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COMPARATIVE ANALYSIS OF POLYNOMIAL FEATURES FOR TEXTURE CLASSIFICATION

Ahmad M. Alenezi[1 §], I.S. Rakhimov[2]

[1]Department of Mathematics
The Higher Institute of Telecommunication and Navigation
PAAET, Shuwaikh, KUWAIT

[1,2]Department of Mathematics
Faculty of Science and Institute for Mathematical Research Universiti Putra Malaysia
UPM Serdang Selangor D.E., MALAYSIA

Abstract: In the paper the feature extraction capability of Krawtchouk and Chebyshev polynomials are tested using Kylberg texture set. The original feature extraction technique based on the combination of either Chebyshev or Krawtchouk moments and low order statistical moments is proposed. Among possible configurations of statistical moments, the most suitable for each type of polynomial moments are chosen. Support Vector Machine is used as primary classification method.

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1. Introduction

The human's eye has a remarkable ability to discern between different texture types almost effortlessly. Although for computers it is a challenge. The problem of automatic texture classification arises in numerous areas such as agriculture,

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§Correspondence author

industry, ecology, medical diagnostics [1], etc. In this paper Support Vector Machine is used as the classifier. Features extraction is the controlling factor the Support Vector Machine (SVM) performance, and thus lots of attention and research previously focused on features extraction. The features could be separated into two groups, the local features, and global features. The local features scope is very narrow and limited to a particular location without the need to worry about surroundings while the global features, on the contrary, takes the surroundings into account.

2. Feature Extraction Techniques

2.1. Haralick

Haralick features [11], also known as Gray Level Co-occurrence Matrix (GLCM) features, are the one of the most widely used texture features. A co-occurrence matrix, also referred to as a co-occurrence distribution, is defined over an image to be the distribution of co-occurring values at a given offset. Using a statistical approach such as co-occurrence matrix represents the distance and angular spatial relationship over an image sub-region of specific size.Given an image I, of size $N \times N$, the co-occurrence, matrix P can be defined as

$$P_{ij} = \sum_{x=1}^{N} \sum_{y=1}^{N} \begin{cases} 1 & \text{if } I(x,y) = i \quad \text{and} \quad I(x+\Delta_x, y+\Delta_y) = j \\ 0 & \text{if otherwise.} \end{cases} \tag{1}$$

Here, the offset (x, y), is specifying the distance between the pixel-of-interest and its neighbor.

2.2. LBP

The Local Binary Patterns (LBPs) method was proposed by Ojala et al. [3] to encode the pixel-wise information in the texture images. At a center pixel , each neighboring pixel is assigned with a binary label, which can be either '0' or '1', depending on whether the center pixel has higher intensity value than the neighboring pixel. The neighboring pixels are the angularly evenly distributed sample points over a circle with radius centered at the center pixel An exhaustive survey of feature extraction techniques for textures can be found in [4]

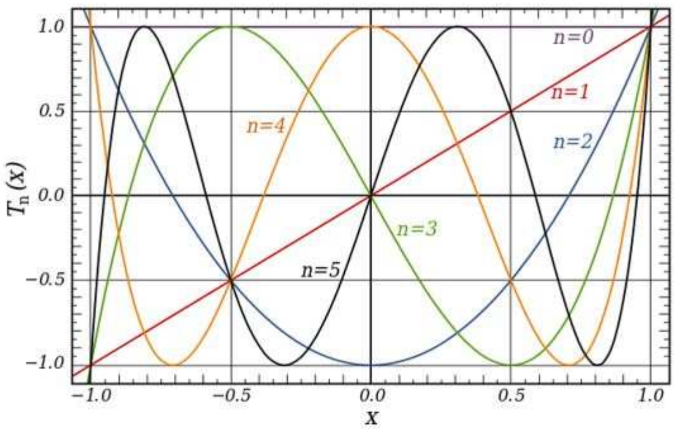

Figure 1: Examples of Chebyshev polynomials

3. Polynomial Features Analysis

3.1. Brief Review of Polynomials

Both Chebyshev as well as Krawtchouk polynomials form orthonormal system. The convenient way to obtain Chebyshev polynomials is from the generating function [5]

$$\begin{aligned} g_1(t,x) &\equiv \frac{1-t^2}{1-2xt+t^2} \\ &= T_0(x) + 2\sum_{k=1}^{\infty} T_k(x)\cdot t^k \end{aligned} \tag{2}$$

where $T_k(x)$ are the sought-for polynomials and t is a dummy variable.

The polynomials are orthogonal with respect to the inner product defined as follows

$$\int_{-1}^{1} T_n(x)T_m(x)\frac{1}{\sqrt{1-x^2}}dx = \delta_{nm} \tag{3}$$

The Chebyshev moments for a $N \times M$ image $f(x,y)$ are conveniently defined as

$$C_{nm} = \sum_x \sum_y f(x,y)T_n(x)T_m(y) \tag{4}$$

One of the approaches to use Chebyshev moments as a tool to extract features for texture classification is proposed in [6] Krawtchouk polynomials are also form an orthogonal system but their distinctive feature is that they are naturally discrete so that there is no need to care about errors caused by discretization as in the case of Chebyshev polynomials, which need to be discretized [5].

$$K_n^{(p)}(x,N)=\sum_i \binom{N-x}{n-i}(-1)^i p^i(1-p)^{n-i} \tag{5}$$

These polynomials are depicted in the Fig. 2

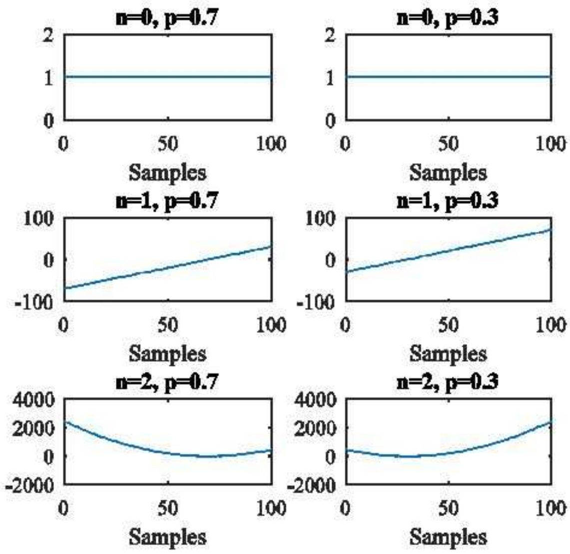

Figure 2: Examples of Krawtchouk polynomials

Krawtchouk polynomials obey orthogonality relation

$$\sum_x w^{(p)}(x,N)\cdot K_n^{(p)}(x,N)\cdot K_m^{(p)}(x,N)=\rho(n,p,N)\delta_{nm} \tag{6}$$

where

$$w^{(p)}(x,N)=\binom{N}{x}p^x(1-p)^{N-x} \tag{7}$$

is a weighting function and

$$\rho(n,p,N)=(-1)^n\Big(\frac{1-p}{p}\Big)^n\frac{n!\Gamma(-N)}{\Gamma(n-N)} \tag{8}$$

is the squared norm.

It is common [7, 8] to incorporate norm (8) and weighting function (7) with polynomials (5) itself getting normed Krawtchouk polynomials.

$$k_n^{(p)}(x,N) = k_n^{(p)}(x,N)\sqrt{\frac{w^{(p)}(x,N)}{\rho(n,p,N)}} \tag{9}$$

Some examples of weighted polynomials are depicted in the Fig. 3

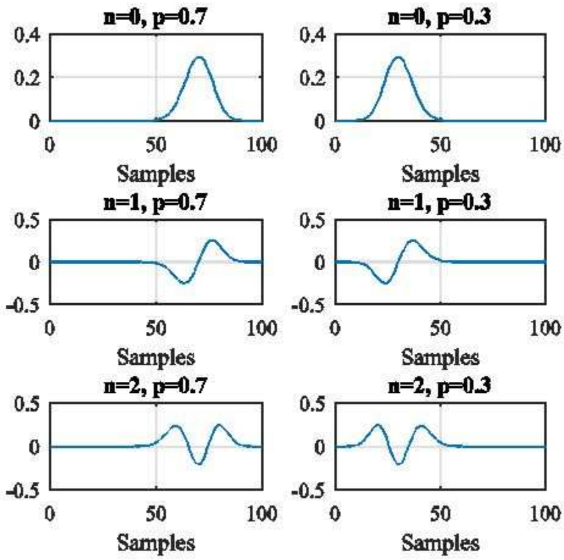

Figure 3: Examples of weighted Krawtchouk polynomials

As it clearly can be seen from the Fig. 4 weighted Krawtchouk polynomials are highly localized. Moreover parameters and allow one to change the domain of localization over the image.

Krawtchouk moments Q_{nm} of an image $f(x,y)$ with $x = 0\cdots, N-1$; and $y = 0, \cdots, M-1$; are defined as [7]

$$Q_{nm} = \sum_x \sum_y f(x,y) K_n^{(p_1)}(x, N-1) K_m^{(p_2)}(x, M-1) \tag{10}$$

An exhaustive treatment of polynomial moments and their applications is given in [9]

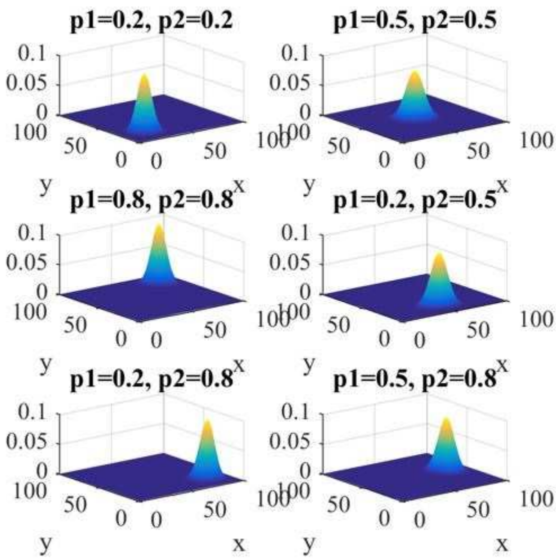

Figure 4: Weighted Krawtchouk polynomials dependence on p_1 and p_2

3.2. General Feature Extraction Procedure

Taking each $N \times N$ image from the texture set it is partitioned in $d \times d$ patches ($d < N$ and d divides N). Each patch is then processed as follows

1. Calculate either Chebyshev or Krawtchouk moments for each patch using formula (4) or (10) correspondingly;

2. Stack obtained spectra in a $3D$ array as depicted in the Fig. 5;

3. Calculate combination of low order statistical moments along the $3rd$ dimension of previously formed $3D$ array;

4. Form a feature vector using calculated statistical moments for each pixel.

In this paper only four statistical moments are used to extract features: mean, standard deviation, skewness and kurtosis. They are calculated via convenient formulas

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{11}$$

$$\sigma = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2 \tag{12}$$

$$s = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^3}{\left(\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2}\right)^3} \tag{13}$$

$$k = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^4}{\left(\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2}\right)^2} \tag{14}$$

There are $2^4 = 16$ possible combinations of 4 statistical moments among which only score best combinations are eligible to be used to form a feature vector.

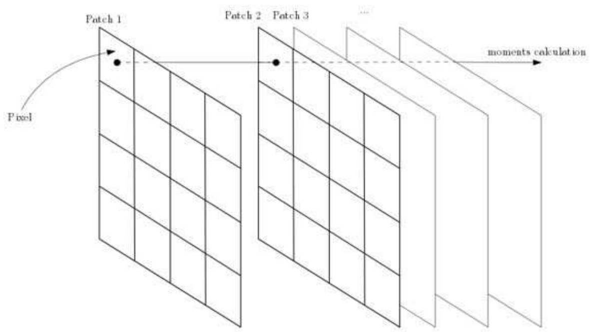

Figure 5: Extracted patches stacked in $3D$ array (arrow depicts the dimension along which statistical moments calculation is performed)

In the Fig. 6 and 7 the calculated statistical moments are depicted. It is clear that for Chebyshev polynomials variation and kurtosis behave in a much regular way than remaining statistical moments do, while for Krawtchouk polynomials only standard deviation shows regular behavior.

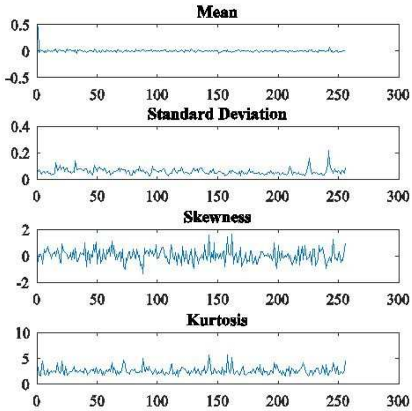

Figure 6: Changing of statistical moments for Chebyshev polynomials spectra along patches

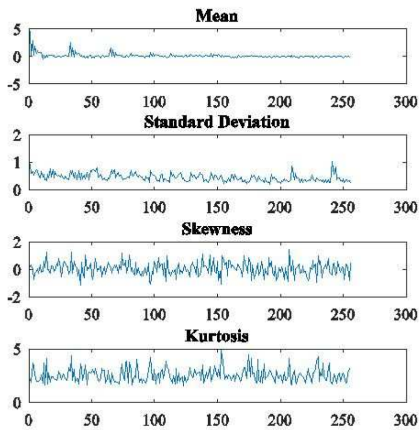

Figure 7: Changing of statistical moments for Krawtchouk polynomial spectra along patches

TABLE I
RESULTS FOR DIFFERENT RUNS OF THE ALGORITHM

No.	Chebyshev features accuracy, %	Krawtchouk features accuracy, %
1	89.62	88.56
2	88.84	87.83
3	89.68	88.73
4	90.63	89.45
5	89.50	88.06
Average:	89.56	88.53

After having been calculated statistical moments for each patch they are stored in a feature vector for every image in the set. In Fig. 8 the process of feature vector formation is depicted for a particular image partitioned into patches. The letters m, d, s, and k stand for mean, standard deviation, skewness, and kurtosis respectively.

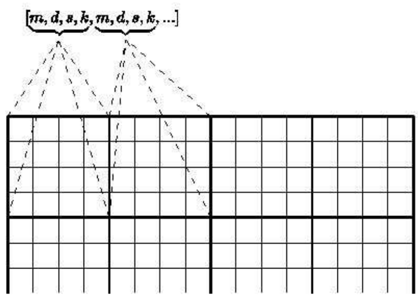

Figure 8: Feature vector forming

4. Experimental Analysis

4.1. Dataset Description

Kylberg texture set consists of 28 classes 160 images each (examples are depicted in Fig. 9). We part it in training and testing sets just randomly selecting 60% of images to be in training and testing set otherwise.

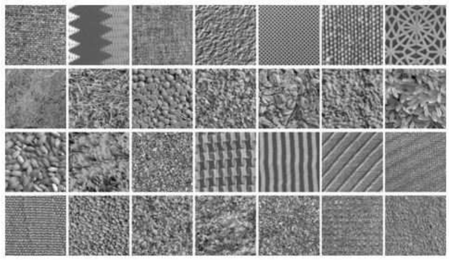

Figure 9: Exemplary samples from Kylberg texture set

4.2. Classification Results

Proposed approach was tested running algorithm number of times using either Chebyshev or Krawtchouk features. In order to justify comparison each run random sampling was performed while different features were extracted from the same samples. Results for each run are shown in the Table 1.

Examples of how features perform on each class are shown in the Fig. 10 for Chebyshev features and Fig. 11 for Krawtchouk features.

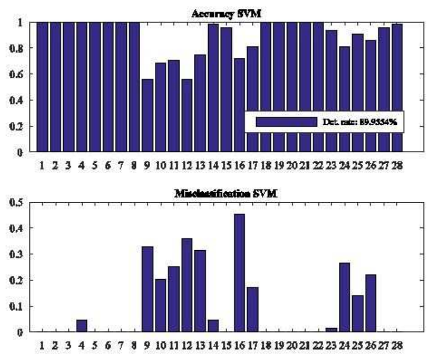

Figure 10: Results of SVM classification using Chebyshev features with 16x16 patch size

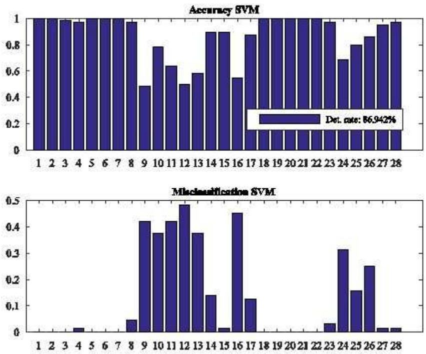

Figure 11: . Results of SVM classification using Karwtchouk features 16x16 patch size

Proposed approach performs better using Chebyshev moments because it has oscillatory structure unlinke Krawtchouk polynomials which are much more localized.

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