa_1 > 1. \end{aligned} \tag{19}$$

where,

$$\delta_1 = \frac{1}{27}\left(9\kappa_0\kappa_1 - 2\kappa_0^3 - 2(\kappa_0^2 - 3\kappa_1)^{3/2}\right)$$

$$\delta_2 = \frac{1}{27}\left(9\kappa_0\kappa_1 - 2\kappa_0^3 + 2(\kappa_0^2 - 3\kappa_1)^{3/2}\right)$$

Thus the inequalities (18) and (19) are utilized to define the stability region for specific values of the various parameters such as oblateness, radiation pressure and so on. These conditions for stability are analogus to Ragos and Zagouras[9] when oblateness of the primaries and infinitesimal are neglected.

5. Graphical Exploration

In this section the analytical results obtained in the previous sections are numerically explored and graphically presented around two binary systems Luyten-726 and Sirius. Figures 1 and 2 are depicting the variation in the coordinates of the out of plane equilibrium point L_6 around Luyten-726 and Sirius, when they are assumed as function of oblateness of infinitesimal for varying range of radiation factor, respectively. It was observed that both $x-$ and $z-$ coordinates varies with the variation in the oblateness of the infinitesimal.

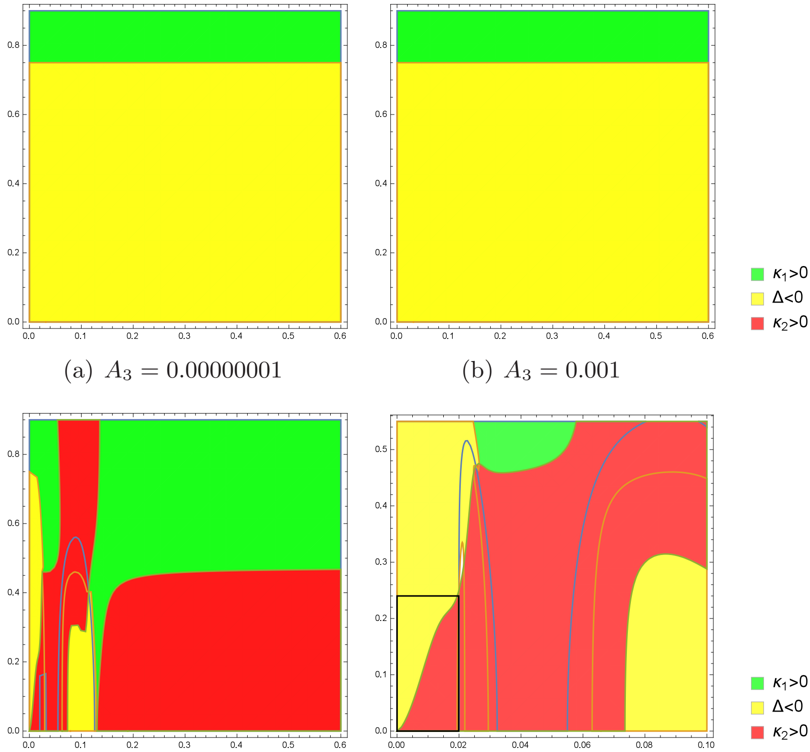
Taking the particular values $a = 0.9996$, $q_1 = 0.9998$, $q_2 = 0.8999$, $A_1 = 0.001$ and $A_2 = 0.06$, the possible region of stability has been plotted in the $\mu - e$ plane, varying the values of A_3 according to the stability criteria presented by (16).

The area of intersection of all the three inequalities define the range of μ and e for which the equilibrium points are stable. As shown by figures 4 and 6, the out of plane equilibrium points are unstable when $q_1 < 0$even for increased value of oblateness of the third body. However, it is observed that for $q_1 > 0$ the out of plane equilibrium points may become stable for high value of oblateness of the third body. This is shown in Sub-figures 6c and 6d which depicts the $\mu - e$ plane when $A_3 = 0.05$ and its zoomed part showing the region where all the three stability conditions are coincident. In the graph this region is shown by dark orange colour.

6. Conclusion and Discussion

The location and stability of out of plane equilibrium points in the photogravitational elliptical restricted three-body problem, where the primaries and the infinitesimal are taken to be oblate spheroid, has been investigated. Also the primaries are assumed to be luminous body. The position of equilibrium points are analyzed in the form of series and found that inclusion of oblateness factor of the infinitesimal along with the eccentricity of the orbits, semi-major axis, radiation and oblateness parameters of both the primaries affect the position of the out-of-plane equilibrium points.

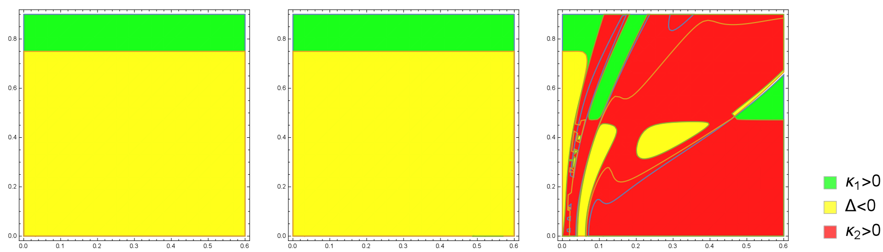
The study of locations and stability of out of plane equilibrium points when both the primaries are radiating oblate spheroids was undertaken by Singh and Umar [12]. Our paper extend the work by assuming the non-sphericity of the third participating body as well. The stability region in the $\mu - e$ plane has been plotted using the conditions given by the inequalities (16) and represented



(a) $A_3 = 0.00000001$ (b) $A_3 = 0.001$

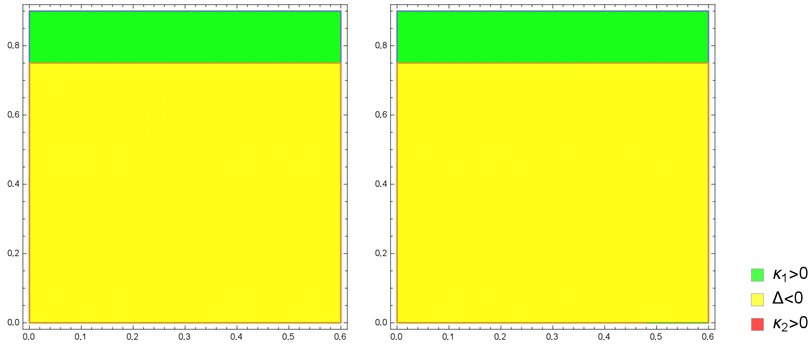
(d) $A_3 = 0.05$ (e) Zoom part of the figure taking $A_3 = 0.05$

Figure 3: The region plot denoting the area satisfying the stability conditions as given in (16) in the $\mu - e$ plane for $q_1 > 0,\ q_2 > 0$.

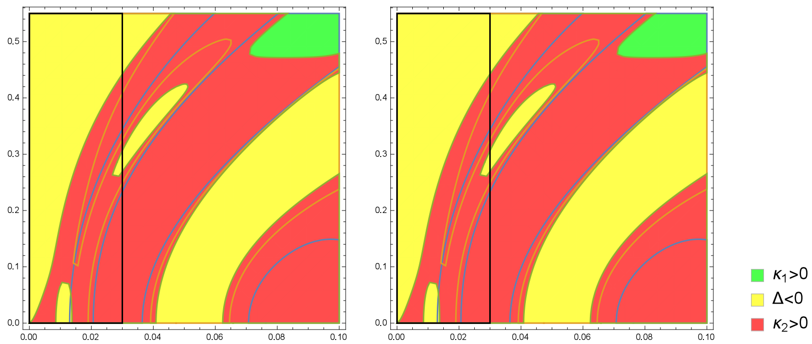


(a) $A_3 = 0.00000001$ (b) $A_3 = 0.001$ (c) $A_3 = 0.05$

Figure 4: The region plot denoting the area satisfying the stability conditions as given in (16) in the $\mu - e$ plane for $q_1 < 0,\ q_2 > 0$.

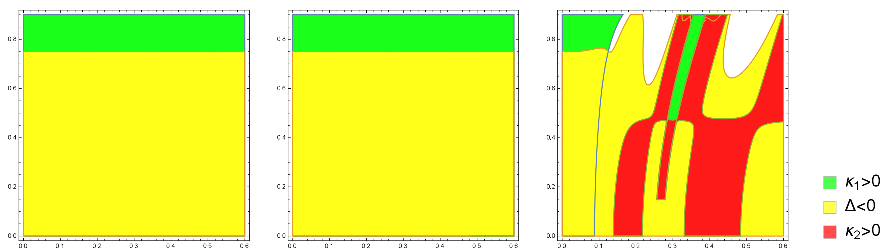


(a) $A_3 = 0.00000001$ (b) $A_3 = 0.001$



(d) $A_3 = 0.1$ (e) Zoom part of the figure taking $A_3 = 0.1$

Figure 5: The region plot denoting the area satisfying the stability conditions as given in (16) in the $\mu - e$ plane for $q_1 > 0,\ q_2 < 0$.



(a) $A_3 = 0.00000001$ (b) $A_3 = 0.001$ (c) $A_3 = 0.05$

Figure 6: The region plot denoting the area satisfying the stability conditions as given in (16) in the $\mu - e$ plane for $q_1 < 0,\ q_2 < 0$.

in figures 3 - 6. We arrived at the conclusion that the out of plane equilibrium points remains unstable when $q_1 < 0$, however when $q_1 > 0$, the points may become stable for very small range of μ and e when the third body is having high value of oblateness.

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