International Journal of Pure and Applied Mathematics
Volume 119 No. 2 2018, 309-320
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v119i2.4
ijpam.eu

COMPUTER MODELING OF MOVEMENT AND DERAILMENT OF THE TRAIN CAR WHEELSET

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Abstract: Ensuring safety for train movement and switching are the priorities for rail transport, because crashes and accidents caused by derailment are not yet possible to be eliminated completely. Therefore the theme is undoubtedly relevant. The main goal achieved by the authors is concluded in development of computer modeling of wage wheels movement and derailment. It was defined that out of two wheels connected by rigid axis, the one starts slipping where the moment of friction at skidding is less. In order to achieve the set goal the authors used the methods of analysis, modeling and comparison. The authors found that the obtained algorithms for calculation of the reactions between the wheels and rails at draught and breaking will be the same.

AMS Subject Classification: 68M01, 68M20
Key Words: wheel rolling, wheel slipping, traction effort, braking, breakaway

1. Introduction

The most frequent violations of safety causing derailment, especially at rolling

Received: April 11, 2018
Revised: June 20, 2018
Published: July 4, 2018

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of the wheel flange onto the top of rail, take place at curved sections with the presence of deviations at maintenance of the driving trains (abrading) and upper rail structure (inequalities of rail strings). Estimation of the operation safety requires an elaborate computer model, which will describe the physical processes taking place during the wage wheel rail interaction at derailment in the most precise way. Such a computer model allows not only precisely estimating the influence of the driving trains abradings at moving with various velocities through the inequalities of the rail strings as the deviations of the rail maintenance standards, but also choosing optimal correlation of their characteristics (rail gage, wheel profile, and permitted deviations in maintenance). Creation of such a computer model requires the idea of the dynamic force interaction at the movement and derailment considering the rolling, slipping and sliding of the wheels along the side edges.

Analysis of the research review [1, 2, 3, 4, 5] on the modeling of the wheel wage movement showed that the majority of the papers use the theory of reep (elastic sliding or slipping) according to Carter (1916), Mller (1960) or Kalker (1967) with various elaborations of the movement and contact equations according to Hertz (1882), Possehl, Butfua and Matsudaira (1960), Cooperider (1962), De Pater (1963), Van Bommel (1964), Wickens (1965), Knot and Hoon (1985), Shiller (1990), Pascal and Sauvage (1991), True (1994), Kick and Piotrovsky (1996), Ayass, Maupu and Sholle (2000), Schupp (2003), Pollack (2003) and the Russian engineers (since 1880 till present), as well as such software as CONTACT, USETAB, PATH, NedTrain, FASTSIM and complexes VOCO, VOCOLIN, VOCODYN, MEDYNA, VAMPIRE, NUCARS, GENSYS, SIMPACK, ADAMS/Real, UM (Universal Mechanism. Belarusian State Technological University), DIONIS (DIONiS – Dynamics, optimization, loading and statics. Moscow State University of Railway Engineering).

2. Characteristics of N. Joukowsky Principle

However, at the mathematical description of the wage wheel and track force interactions the old principle of N.Joukowsky is often forgotten, showing which of the wheels in the wage wheel slips or slides and under what conditions and where there is no sliding. At the same time in the process of numerical integration in time of the differential equations the effects and situations arise, not reflecting explicitly the physics of the mechanical process, e.g. such as movement with zero velocity and wheel unloading, as well as with the stagger, lift and shift in the curved track etc.

According to Professor N.Joukowsky [3] the principle of slipping wheels definition in the wage wheels is concluded in finding the correlations between the moments of slipping frictions. Out of two wheels, connected by the axis, the one starts slipping where the moment of friction at skidding is less. Force connections of the wage wheels with the rails are defined by the conditions of their mutual contacting combined with the processes of rolling and slipping of the wheels, as well as sliding of the wheels along the side edges.

Definition of Conditions for the Contact of the Wheel Flanges with the Side Edge. As known, the conditions for contacting of the wheel edges and rails are defined by the shape of the flange and rail surface and the angle of attack. The angle of attack is the angle that forms the wage wheels axis in each point of the track with the normal to the position of the wheel face surface (in curved track sections the normal to the lateral surface of the exterior rail). At the same time the normal to the lateral surface of the rail should be in the rail section passing the middle of the wheel flange contact area with the lateral edge of the rail. In the ideal circular curve the normal is the radius of the curve. At the inequalities of the track in the plan it is necessary to define the normal position at each point of the inequality every time. As an example, let us consider two peculiar types of track inequalities in the plans of the curved track sections: angle in conjunction (equal to $2\phi_c$) and smooth inequality, described by the following equation (Figure 1):

$$\eta\left(S_i\right)=\frac{a}{2}(1-\cos\frac{2\pi S_i}{L_i}), \tag{1}$$

where S_i is current absciss of inequality measured from the beginning of inequality through the arc of the circular curve; L_i is length of inequality; a is maximum value of the inequality vertical axis (along the exterior rail of circular curve).

If (see Figure 1) to accept that the angle formed by the normal to the circular curve in A point of the inequality beginning is equal to zero, while the angle of wabbling of the wage wheels in relation to the normal in this point (i.e. to the radius of the circular curve) as equal to ϕ_{wab}, then the angle of attack at the beginning of smooth inequality $\phi_{clim}=\phi_{wab}$ (in moving axes). At further movement along the inequality from A point, the angle formed by the tangent to the inequality will be equal to

$$\frac{d\eta_i}{dS_i}=\frac{a\pi}{L_i}\sin\frac{2\pi S_i}{L_i}. \tag{2}$$

This angle as seen from Figure 1 is equal to the angle between the normal in this point and the circular curve radius. At some arbitrary at the inequality in B point the angle formed by the radius some drawn to this point and axis Y is equal to the following:

$$\phi_w = \frac{S_i(B)}{R} \tag{3}$$

Thus the angle formed by the normal in some B point of the inequality with the Y axis will be as follows:

$$\phi_w = {}^{S_i(B)}\!/_{R} + {}^{d\eta_i}\!/_{dS_i} \tag{4}$$

While the angle of attack of the wheels in each point of this inequality will be as follows:

$$\psi_{c\lim} = \phi_{wab} + \phi_b. \tag{5}$$

For "angle in conjunction" type inequality of the track (with the angle of curve $2\phi_c$ – see Figure1) let us define the position of the normal within the inequality in relation to which the climbing angles are set. If the initial angle of normal to Y axis in the beginning of the inequality (Figure 1) is equal to ϕ_i, then within the straight section from A to this angle of normal in absolute coordinate system is preserved. Within the section from to the normal declination angle in the absolute coordinate system will be equal to $\phi_i + 2\phi_c$. While in the moving axes considering the influence angle of wage wheels in relation to the radiuses of the circular curve, the angle of attack will be within the A section $\psi_{clim} = \phi_{wab} + \phi_A$, while within the section:

$$\psi_{c\lim} = \phi_{wab} + \phi_A + 2\phi_c. \tag{6}$$

3. Definition of Rolling and Slipping of the Wheels in Motion

Knowing the angle of attack using the formula of P.G. Koziychuk [4] one may define the deflection of the flange contact with the top of rail - b, usually called "climbing of the wheel". Value "b" is as follows:

$$b = (r + t)\, tg\tau \cdot tg\phi_{c\lim}, \tag{7}$$

where r is radius of the wheel; t is the distance along the vertical from the top of rail to the point of pressing of the flange to the top of wheel; τ is the deflection angle of the cone part of flange to the horizon; ϕ_{clim} is the angle of attack.

Rolling and Slipping of the Wheels at the Wage Wheels Movement along the Rails in the Curves at Coasting Retardation. While moving of the wage

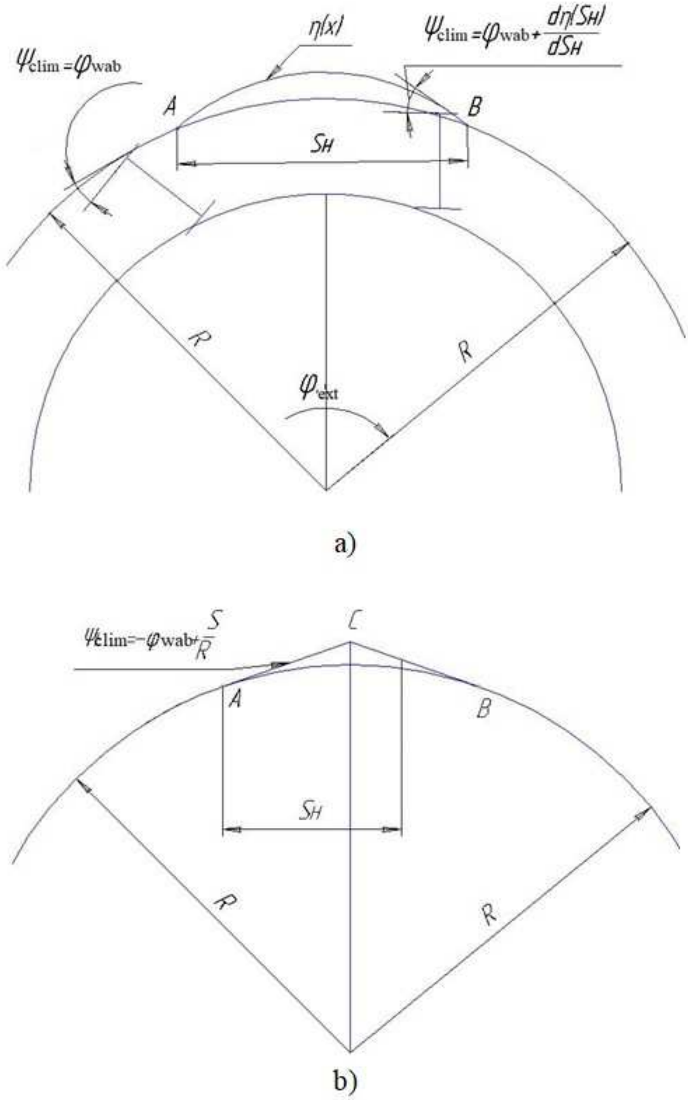

Figure 1: Scheme to the Definition of the Angle of Wage Wheels Climbing on the Isolated Inequalities in the Plan along the External Rail of Circular Curve:) on Smooth Surface; b) on Conjunction ("Angle") Inequality in the Rail Conjunction.

wheels along the rails with the wheels rigidly connected between each other by the axis, at each point of time there is inevitable slipping of some wheels wage through the length and breadth of the wheels. Especially significant are such slippings of the wheels while rolling stock moving along curved sections with small radius. The reasons of such "slipping" were considered by N.Joukowsky [3]; he also pointed at the method providing an opportunity to solve the set task of which wheel slips and at which conditions.

In case of movement of the rolling stock at the "coasting retardation" (i.e. without draught or braking moment of the forces at its wheels) it is possible to define which wheels of the wage wheels slips at this step of integration in time, while which one rolls without sliding, as shown by N.Joukowsky [3], by the fact which of the sliding frictional force moments correlation M takes place at the wheel of the wage wheels.

4. Definition of Possible Correlation Values

Let us consider i-wheels moving along the exterior rail of the curve, while $(i+1)$-wheels – along the internal one. The following three types of such correlations are possible [5]:

$$M_i > M_{i+1};\ M_i = M_{i+1};\ M_i < M_{i+1}. \tag{8}$$

Let's consider these possible values of correlations one by one:

1) $M_i > M_{i+1}$;

Out of two wheels connected by rigid axis, the one starts slipping where the moment of friction at skidding is less. If the loads at the i and $i+1$ wheels will be equal to $_{ki}$ and $_{ki+1}$, their radiuses will be equal to r_i and r_{i+1}, while the coefficients of their sliding frictions along the rails are equal to μ, then the moments of the sliding frictional forces for these wheels along the rails will be thus the same.

So, if, e.g. $M_i > M_{i+1}$, then $(i+1)$ wheel starts slipping. While if $M_{i+1} > M_i$, then i-wheel will slip. However, it does not mean that in these cases some wheel will be skidding, it will just slip.

According to the principle of N.Joukowsky [3], let us define which wheel of the wage wheels will slip. In case depicted at Figure 1, except for the vertical load $_k$, the wheels may be also influenced by directing forces Y_i with the "climbing" equal to b_i. Thus, the moment of all the forces retaining the i-wheel from slipping along the exterior rail is equal to [5]:

$$M_i = P_{ki}\mu_{sl}r_i + Y_n\mu_{fl}b_i + P_{ki}f_k, \tag{9}$$

where, b_i – wheel flange climbing on the rail; μ_{sl} coefficient of sliding friction of the wheel along the rail in its rolling plain; f_k – coefficient of sliding friction of the wheel along the rail; μ_{fl} – coefficient of sliding friction of the wheel flange along the side edge.

At the $(i+1)$-wheels of the same wage wheels it is equal as follows [5]:

$$M_{in} = P_{ki+1}\mu_{sl}r_{in} + Y_{ni+1}\mu_{fl}b_{i+1} + P_{ki+1}f_k, \tag{10}$$

(3). If $M_i > M_{i+1}$,

Then the $(i+1)$- wheel slips, while $i2$- wheel rolls without sliding, and the number of the wage wheels turns at rolling of the wheel at the small section of A arc at the $d\alpha$ angle will be as follows:

$$n_{turn} = \frac{(\rho + S_1)\, d\alpha}{2\pi r_i}, \tag{11}$$

where ρ – radius of the curved track section; S_1 – half of the gage. Provided that $(i+1)$-wheel rolls over to the total length of track:

$$S_{i+1} = 2\pi r_{i+1} * n_{ob} = 2\pi r_{i+1}\frac{(p + S_1)d\alpha}{2\pi r_i} = r_{i+1}\frac{(p + S_1)d\alpha}{r_i} \tag{12}$$

As the length of the section $CD = (\rho + S_1)d\alpha$ (Figure 2) is more than S_{i+1}, then $(i+1)$-wheel will slide backwards (clockwise) at this section with the following length:

$$\Delta W_{i+1} = (\rho - S_1)d\alpha - \frac{r_{i+1}}{r_1}(\rho + S_i)d\alpha \tag{13}$$

Relative sliding of the $(i+1)$-wheel at the D section is as follows:

$$\xi_{i+1} = \frac{\Delta W}{CD} = \frac{1}{(\rho - S_i)d\alpha}\{(\rho - S_i)d\alpha - \frac{r_{i+1}}{r_i}(\rho + S_i)d\alpha\} \tag{14}$$

thus

$$\xi_{i+1} = \left\{\frac{\rho + S_i}{\rho - S_i} * \frac{r_{i+1}}{r_i} - 1\right\} \tag{15}$$

$$\xi_i = 0.$$

2) The case when $M_i < M_{i+1}$;

At the same time, the i-wheel slips, while the $(i'+1)$-wheel does not slip. Likewise, we will obtain the formula for the calculation in relation to the i-wheel:

$$\xi_i = \frac{\Delta W_i}{AB} = 1 - \frac{\rho - S_i}{\rho + S_i} * \frac{r_i}{r_{i+1}}; \tag{16}$$

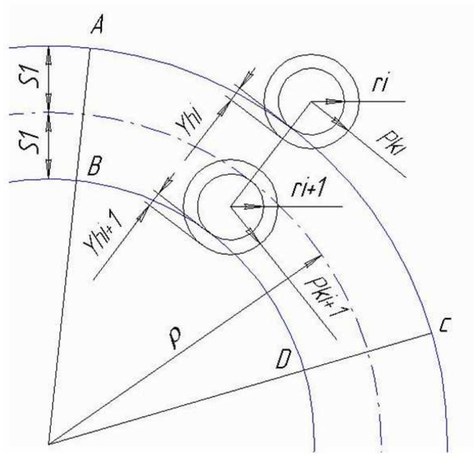

Figure 2: The Scheme to the Calculation of Slipping Wheels out of the Wage Wheels Towards the External and Internal Rails in the Curved Section of the Track

In this case $\xi_{i+1} = 0$.

3) The case when $M_i = M_{i+1}$ (within the calculation accuracy). It is quite obvious that in this case $\xi_i = \xi_{i+1} = 0$.

The calculated values of relative slidings will take place in the wheel rolling plain. Let us call them modal values of relative longitudinal frictional sliding of the wheels and denote as ξ_{0imod} and $\xi_{0i+1mod}$. However, due to the effect of horizontal transverse forces on the wage wheels, they may also slip perpendicular to their rolling plain, i.e. in the direction parallel to the wage wheels axis. It is obvious that according to the common transverse shifts of the wheel rolling surface in relation to the rail, one may also define relative transverse frictional sliding of the wheel along the rail: $\xi = \frac{d\Delta}{Vdt}$, (16), where V is the velocity of the wage wheels movement along the track. Knowing that, ξ_{0imod} and $\xi_{0i+1mod}$ find the corresponding interaction forces between the wheels and rails. Let us define the modal forces of resistance to the longitudal and transverse shifts of the wheels along the rails

$$F_{o\;\mathrm{mod}\;i} = \frac{M_i}{r_i}$$

$$F_{o\;\mathrm{mod}\;i+1} = \frac{M_{i+1}}{r_{i+1}} + P_{ki+1}\;\mathrm{sgn}\;a\xi_{o\;\mathrm{mod}\;i+1} \tag{17}$$

$$F_{n\;\mathrm{mod}\;i} = P_{ki}\mu_{non}\;\mathrm{sgn}\;a\xi_{n\;\mathrm{mod}\;i}$$

$$F_{n\ \mathrm{mod}\ i+1} = P_{ki+1}\mu_{non}\ \mathrm{sgn}\ a\xi_{n\ \mathrm{mod}\ i}$$

Here, M_i and M_{i+1} are earlier defined moments of frictional forces, r_i and r_{i+1} are radiuses of the wheels per each time moment in the points of their contact with the top of rail, P_{ki} and P_{k+1i} are current values of vertical loads of the wheels on the rails, while sgn $a(\xi_{0imod})$ and sgn$a(\xi_{0i+1mod})$ are the function of the creeping forces for elastic sliding of the wheels the length and the breadth of the rails. Using the values of the force vectors modules F_{0mod} and F_{nmod}, let us calculate the values of their projections for the X and Y axes:

$$\left.\begin{aligned} F_{xi} &= F_{0\ \mathrm{mod}\ i} * \cos\psi_i + F_{n0\ \mathrm{mod}\ i}\sin\psi_i \\ F_{xi+1} &= F_{0\ \mathrm{mod}\ i+1} * \cos\psi_{i+1} + F_{n\ \mathrm{mod}\ i+1}\sin\psi_{i+1} \\ F_{yi} &= F_{0\ \mathrm{mod}\ i} * \sin\psi_i + F_{n\ \mathrm{mod}\ i}\cos\psi_i \\ F_{yi+1} &= F_{0\ \mathrm{mod}\ i} * \sin\psi_{i+1} + F_{n\ \mathrm{mod}\ i+1}\cos\psi_{i+1} \end{aligned}\right\} \tag{18}$$

Values F and F are further used in the initial system of differential equations, describing the movement of each wage wheels of the rolling stock [5].

The 2nd case, when $M_i = M_{i+1}$ (within the calculation accuracy),

$$\xi_{0imod} = \xi_{0i+1mod} = 0 \tag{19}$$

The 3rd case, when $M_i < M_{i+1}$, one wheel rolls without friction sliding along the internal rail of the curve, while the second wheel, which moves along the external rail, slips along the rail. It is as follows

$$\xi_{0\ \mathrm{mod}\ i+1} = 0; \xi_{0\ \mathrm{mod}\ i} = 1 - \frac{\rho - S_i}{p + S_i} * \frac{r_i}{r_{i+1}}; \xi_{non\ \mathrm{mod}} = \frac{d\Delta_{pki}}{V dt}. \tag{20}$$

Modal values of the friction forces of the wheel movement along the rails are defined according to the following formulas:

$$F_{0\ \mathrm{mod}\ i+1} = \frac{M_{i+1}}{r_{i+1}}; F_{0\ \mathrm{mod}\ i} = \frac{M_i}{r_i} + P_{ki}\mu_{np}\ \mathrm{sgn}\ a\xi_{0\ \mathrm{mod}};$$

$$F_{n\ \mathrm{mod}\ i} = P_{ki}\mu_{non}\ \mathrm{sgn}\ a\xi_{non\ \mathrm{mod}\ i}; \tag{21}$$

$$F_{non\ \mathrm{mod}\ i+1} = P_{ki+1}\mu_{non}\ \mathrm{sgn}\ a\xi_{non\ \mathrm{mod}\ i+1}.$$

The components of the friction forces of the wheels according to the rails by the and Y axes are calculated the same way.

5. Peculiarities of the Rolling and Slipping Calculation at the Wage Wheels Movement in the Curves at their Braking

Rolling and slipping of the Wheels at the Wage Wheels Movement at their Braking. In the draught and braking modes, the moments of external forces are applied to the wheels of rolling stocks. While at braking – of the friction forces of the brake-blocks along the surface of the wheel M_{br} rolling.

In these cases the maximum possible total of all the moments of the rolling resistance forces of the wheels may be represented as follows:

$$\begin{aligned} M_{ires} &= P_{ki} f_k + Y_{Hi} b_i \mu_{rp} + M_{br}, \\ M_{i+1res} &= P_{ki+1} + Y_{Hi+1} b_{i+1} \mu_{rp} + M_{br}. \end{aligned} \tag{22}$$

By using this formula, one may define which wheel of the wage wheels, i or $(i + 1)$ rolls and which of them slides or breaks into sliding per each point of time t. For this purpose the values of the moments of forces are defined $\bar{M}_i = P_{ki}\psi_{ad} - M_{ires}$ and $\bar{M}_{i+1} = P_{ki+1}\psi_{ad} - M_{i+1res}$, defining the possibility of the wheel friction sliding. Here ψ_{ad} is coefficient of adhesion.

It is quite obvious that if M_i and M_{i+1} are below zero, then the wheels of these wage wheels either sliding in this point of time or skidding along the rails.

While in case $M_i < 0$, and $M_{i+1} > 0$, at his point of time i-wheel slips, while $(i + 1)$ one rolls without sliding. However, this mode of movement is “transitional”, because the breakage of adhesion at one wheel of the wage wheels causes the breakage of adhesion at the other wheel of the same wage wheels.

But if $M_i > 0$ and $M_{i+1} > 0$, then one of the wheels may roll at this moment and another will slide with friction.

6. Calculation of Possible Variants of Rolling Stock Movement with Service Braking

Below the exact case is considered, when rolling stock is moving with the service braking for maintaining of some constant speed of the movement (e.g., at the long descent section) provided that condition $M_i > 0$ and $M_{i+1} > 0$ is observed.

Therewith the following three variants of the movement are possible:

) First case $M_{i+1} < M_i$;

Notably, i-(uneven) wheels move along the external, while $(i + 1)$-(even) wheels move along the internal rail of the curve. It is quite obvious that the wheel will slip having less limit moment of friction sliding, i.e. in this case it is the (i+1)-wheel. As it was shown above, in this case

$$\xi_{0\ \mathrm{mod}\ i} = 0; \xi_{0\ \mathrm{mod}\ i+1} = \frac{\rho + S_i}{\rho - S_i} * \frac{r_{i+1}}{r_i} - 1; \xi_{non\ \mathrm{mod}} = \frac{d\Delta_{pk}}{V dt}. \quad (23)$$

For further calculations the following common formulas are used in this case:

$$\begin{aligned} &F_{0\ \mathrm{mod}\ i+1} = \frac{M_{i+1}}{r_{i+1}}; F_{0\ \mathrm{mod}\ i} = \frac{M_i}{r_i} + P_{ki}\mu_{np}\ \mathrm{sgn}\ a\xi_{0\ \mathrm{mod}\ i};\\ &F_{n\ \mathrm{mod}\ i} = P_{ki}\mu_{non}\ \mathrm{sgn}\ a\xi_{non\ \mathrm{mod}\ i};\\ &F_{non\ \mathrm{mod}\ i+1} = P_{ki+1}\mu_{non}\ \mathrm{sgn}\ a\xi_{non\ \mathrm{mod}\ i+1}.\end{aligned} \quad (24)$$

$$\left.\begin{aligned} &F_{xi} = F_{0\ \mathrm{mod}\ i}\cos\psi_i + F_{n0\ \mathrm{mod}\ i}\sin\psi_i,\\ &F_{xi+1} = F_{0\ \mathrm{mod}\ i+1} * \cos\psi_{i+1} F_{n\ \mathrm{mod}\ i+1}\sin\psi_{i+1},\\ &F_{yi} = F_{0\ \mathrm{mod}\ i}\sin\psi_i + F_{n\ \mathrm{mod}}\ \cos\psi_i,\\ &F_{yi+1} = F_{0\ \mathrm{mod}\ i+1}\sin\psi_{i+1} + F_{n\ \mathrm{mod}\ i+1}\cos\psi_{i+1}\end{aligned}\right\} \quad (25)$$

b) The second case $M_{i+1} = M_i$

Here, if M_{i+1}, within the calculation accuracy it is equal to M_0, then $\xi = \frac{d\Delta_{kp}}{V dt}$, and the same formulas are used for F_x and F_y, suggesting that $\xi_i = \xi_{i+1} = 0$.

c) The third case.

Here the formulas of p.2 are used.

4. Rolling and Slipping of the Wheels at Movement of the Wage Wheels in Curves in the Draught Mode [5]. Let us denote the draught moment of forces at the axe of wage wheels as $2M_{dr}$, i.e. let us consider the total of the draught moments at the wheels M_{idr} and M_{i+1dr} to be equal to $2Mdr$. We need to define $\bar{M}(|M_{dr}| - M_{res}) - P_k\psi_{ad}$ for each wheel of the wage wheels and based on it to state the wheel with greater value of this moment of forces. The friction sliding will take place at the wheel out of the wage wheels, where M will be more. At the same time, no wheel should have negative M, because it will cause wheel skidding.

If $M_{i+1} < M_i$, then the wheel out of the wage wheels will slide with less difference between the efficient draught force $(|M_{dr}| - M_{res})$ and $P_k\psi_{ad}$, i.e. in this case it is the $(i+1)$-wheel.

7. Conclusion

Thus, the obtained calculation algorithms of the reactions between the wheels and rails at draught and braking are the same. At the definition of reaction between the wheels and rails at draught on the rolling stock wheels one may apply the same formulas used in p.2 and cases 1, 2 and 3.

The presented methods of force interaction computation are applied at the Rolling Stock Economy Department of the Moscow State University of Railway Engineering in the DIONiS software. This software is applied for conduction of research and experiments with the purpose of multivariant quantitative and qualitative estimate of dynamic qualities and safety of the rolling stock movement as part of cargo trains in maintenance of driving trains track structure. The software allows precisely predicting real derailments, dynamic uploading, stress and strain condition, reliability, abradings, run life of every interacting detail and the driving train node, draw-and-buffer gears and automotive brakes.

The suggested solutions open new opportunities for more objective and precise estimate in definition of the wheel and the point of derailment at investigation of the violations in the safety of operation, accidents and crashes, as well as to reveal technical causes and people responsible for derailment in a more precise way.

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