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COMPUTATION OF TOPOLOGICAL INDICES OF WINDMILL GRAPH

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Abstract: In this paper, we compute ABC index, ABC_4 index, Sum connectivity index, Randic connectivity index, GA index and GA_5 index of windmill graph.

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Key Words: ABC index, ABC_4 index, Sum connectivity index, Randic connectivity index, GA index and GA_5 index

1. Introduction

The Windmill graph is the graph obtained by taking n copies of the complete

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graph K_m with a vertex in common. It is denoted by ${W_m}^{(n)}$ and consisting of n copies of K_m. The Windmill graph is also called as friendship graph if $m = 3$. Windmill graph ${W_m}^{(n)}$ contains $(m-1)n+1$ vertices and $\frac{mn(m-1)}{2}$ edges as shown in the Figure 1 to 3.

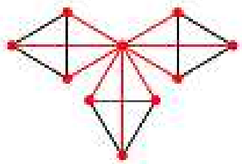

Figure 1

$W_4^{(3)}$

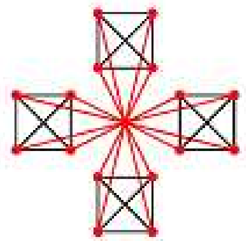

Figure 2

$W_5^{(4)}$

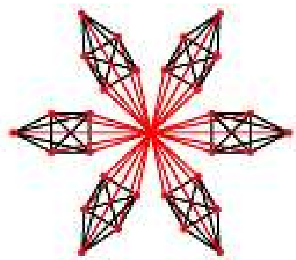

Figure 3

$W_6^{(6)}$

A topological index $Top(G)$ of a graph G, is a number with the property that for every graph H isomorphic to G, $Top(H) = Top(G)$. So, a topological index is a real number derived from the structure of a graph, which is invariant under graph isomorphism. Topological indices are numerical parameters of a graph which characterize its topology and are usually graph invariants. Also topological indices are the molecular descriptors that describes the structures of chemical compounds and it help us to predict certain physico-chemical properties like boiling point, enthalpy of vaporization, stability etc.

All graphs considered in this paper are finite, connected, loop less and without multiple edges. Let $G = (V, E)$ be a graph with n vertices and m edges. The degree of a vertex $u \in V(G)$ is denoted by d_u and is the number of vertices that are adjacent to u. The edge connecting the vertices u and v is denoted by uv.

Using these terminologies, certain topological indices are defined in the following manner.

The atom-bond connectivity index, ABC index is one of the degree based molecular descriptor, which was introduced by Estrada et al. [5] in late 1990's. Some upper bounds for the atom-bond connectivity index of graphs can be found in [2], The atom-bond connectivity index of chemical bicyclic graphs, connected graphs can be seen in [3, 15]. For further results on ABC index of trees see the papers [8, 10, 14, 16] and the references cited there in.

Definition 1.1. Let $G = (V, E)$ be a molecular graph and d_u is the degree of the vertex u, then ABC index of G is defined as,

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}.$$

The fourth atom bond connectivity index, $ABC_4(G)$ index was introduced by M. Ghorbani et al. [9] in 2010. Further studies on $ABC_4(G)$ index can be found in [6, 7].

Definition 1.2. Let G be a graph, then its fourth ABC index is defined as , $ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}$, where S_u is sum of the degrees of all neighbors of vertex u in G. In other words, $S_u = \sum_{uv \in E(G)} d_v$, Similarly for S_v.

The first and oldest degree based topological index is Randic index [12] denoted by $\chi(G)$ and was introduced by Milan Randic in 1975. It provides a quantitative assessment of branching of molecules.

Definition 1.3. For the graph G Randic index is defined as,
$\chi(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u d_v}}$.

Sum connectivity index belongs to a family of Randic like indices and it was introduced by Zhou and Trinajstic [18]. Further studies on Sum connectivity index can be found in [19, 20].

Definition 1.4. For a simple connected graph G, its sum connectivity index $S(G)$ is defined as, $S(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u + d_v}}$.

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The Geometric-arithmetic index, $GA(G)$ index of a graph G was introduced by D. Vukicevic et.al[13]. Further studies on GA index can be found in [1, 4, 17]

Definition 1.5. Let G be a graph and $e = uv$ be an edge of G then, $GA(G) = \sum_{e=uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u d_v}$.

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The fifth Geometric-arithmetic index, $GA_5(G)$ was introduced by A.Graovac et al [11] in 2011.

Definition 1.6. For a Graph G, the fifth Geometric-arithmetic index is defined as $GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v}$, Where S_u is the sum of the degrees of all neighbors of the vertex u in G, similarly S_v.

2. Main results and Discussion

Theorem 2.1. *The Atom bond connectivity index of Windmill graph is* $ABC({W_m}^{(n)}) = \frac{n(m-2)^{\frac{3}{2}}}{\sqrt{2}} + \sqrt{n[(m-1)(n+1)-2]}$.

Proof. Consider the Windmill graph ${W_m}^{(n)}$. We partition the edges of ${W_m}^{(n)}$ into edges of the type $E_{(d_u,d_v)}$ where uv is an edge and d_u is the degree of the vertex u. In ${W_m}^{(n)}$, we get edges of the type $E_{(m-1,m-1)}$ and $E_{[m-1,(m-1)n]}$. Edges of the type $E_{(m-1,m-1)}$ and $E_{[m-1,(m-1)n]}$ are colored in black and red respectively as shown in the Figure [4]. The number of edges of these types are given in the table 1.

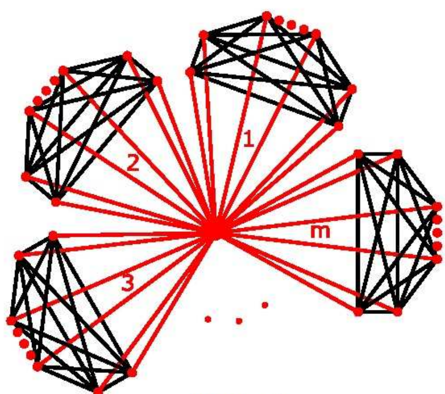

Figure 4

${W_m}^{(n)}$

Table 1: Edge partition based on degrees of end vertices of each edge

Edges of the type $E_{(d_u,d_v)}$	**Number of edges**
$E_{(m-1,m-1)}$	$\frac{(m-1)(m-2)n}{2}$
$E_{[m-1,(m-1)n]}$	$(m-1)n$

We know that $ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$

i.e., $ABC(W_m{}^{(n)}) = |E_{(m-1),(m-1)}| \sum_{uv \in E_{(m-1),(m-1)}(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$

$$+ |E_{(m-1),(m-1)n}| \sum_{uv \in E_{(m-1),(m-1)n}(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$$

$$= \frac{(m-1)(m-2)n}{2} \sqrt{\frac{m-1+m-1-2}{(m-1)(m-1)}}$$

$$+ (m-1)n \sqrt{\frac{(m-1)+(m-1)n-2}{(m-1)(m-1)n}}$$

[From table(1) and figure(4)]

$$= \frac{(m-1)(m-2)n}{2} \sqrt{\frac{2m-4}{(m-1)^2}} + (m-1)n \sqrt{\frac{(m-1)+(m-1)n-2}{(m-1)(m-1)n}}$$

$$\therefore ABC(W_m{}^{(n)}) = \frac{n(m-2)^{\frac{3}{2}}}{\sqrt{2}} + \sqrt{n[(m-1)(n+1)-2]}.$$

□

Theorem 2.2. *The Randic Index of Windmill graph is* $\chi(W_m{}^{(n)}) = \frac{(m-2)n + 2\sqrt{n}}{2}$.

Proof. We know that $\chi(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u d_v}}$

i.e., $\chi(W_m{}^{(n)}) = |E_{(m-1),(m-1)}| \sum_{uv \in E_{(m-1),(m-1)}(G)} \frac{1}{\sqrt{d_u d_v}} +$

$$|E_{(m-1),(m-1)n}| \sum_{uv \in E_{(m-1),(m-1)n}(G)} \frac{1}{\sqrt{d_u d_v}}$$

$$= \frac{(m-1)(m-2)n}{2} \cdot \frac{1}{\sqrt{(m-1)(m-1)}} + (m-1)n \frac{1}{\sqrt{(m-1)(m-1)n}}$$

[From table(1) and figure(4)]

$$\chi(W_n^{(m)}) = \frac{(m-2)n + 2\sqrt{n}}{2}. \qquad \square$$

Theorem 2.3. *The Geometric-arithmetic index of Windmill graph is* $GA(W_m{}^{(n)}) = \dfrac{(m-1)(m-2)n}{2} + \dfrac{2(m-1)n^{\frac{3}{2}}}{(n+1)}$.

Proof. $GA(G) = \sum\limits_{uv \in E(G)} \dfrac{2\sqrt{d_u d_v}}{(d_u + d_v)}$

$$GA(W_m{}^{(n)}) = |E_{(m-1),(m-1)}| \sum_{uv \in E_{(m-1),(m-1)}(G)} \frac{2\sqrt{d_u d_v}}{(d_u + d_v)} +$$

$$|E_{(m-1),(m-1)n}| \sum_{uv \in E_{(m-1),(m-1)n}(G)} \frac{2\sqrt{d_u d_v}}{(d_u + d_v)}$$

[From table(1) and figure(4)]

$$= \frac{(m-1)(m-2)n}{2}\left[\frac{2\sqrt{(m-1)(m-1)}}{(m-1)+(m-1)}\right] + (m-1)n\left[\frac{2\sqrt{(m-1)(m-1)n}}{(m-1)+(m-1)n}\right]$$

$$= (m-1)(m-2)n\frac{(m-1)}{2(m-1)} + \frac{2(m-1)n(m-1)\sqrt{n}}{(m-1)(n+1)}$$

$$\therefore GA(W_m{}^{(n)}) = \frac{(m-1)(m-2)n}{2} + \frac{2(m-1)n^{\frac{3}{2}}}{(n+1)}. \qquad \square$$

Theorem 2.4. *The sum connectivity index of Windmill graph is* $S(W_m{}^{(n)})$ $= n(\sqrt{m-1})\left[\dfrac{(m-2)}{2^{\frac{3}{2}}} + \dfrac{1}{\sqrt{n+1}}\right]$.

Proof. $S(G) = \sum\limits_{uv \in E(G)} \dfrac{1}{\sqrt{d_u + d_v}}$

$$S(W_m{}^{(n)}) = |E_{(m-1),(m-1)}| \sum_{uv \in E_{(m-1),(m-1)}(G)} \frac{1}{\sqrt{d_u + d_v}} +$$

$$|E_{(m-1),(m-1)n}| \sum_{uv \in E_{(m-1),(m-1)n}(G)} \frac{1}{\sqrt{d_u + d_v}}$$

[From table(1) and figure(4)]

$$= \frac{(m-1)(m-2)n}{2\sqrt{2m-2}} + \frac{(m-1)n}{\sqrt{mn-n+m-1}}$$
$$= n(\sqrt{m-1})\left[\frac{(m-2)}{2^{\frac{3}{2}}} + \frac{1}{\sqrt{n+1}}\right]. \qquad \square$$

Theorem 2.5. *The fourth atom bond connectivity index of Windmill graph is*

$$ABC_4(D_m^{(n)}) = \frac{n(m-2)\sqrt{m^2+mn-3m-n+1}}{(m+n-2)\sqrt{2}} + \sqrt{\frac{n[(m-1)(mn+m-2)-2]}{(m-1)(m+n-2)}}.$$

Proof. We partition the edges of ${W_m}^{(n)}$ into edges of the type $E^*_{(S_u,S_v)}$ where uv is an edge and S_u is the sum of the degrees of all neighbors of vertex u in G. In other words, $S_u = \sum_{uv\in E(G)} d_v$, Similarly for S_v. If $n \geq 2$, in ${W_m}^{(n)}$ we get edges of the type $E^*_{(m-1)n+(m-1)(m-2),(m-1)n+(m-1)(m-2)}$ and $E^*_{(m-1)n+(m-1)(m-2),(m-1)^2n}$. Edges of the type

$$E^*_{(m-1)n+(m-1)(m-2),(m-1)n+(m-1)(m-2)}$$

and

$$E^*_{(m-1)n+(m-1)(m-2),(m-1)^2n}$$

are colored in black and red respectively as shown in the Figure [5]. The number of edges of these types are given in the table 2.

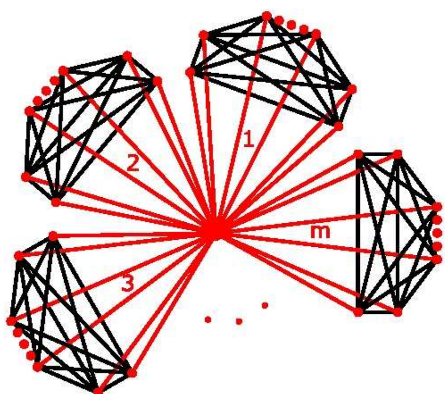

Figure 5

${W_m}^{(n)}$

Table 2

Edge partition based on degree sum of neighbors of end vertices of each edge.

Edges of the type	Number of edges
$E^*_{(m-1)n+(m-1)(m-2),(m-1)n+(m-1)(m-2)}$	$\frac{(m-1)(m-2)n}{2}$
$E^*_{(m-1)n+(m-1)(m-2),(m-1)^2n}$	$(m-1)n$

We know that $ABC_4(G) = \sum_{uv\in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}$

i.e., $ABC_4(W_m{}^{(n)}) = |E^*_{(m-1)n+(m-1)(m-2),(m-1)n+(m-1)(m-2)}|$

$$\sum_{uv\in E^*_{(m-1)n+(m-1)(m-2),(m-1)n+(m-1)(m-2)}(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}$$

$$+ |E^*_{(m-1)n+(m-1)(m-2),(m-1)^2n}| \sum_{uv\in E^*_{(m-1)n+(m-1)(m-2),(m-1)^2n}(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}$$

[From table(2) and figure(5)]

$$= \frac{(m-1)(m-2)n}{2} \sqrt{\frac{2[(m-1)n + (m-1)(m-2)] - 2}{[(m-1)n + (m-1)(m-2)]^2}}$$

$$+ (m-1)n\sqrt{\frac{(m-1)n + (m-1)(m-2) + (m-1)^2n - 2}{[(m-1)n + (m-1)(m-2)](m-1)^2n}}$$

$$\therefore ABC_4(W_m{}^{(n)}) = \frac{n(m-2)\sqrt{m^2 + mn - 3m - n + 1}}{(m+n-2)\sqrt{2}} + \sqrt{\frac{n[(m-1)(mn+m-2) - 2]}{(m-1)(m+n-2)}}.$$

□

Theorem 2.6. *The fifth Geometric-arithmetic index of Windmill graph is* $GA_5(W_m{}^{(n)}) = \frac{m^2n - 3mn + 2n}{2} + \frac{\sqrt{[(m-1)n]^3(m+n-2)}}{mn + m - 2}$

Proof. We know that $GA_5(G) = \sum_{uv\in E(G)} \frac{2\sqrt{S_u S_v}}{(S_u + S_v)}$

$GA_5(W_m{}^{(n)}) = |E^*_{(m-1)n+(m-1)(m-2),(m-1)n+(m-1)(m-2)}|$

$$\sum_{uv\in E^*_{(m-1)n+(m-1)(m-2),(m-1)n+(m-1)(m-2)}(G)} \frac{2\sqrt{S_u S_v}}{(S_u + S_v)} +$$

$$|E^*_{(m-1)n+(m-1)(m-2),(m-1)^2n}| \sum_{uv\in E^*_{(m-1)n+(m-1)(m-2),(m-1)^2n}(G)} \frac{2\sqrt{S_uS_v}}{(S_u+S_v)}$$

$$= \frac{(m-1)(m-2)n}{2}\left[\frac{2\sqrt{[(m-1)n+(m-1)(m-2)]^2}}{2[(m-1)n+(m-1)(m-2)]}\right] +$$

$$(m-1)n\left[\frac{2\sqrt{[(m-1)n+(m-1)(m-2)](m-1)^2n}}{2[(m-1)n+(m-1)(m-2)+(m-1)^2n]}\right]$$

$$= \frac{m^2n-3mn+2n}{2} + \frac{2n(m-1)\sqrt{n(m-1)(m+n-2)}}{mn+m-2} \qquad \square$$

Conclusion:

The general formula for ABC index, ABC_4 index, Randic connectivity index, Sum connectivity index, GA index and GA_5 index of Windmill graph is computed here analytically.

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