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TEMPERATURE FIELD OF A FLUID OVER A STRETCHING SHEET WITH UNIFORM HEAT FLUX BY VARIATIONAL HOMOTOPY PERTURBATION METHOD

B. Vennila
Department of Mathematics
Faculty of Science & Humanities
SRM University
Kattankulathur, 603 203, Tamil Nadu, INDIA

Abstract: In the present paper, the temperature distribution in the flow of a viscous incompressible fluid caused by stretching sheet with uniform heat flux has been analyzed. A new kind of technique which is called the variational Homotopy perturbation method has been employed for finding the solutions. The solutions for velocity and temperature distribution are attained by this method. The series solutions are obtained for the non linear equations caused by temperature field over a stretching sheet and the results are compared with the exact solutions, the accuracy is studied by this variational Homotopy perturbation method.

AMS Subject Classification: 74BXX, 76DXX
Key Words: variational iterative method, prandtl number, similarity transforms, series solution, kinematic viscosity

1. Introduction

The study of heat transfer to a fluid flowing in a channel has applications in technological fields, heat exchanger, reactor cooling etc. All these investigations are restricted to hydrodynamic flow and heat transfer problems, recently

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these problems have become more important to industry. Due to its wide range of applications, the stretching sheet problems have been studied by a number of researchers. Most solutions available are based on numerical techniques such as kellor box method, Runge-Kutta method and finite element method. Duttal et al [1] discussed incomplete gamma function to study the behavior of temperature distribution over a stretching sheet. Noor et al [2] solved the higher dimensional initial boundary value problems by variational Homotopy perturbation method.In Matinbar et al [3] used variational homotopy perturbation method for Fishers equations.There are few investigators who have tried to study the flow of fluid over a stretching sheet and their behavior under different conditions. He [4],[5],[6],[7],[8] and [9] introduced the homotopy perturbation method, which is developed by combining the standard homotopy and perturbation method. In these methods the solution is given in an infinite series usually converging to an accurate solution.Due to numerous industrial processes the boundary layer concept for flow of an incompressible fluid over a stretching sheet is quite popular among the researchers recently. A broad range of analytical and numerical methods have been used in the analysis of these scientific models. An effective method is required to analyze the mathematical model which provides solutions conforming to physical reality.Most of the interest denoted to the heat transfer in Engineering applications is the study of the thermal response of the conduit wall and fluid temperature (Dirichlet problem) and uniform heat flux (Neumann problem). Wazwaz [10] discussed the decomposition method for solving higher dimensional initial boundary value problems of variable coefficients. He [11],[12],[13],[14],[15],[16] and [17] studied homotopy perturbation techniques for some kind of nonlinear problems. Nure Syuhada Ismail et al [18] investigatedthe effect of surface tension gradient and heat transfer in a parallel stream with constant surface heat flux by using stability analysis. Shivaraman et al [19] analyzed Marangoni effects on forced convection of power law fluids in thin film over an unsteady horizontal stretching surface with heat source. Bachok et al [20] discussed the boundary layer flow and heat transfer of a nanofluid over an exponentially shrinking sheet.

The aim of this paper is to investigate the velocity and temperature distribution in the flow of a viscous incompressible fluid caused by stretching sheet and comparing with the exact solutions.

2. Variational Homotopy Perturbation Method (VHPM)

To express the basic idea of the modified variational iteration method, we consider the following general differential equation

$$Lu + Nu = g(x)$$

where L is a linear operator, N is a nonlinear operator,and g(x) the forcing term. According to variational method, we can create a correction functional as follows:

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda(\xi)(Lu_n(\xi) + Nu_n(\xi) - g(\xi))d(\xi)$$

where λ is a Lagrange multiplier, which can be identified optimally by variational iteration method.The subscripts n denote the nth approximation, $\tilde{f}_n$ is considered as a restricted variation. That is, $\delta\tilde{f}_n = 0$. Now, we apply the homotopy perturbation method,

$$\begin{aligned}\textstyle\sum_{n=0}^{\infty} p^n f_n = u_0(x) + p\int_0^x \lambda(\xi)\Big(\sum_{n=0}^{\infty} p^{(n)} L(u_n)\\ + \textstyle\sum_{n=0}^{\infty} p^{(n)} N(u_n)\Big)d\xi - \int_0^x \lambda(\xi)g(\xi)d\xi,\end{aligned}$$

which is the variational homotopy perturbation method (VHPM) and is formulated by the coupling of variational iteration method and Adomian's polynomials. A comparison of like powers of p gives solutions of various orders.

3. Mathematical formulation of the problem

Consider the case of a flat sheet issuing from a thin slit at $x = 0, y = 0$, and subsequently being stretched, as in a polymer processing application. The flow caused by the stretching of this sheet is assumed to be laminar. Assuming boundary layer approximations, the equations of continuity, momentum and heat transfer in the usual notation are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{\sigma}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u and v are the velocity components in the x and y directions respectively, σ is the Prandtl number and ν is the kinematic viscosity subject to the boundary conditions

$$u = \alpha(x), v = 0, -\lambda\frac{\partial T}{\partial y} = A' \text{ at } y = 0, u \to 0, T \to T_\infty \text{ as } y \to \infty \tag{4}$$

Define a stream function

$$\psi = -(\alpha\nu)^{1/2} x f(\eta), \eta = (\alpha/\nu)^{1/2} y \tag{5}$$

$$u = \alpha x f'(\eta), v = -(\alpha\nu)^{1/2} f(\eta) \tag{6}$$

which are consistent with equations (1) and (2).

4. SOLUTION OF THE PROBLEM

Substitution of equation (4), (5) and (6) in (1) and (2) gives

$$f'^2(\eta) - f(\eta)f''(\eta) = f'''(\eta) \tag{7}$$

$$g''(\eta) - \sigma f(\eta)g'(\eta) = 0 \tag{8}$$

Subject to the boundary conditions

$$f(0) = 0, f'(0) = 1, f'(\infty) = 0 \tag{9}$$

$$g'(0) = -1, g(\infty) = 0 \tag{10}$$

In this section, we mainly solve our equation (7) and (8) with boundary conditions given in (9) and (10), by He's variational iterative method. The initial guess for f & g is given below

$$f_0(\eta) = \eta + \frac{\alpha_1\eta^2}{2} \tag{11}$$

$$g(0) = 1 + \alpha_2\eta \tag{12}$$

where $f''(0) = \alpha_1 < 0$ and $g'(0) = \alpha_2 < 0$. To solve (7), (8), (9) & (10), with the help of Variational iterative method, we create a correctional functional which is given by

$$f_{n+1}(\eta) = f_n(\eta) + \int_0^\eta \lambda_1(\xi)\left(\frac{\partial^3 f_n(\xi)}{\partial\xi^3} - (\tilde{f}'_n(\xi))^2 + \tilde{f}_n(\xi)\tilde{f}''_n(\xi)\right)d\xi \tag{13}$$

$$g_{n+1}(\eta) = g_n(\eta) + \int_0^\eta \lambda_2(\xi)\left(\frac{\partial^2 g_n(\xi)}{\partial\xi^2} + \sigma\tilde{f}_n(\xi)\frac{\partial\tilde{g}_n(\xi)}{\partial\xi}\right)d\xi \tag{14}$$

Making the correction functional stationary, the Lagrange multipliers can easily be identified

$$\lambda_1 = -\frac{1}{2}(\xi-\eta)^2,\ \lambda_2 = (\xi-\eta) \tag{15}$$

Consequently,

$$f_{n+1}(\eta) = f_n(\eta) - \frac{1}{2}\int_0^\eta (\xi-\eta)^2\left(\frac{\partial^3 f_n(\xi)}{\partial\xi^3} - (\tilde{f}'_n(\xi))^2 + \tilde{f}_n(\xi)\tilde{f}''_n(\xi)\right)d\xi \tag{16}$$

$$g_{n+1}(\eta) = g_n(\eta) + \int_0^\eta (\xi-\eta)\left(\frac{\partial^2 g_n(\xi)}{\partial\xi^2} + \sigma\tilde{f}_n(\xi)\frac{\partial\tilde{g}_n(\xi)}{\partial\xi}\right)d\xi \tag{17}$$

Applying the variational homotopy perturbation method (VHPM), we get

$$\begin{aligned} f_0 + pf_1\cdots &= f_0(\eta) - \frac{p}{2}\int_0^\eta (\xi-\eta)^2\left(\frac{\partial^3 f_0}{\partial\xi^3} + p\frac{\partial^3 f_1}{\partial\xi^3}\cdots\right) \\ &- \left(\frac{\partial f_0}{\partial\xi} + p\frac{\partial f_1}{\partial\xi} + \cdots\right)^2 \\ &+ (f_0 + pf_1 + \cdots)\left(\frac{\partial^2 f_0}{\partial\xi^2} + p\frac{\partial^2 f_1}{\partial\xi^2} + \cdots\right)d\xi \end{aligned} \tag{18}$$

$$\begin{aligned} g_0 + pg_1 + \cdots = g_0(\eta) + p\int_0^\eta (\xi-\eta)\Big(\Big(\frac{\partial^2 g_0(\xi)}{\partial\xi^2} + p\frac{\partial^2 g_1(\xi)}{\partial\xi^2} + \cdots\Big) \\ + \sigma(f_0(\xi) + pf_1(\xi) + \cdots)\Big(\frac{\partial g_0(\xi)}{\partial\xi} + p\frac{\partial g_1(\xi)}{\partial\xi} + \cdots\Big)\Big)d\xi \end{aligned} \tag{19}$$

Comparing the coefficient of like powers of p, we get

$$p^{(0)} = f_0(\eta) = \eta + \frac{\alpha_1\eta^2}{2} \tag{20}$$

$$p^{(1)} = f_1(\eta) = \eta + \frac{\alpha\eta^2}{2} + \frac{\eta^3}{6} + \frac{\alpha\eta^4}{24} + \frac{\alpha^2\eta^5}{120} \tag{21}$$

$$\begin{aligned} p^{(2)} &= f_2(\eta) = \eta + \frac{\alpha_1\eta^2}{2} + \frac{\eta^3}{6} + \frac{\alpha_1\eta^4}{24} + \frac{\alpha_1^2\eta^5}{120} + \frac{\alpha_1\eta^6}{720} + \frac{\alpha_1^2\eta^7}{5040} \\ &+ \frac{\alpha_1^3\eta^8}{40320} + \frac{\alpha_1^2\eta^9}{362880} + \frac{\alpha_1^3\eta^{10}}{3628800} + \frac{\alpha_1^4\eta^{11}}{39916800} + \cdots \end{aligned} \tag{22}$$

The series solution is given by

$$f(\eta) = \lim_{n\to\infty} f_n \tag{23}$$

$$\begin{aligned} f(\eta) &= \eta + \frac{\alpha_1\eta^2}{2} + \frac{\eta^3}{6} + \frac{\alpha_1\eta^4}{24} + \frac{\alpha_1^2\eta^5}{120} + \frac{\alpha_1\eta^6}{720} + \frac{\alpha_1^2\eta^7}{5040} + \frac{\alpha_1^3\eta^8}{40320} \\ &+ \frac{\alpha_1^2\eta^9}{362880} + \frac{\alpha_1^3\eta^{10}}{3628800} + \frac{\alpha_1^4\eta^{11}}{39916800} + \cdots \end{aligned} \tag{24}$$

$$p^{(0)} = g_0 = 1 + \alpha_2\eta \tag{25}$$

$$p^{(1)} = g_1 = 1 + \alpha_2\eta - \frac{\sigma\alpha_2\eta^3}{6} - \frac{\sigma\alpha_1\alpha_2\eta^4}{24} \tag{26}$$

The series solution is given by

$$g(\eta) = \lim_{n\to\infty} g_n \tag{27}$$

$$\begin{aligned} g(\eta) &= 1 + \alpha_2\eta - \frac{\sigma\alpha_2\eta^3}{6} - \frac{\sigma\alpha_1\alpha_2\eta^4}{24} - \frac{\sigma\alpha_2\eta^5}{120} + \frac{\sigma^2\alpha_2\eta^6}{240} \\ &- \frac{\sigma\alpha_1\alpha_2\eta^6}{720} - \frac{\sigma^2\alpha_1\alpha_2\eta^6}{72} + \frac{\sigma\alpha_1^2\alpha_2\eta^7}{504} - \frac{\sigma\alpha_1^2\alpha_2\eta^7}{5040} \end{aligned} \tag{28}$$

5. Results and Discussion

Table 1: The comparison results for velocity of the VHPM with the exact solution

Exact solution		VHPM	
η	$f(\eta)$	η	$f(\eta)$
0.1	0.0952	0.1	0.0997
0.2	0.1813	0.2	0.1993
0.3	0.2592	0.3	0.3000
0.4	0.3297	0.4	0.4026
0.5	0.3935	0.5	0.5081
0.6	0.4512	0.6	0.6175

From the table 1 it can be seen that present solution method VHPM results are better than the exact solution results. Therefore by observing the results obtained by VHPM and exact solution method we found that the series solution obtained byVHPM converges faster than the exact solution in the studied case.

Table 2: The comparison resultsfor heat flux of the VHPM with the exact solution

Exact solution		VHPM	
η	$g(\eta)$	η	$g(\eta)$
0.1	-0.9952	0.1	-0.9952
0.2	-0.9815	0.2	-0.9814
0.3	-0.9600	0.3	-0.9601
0.4	-0.9321	0.4	-0.9330
0.5	-0.8990	0.5	-0.9023
0.6	-0.8617	0.6	-0.8711

Table 2 shows that the present solution method VHPM results coincide with exact solution results for temperature distribution. Numerical results tell that the VHPMis easy to apply and reduces the computational work .we find that agreement is perfectly good.

6. Conclusion

From the above discussion, we arrive at the conclusions that the variational homotopy perturbation method has been successfully applied to offer the series solution of the boundary layer equations of the two dimensional flow over a stretching sheet with uniform heat flux and the results obtained for velocity and temperature distribution were compared with exact solutions.It can be concluded that the solutions are very close to exact solution and perfect by variational homotopy perturbation method (VHPM).

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