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REMARKS ON η-ρ-CONTINUITY WHERE $\rho \in \{L, M, R, S\}$ AND $\eta \in \{$pre, semi,$\alpha, \beta\}$

Premakumari[1], Evanzalin Ebenanjar[2 §]
[1]Shri Sakthikailassh Women's College
Salem, 636003, TamilNadu, INDIA
[2]Department of Mathematics
Karunya University
Coimbatore, 641114, Tamilnadu, INDIA

Abstract: The purpose of this paper is to disprove certain results on pre-ρ-continuity, semi-ρ-continuity, α-ρ-continuity and β-ρ-continuity that were introduced in 2013 and 2015 by Priyadarshini and Selvi.

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1. Introduction

Topology plays a vital role in Mathematics. The applications of topology depend upon the properties of open sets, closed sets, interior operator and closure operator. Stone [1] introduced regular open sets and regular closed sets. Levine [2] studied semi-open sets and semi-closed sets. Njastad [3] introduced and studied α-open sets and α-closed sets. Mashhour.et.al. [4] introduced pre-open sets

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§Correspondence author

and pre-closed sets. Abd El Monsef.et.al [5] introduced β-open sets. Selvi.et.al. [6] introduced ρ-continuity between a topological space and a non-empty set where $\rho \in$ {L,M,R,S}. The purpose of this paper is to disprove certain results on pre-ρ-continuity, semi-ρ-continuity, α-ρ-continuity and β-ρ-continuity that were introduced in 2013 and 2015 by Priyadarshini and Selvi.

2. Preliminaries

The following definitions will be useful in sequel.

Definition 1. (Definition 4.1.1 of [7]) A function $f : (X, \tau) \to (Y, \sigma)$ is called pre-continuous if the inverse image of each open set in Y is pre-open set in X.

Definition 2. (Definition 4 of [2]) A function $f : (X, \tau) \to (Y, \sigma)$ is called semi-continuous if the inverse image of each open set in Y is semi-open set in X.

Definition 3. (Definition 1.1 of [8]) A function $f : (X, \tau) \to (Y, \sigma)$ is called α-continuous if the inverse image of each open set in Y is α-open set in X.

Definition 4. (Definition 2.1 of [5]) A function $f : (X, \tau) \to (Y, \sigma)$ is called β-continuous if the inverse image of each open set in Y is β-open set in X.

Definition 5. Let $f : (X, \tau) \to Y$ be a function. Then f is

(i) L-continuous [6] if $f(f^{-1}(A))$ is open in (X, τ) for every open set A in (X, τ).

(ii) M-continuous [6] $f(f^{-1}(A))$ is closed in (X, τ) for every closed set A in (X, τ).

Definition 6. Let $f : X, \to (Y, \sigma)$ be a function. Then f is

(i) R-continuous [6] if $f(f^{-1}(B))$ is open in (Y, σ) for every open set B in (Y, σ).

(ii) S-continuous [6] $f(f^{-1}(B))$ is closed in (Y, σ) for every closed set B in (Y, σ).

3. Pre-ρ-Continuity

In this section a counter example is given to establish that the conclusions of Theorem 3.6 (i), (ii); Corollary 3.7; Theorem 3.8; Theorem 4.1 and Theorem 4.2 of [9] fail to hold.

Definition 7. (Definition 3.1 of [9]) Let $f : (X, \tau) \to Y$ be a function. Then f is

(i) pre-L-continuous if $f^{-1}(f(A))$ is open in (X, τ) for every pre-open set A in (X, τ).

(ii) pre-M-continuous if $f^{-1}(f(A))$ is closed in (X, τ) for every pre-closed set A in (X, τ).

Definition 8. (Definition 3.2 of [9]) Let $f : X \to (Y, \sigma)$ be a function. Then f is

(i) pre-R-continuous if $f(f^{-1}(B))$ is open in (Y, τ) for every pre-open set B in (Y, τ).

(ii) pre-S-continuous if $f(f^{-1}(B))$ is closed in (Y, τ) for every pre-closed set B in (Y, τ).

The following example will be useful for disproving certain results of Priyadharshini et.al. [9].

Example 9. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function where $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$, $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{1\}, Y\}$. It is easy to show the following.*
$PO(X, \tau) = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\} = SO(X, \tau) = \alpha O(X, \tau) = \beta O(X, \tau)$.
$PC(X, \tau) = \{\phi, \{b, c\}, \{c\}, \{b\}, X\} = SC(X, \tau) = \alpha C(X, \tau) = \beta C(X, \tau)$.
$PO(Y, \sigma) = \{\phi, \{1\}, \{1, 2\}, \{1, 3\}, Y\} = SO(Y, \sigma) = \alpha O(Y, \sigma) = \beta O(Y, \sigma)$.
$PC(Y, \sigma) = \{\phi, \{2, 3\}, \{3\}, \{2\}, Y\} = SC(Y, \sigma) = \alpha C(Y, \sigma) = \beta C(Y, \sigma)$.

Definition 10. (Definition 2.10 of [9]) Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then f is pre-open(resp.pre-closed) if $f(A)$ is pre-open(resp.pre-closed) in (Y, σ) for every pre-open (resp.pre-closed) set A in (X, τ).

Result 11. *(Theorem 3.6 (i) and (ii) of [9]*

(i) Every injective function $f : (X, \tau) \to (Y, \sigma)$ is pre-L-continuous and pre-M-continuous.

(ii) Every surjective function $f : (X, \tau) \to (Y, \sigma)$ is pre-R-continuous and pre-S-continuous.

Result 12. *(Corollary 3.7 of [9]) If* $f : (X, \tau) \to (Y, \sigma)$ *be bijective function then* f *is pre-L-continuous, pre-M-continuous, Pre-R-continuous and pre-S-continuous.*

Result 13. *(Theorem 3.8 of [9]) Let* $f : (X, \tau) \to (Y, \sigma)$

(i) If f *is L-continuous (resp. M-continuous) then it is pre-L-continuous(resp. pre-M-continuous).*

(ii) If f *is R-continuous(resp. S-continuous) then it is pre-R-continuous(resp. Pre-S-continuous).*

Result 14. *(Theorem 4.1 of [9])*

(i) $f : (X, \tau) \to (Y, \sigma)$ *be pre-open and pre-continuous. Then* f *is pre-L-continuous.*

(ii) $f : (X, \tau) \to (Y, \sigma)$ *be open and pre-continuous. Then* f *is pre-R-continuous.*

Result 15. *(Theorem 4.2 of [9])*

(i) $f : (X, \tau) \to (Y, \sigma)$ *be pre-closed and pre-continuous. Then* f *is pre-M-continuous.*

(ii) $f : (X, \tau) \to (Y, \sigma)$ *be closed and pre-continuous. Then* f *is pre-S-continuous.*

Remark 16. *By taking Example 9, let* $f(a) = \{1\}, f(b) = \{2\}$ *and* $f(b) = \{3\}$. *Clearly* f *is injective and surjective.*

Let $A = \{a, b\}$. *Then* A *is pre-open but not open in* (X, τ). *Since* $f(f^{-1}(A)) = A$ *is not open in* (X, τ), f *is not pre-L-continuous.*

If $A = \{b\}$ *then* A *is pre-closed but not closed in* (X, τ) *that implies* $f(f^{-1}(A)) = A$ *is not closed in* (X, τ). *This shows that* f *is not pre-M-continuous.*

Let $B = \{1, 2\}$. *Then* B *is pre-open but not open in* (Y, σ). *Since* $f(f^{-1}(B)) = B$ *is not open in* (Y, σ), f *is not pre-R-continuous.*

If $B = \{2\}$ *then* B *is pre-closed but not closed in* (Y, σ) *that implies* $f(f^{-1}(B)) = B$ *is not closed in* (Y, σ). *This shows that* f *is not pre-S-continuous.*

The above discussion shows that the conclusions in Result 3.1 and Result 3.2 fail to hold. It can be easily verified that f *is* β*-continuous where* $\beta \in \{L, M, R, S\}$. *but* f *is not pre-L-continuous, not pre-M-continuous, not pre-R-continuous and not pre-S-continuous. This shows that the conclusions of*

Result 3.3 do not hold. It can be easily verified that f is open, pre-open and pre-continuous. Then it follows that the conclusions of Result 3.4 are not true. Again it is true that f is closed and pre-closed. Since f is also pre-continuous, the above discussion shows that Result 3.5 fails to hold.

4. Semi-ρ-Continuity

In this section a counter example is given to establish that the conclusions of Theorem 3.6 (i), (ii); Corollary 3.7; Theorem 3.8; Theorem 4.1 and Theorem 4.2 of [10] fail to hold.

Definition 17. (Definition 3.1 of [10]) Let $f : (X, \tau) \to Y$ be a function. Then f is

(i) semi-L-continuous if $f^{-1}(f(A))$ is open in (X, τ) for every semi-open set A in (X, τ).

(ii) semi-M-continuous if $f^{-1}(f(A))$ is closed in (X, τ) for every semi-closed set A in (X, τ).

Definition 18. (Definition 3.2 of [10]) Let $f : X \to (Y, \sigma)$ be a function. Then f is

(i) semi-R-continuous if $f(f^{-1}(B))$ is open in (Y, τ) for every semi-open set B in (Y, τ).

(ii) semi-S-continuous if $f(f^{-1}(B))$ is closed in (Y, τ) for every semi-closed set B in (Y, τ).

Definition 19. (Definition 2.10 of [10]) Let $f : (X, \tau) \to (Y, \tau)$ be a function. Then f is semi-open (resp.semi-closed) if $f(A)$ is semi-open(resp.semi-closed) in (Y, σ) for every semi-open (resp.semi-closed) set A in (X, τ).

Result 20. *(Theorem 3.6 (i) and (ii) of [10])*

(i) Every injective function $f : (X, \tau) \to (Y, \sigma)$ is semi-L-continuous and semi-M-continuous.

(ii) Every surjective function $f : (X, \tau) \to (Y, \sigma)$ is semi-R-continuous and semi-S-continuous.

Result 21. *(Corollary 3.7 of [10]) If $f : (X, \tau) \to (Y, \sigma)$ be bijective function then f is semi-L-continuous, semi-M-continuous, semi-R-continuous and semi-S-continuous.*

Result 22. *(Theorem 3.8 of [10]) Let $f : (X, \tau) \to (Y, \sigma)$*

(i) If f is L-continuous (resp. M-continuous) then it is semi-L-continuous(resp. semi-M-continuous).

(ii) If f is R-continuous(resp. S-continuous) then it is semi-R-continuous(resp. semi-S-continuous).

Result 23. *(Theorem 4.1 of [10])*

(i) Let $f : (X, \tau) \to (Y, \sigma)$ be semi-open and semi-continuous. Then f is semi-L-continuous.

(ii) Let $f : (X, \tau) \to (Y, \sigma)$ be open and semi-continuous. Then f is semi-R-continuous.

Result 24. *(Theorem 4.2 of [10])*

(i) Let $f : (X, \tau) \to (Y, \sigma)$ be semi-closed and semi-continuous. Then f is semi-M-continuous.

(ii) Let $f : (X, \tau) \to (Y, \sigma)$ be closed and semi-continuous. Then f is semi-S-continuous

Remark 25. *By taking Example 9, let $f(a) = \{1\}, f(b) = \{2\}$ and $f(b) = \{3\}$. Clearly f is injective and surjective.*

Let $A = \{a, b\}$. Then A is semi-open but not open in (X, τ). Since $f(f^{-1}(A)) = A$ is not open in $(X, \tau), f$ is not semi-L-continuous.

If $A = \{b\}$ then A is semi-closed but not closed in (X, τ) that implies $f(f^{-1}(A)) = A$ is not closed (X, τ). This shows that f is not semi-M-continuous.

Let $B = \{1, 2\}$. Then B is semi-open but not open in (Y, σ). Since $f(f^{-1}(B)) = B$ is not open in $(Y, \sigma), f$ is not semi-R-continuous.

If $B = \{2\}$ then B is semi-closed but not closed in (Y, σ) that implies $f(f^{-1}(B)) = B$ is not closed in (Y, σ). This shows that f is not semi-S-continuous.

The above discussion shows that the conclusions in Result 4.1 and Result 4.2 fail to hold. It can be easily verified that f is β-continuous where $\beta \in$\{L,M,R,S\} but f is not semi-L-continuous, not semi-M-continuous, not semi-R-continuous and not semi-S-continuous. This shows that the conclusions of Result 4.3 do not hold. It can be easily verified that f is open, semi-open and semi-continuous. Then it follows that the conclusions of Result 4.4 are not true. Again it is true that f is closed and semi-closed. Since f is also semi-continuous, the above discussion shows that Result 4.5 fails to hold.

5. α-ρ-Continuity

In this section a counter example is given to establish that the conclusions of Theorem 3.6 (i), (ii); Corollary 3.7; Theorem 3.8; Theorem 4.1 and Theorem 4.2 of [11] fail to hold.

Definition 26. (Definition 3.1 of [11]) Let $f : (X, \tau) \to Y$ be a function. Then f is

(i) α-L-continuous if $f^{-1}(f(A))$ is open in (X, τ) for every α-open set A in (X, τ).

(ii) α-M-continuous if $f^{-1}(f(A))$ is closed in (X, τ) for every α-closed set A in (X, τ).

Definition 27. (Definition 3.2 of [11]) Let $f : X \to (Y, \sigma)$ be a function. Then f is

(i) α-R-continuous if $f(f^{-1}(B))$ is open in (Y, τ) for every α-open set B in (Y, τ).

(ii) α-S-continuous if $f(f^{-1}(B))$ is closed in (Y, τ) for every α-closed set B in (Y, τ).

Definition 28. (Definition 2.10 of [11]) Let $f : (X, \tau) \to (Y, \tau)$ be a function. Then f is α-open(resp.α-closed) if $f(A)$ is α-open(resp.α-closed) in (Y, σ) for every α-open (resp.α-closed) set A in (X, τ).

Result 29. *(Theorem 3.6 (i) and (ii) of [11])*

(i) Every injective function $f : (X, \tau) \to (y, \sigma)$ is α-L-continuous and α-M-continuous.

(ii) Every surjective function $f : (X, \tau) \to (y, \sigma)$ is α-R-continuous and α-S-continuous.

Result 30. *(Corollary 3.7 of [11]) If $f : (X, \tau) \to (Y, \sigma)$ be bijective function then f is α-L-continuous, α-M-continuous, α-R-continuous and α-S-continuous.*

Result 31. *(Theorem 3.8 of [11]) Let $f : (X, \tau) \to (Y, \sigma)$*

(i) If f is L-continuous (resp. M-continuous) then it is α-L-continuous(resp. α- M-continuous).

(ii) If f is R-continuous(resp. S-continuous) then it is α-R-continuous(resp. α-S-continuous).

Result 32. *(Theorem 4.1 of [11])*

(i) Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be α-open and α-continuous. Then f is α-L-continuous.

(ii) Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be open and α-continuous. Then f is α-R-continuous.

Result 33. *(Theorem 4.2 of [11])*

(i) Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be pre-closed and α-continuous. Then f is α-M-continuous.

(ii) Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be closed and α-continuous. Then f is α-S-continuous.

Remark 34. *Take the Example 2.9, let $f(a) = \{1\}, f(b) = \{2\}$ and $f(b) = \{3\}$. Clearly f is injective and surjective.*

Let $A = \{a, b\}$. Then A is α-open but not open in (X, τ). Since $f(f^{-1}(A)) = A$ is not open in $(X, \tau)f$ is not α-L-continuous.

If $A = \{b\}$ then A is α-closed but not closed in (X, τ) that implies $f(f^{-1}(A)) = A$ is not closed in (X, τ). This shows that f is not α-M-continuous.

Let $B = \{1, 2\}$. Then B is α-open but not open in (Y, σ). Since $f(f^{-1}(B)) = B$ is not open in $(Y, \sigma), f$ is not α-R-continuous.

If $B = \{2\}$ then B is α-closed but not closed in (Y, σ) that implies $f(f^{-1}(B)) = B$ is not closed in (Y, σ). This shows that f is not α-S-continuous.

The above discussion shows that the conclusions in Result 5.1 and Result 5.2 fail to hold. It can be easily verified that f is α-continuous where $\alpha \in$ {L,M,R,S} but f is not α-L-continuous, not α-M-continuous, not α-R-continuous and not α-S-continuous. This shows that the conclusions of Result 5.3 do not hold. It can be easily verified that f is open, α-open and α-continuous. Then it follows that the conclusions of Result 5.4 are not true. Again it is true that f is closed and α-closed. Since f is also α-continuous, the above discussion shows that Result 5.5 fails to hold.

6. β-ρ-Continuity

In this section a counter example is given to establish that the conclusions of Theorem 3.6 (i), (ii); Corollary 3.7; Theorem 3.8; Theorem 4.1 and Theorem 4.2 of [12] fail to hold.

Definition 35. (Definition 3.1 of [12]) Let $f : (X, \tau) \to Y$ be a function. Then f is

(i) β-L-continuous if $f^{-1}(f(A))$ is open in (X, τ) for every β-open set A in (X, τ).

(ii) β-M-continuous if $f^{-1}(f(A))$ is closed in (X, τ) for every β-closed set A in (X, τ).

Definition 36. (Definition 3.2 of [12]) Let $f : X \to (Y, \sigma)$ be a function. Then f is

(i) β-R-continuous if $f(f^{-1}(B))$ is open in (Y, τ) for every β-open set B in (Y, τ).

(ii) β-S-continuous if $f(f^{-1}(B))$ is closed in (Y, τ) for every β-closed set B in (Y, τ).

Definition 37. (Definition 2.10 of [12]) Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then f is β-open(resp.β-closed) if $f(A)$ is β-open(resp.β-closed) in (Y, σ) for every β-open (resp.β-closed) set A in (X, τ).

Result 38. *(Theorem 3.6 (i) and (ii) of [12])*

(i) Every injective function $f : (X, \tau) \to (Y, \sigma)$ is β - L-continuous and β-M-continuous.

(ii) Every surjective function $f : (X, \tau) \to (Y, \sigma)$ is β-R continuous and β-S-continuous.

Result 39. *(Corollary 3.7 of [12]) If $f : (X, \tau) \to (Y, \sigma)$ be bijective function then f is β-L-continuous, β-M-continuous, β-R-continuous and β-S-continuous.*

Result 40. *(Theorem 3.8 of [12]) Let $f : (X, \tau) \to (Y, \sigma)$*

(i) If f is L-continuous (resp. M-continuous) then it is β-L-continuous(resp. β- M-continuous).

(ii) If f is R-continuous(resp. S-continuous) then it is β-R-continuous(resp. β- S-continuous).

Result 41. *(Theorem 4.1 of [12])*

1. *Let $f : (X, \tau) \to (Y, \sigma)$ be β-open and β-continuous. Then f is β-L-continuous.*

2. *Let $f : (X, \tau) \to (Y, \sigma)$ be open and β-continuous. Then f is β-R-continuous.*

Result 42. *(Theorem 4.2 of [12])*

1. *Let $f : (X, \tau) \to (Y, \sigma)$ be β-closed and β-continuous. Then f is β-M-continuous.*

2. *Let $f : (X, \tau) \to (Y, \sigma)$ be closed and β-continuous. Then f is β-S-continuous.*

Remark 43. *Taking Example 9, let $f(a) = \{1\}, f(b) = \{2\}$ and $f(b) = \{3\}$. Clearly f is injective and surjective.*

Let $A = \{a, b\}$. Then A is β-open but not open in (X, τ). Since $f(f^{-1}(A)) = A$ is not open in $(X, \tau), f$ is not β-L-continuous.

If $A = \{b\}$ then A is β-closed but not closed in (X, τ) that implies $f(f^{-1}(A)) = A$ is not closed in (X, τ). This shows that f is not β-M-continuous.

Let $B = \{1, 2\}$. Then B is β-open but not open in (Y, τ). Since $f(f^{-1}(B)) = B$ is not open in $(Y, \tau), f$ is not β-R-continuous.

If $B = \{2\}$then B is β-closed but not closed in (Y, τ) that implies $f(f^{-1}(B)) = B$ is not closed in (Y, τ). This shows that f is not β-S-continuous.

The above discussion shows that the conclusions in Result 6,1 and Result 6.2 fail to hold. It can be easily verified that f is β-continuous where $\rho \in${L,M,R,S} but f is not β-L-continuous, not β-M-continuous, not β-R-continuous and not β-S-continuous. This shows that the conclusions of Result 6.3 do not hold. It can be easily verified that f is open, β-open and β-continuous. Then it follows that the conclusions of Result 6.4 are not true. Again it is true that f is closed and β-closed. Since f is also β-continuous, the above discussion shows that Result 6.5 fails to hold.

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