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SOME DETERMINISTIC GROWTH CURVES WITH APPLICATIONS TO SOFTWARE RELIABILITY ANALYSIS

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Abstract: The Hausdorff approximation of the shifted Heaviside function $h_{t_0}(t)$ by sigmoidal functions based on the Pham [1] and Song–Chang–Pham [2] cumulative functions is investigated and an expression for the error of the best approximation is obtained in this paper.

The results of numerical examples confirm theoretical conclusions and they are obtained using programming environment Mathematica.

We give real examples with data provided by IBM entry software package [3] using Song–Chang–Pham [2] software reliability model.

AMS Subject Classification: 68N30, 41A46

Key Words: deterministic Pham's model, Song–Chang–Pham model, shifted Heaviside function $h_{t_0}(t)$, Hausdorff approximation, upper and lower bounds

1. Introduction

In this article we study the Hausdorff approximation of the shifted Heaviside function $h_{t_0}(t)$ by sigmoidal functions based on the Pham [1] and Song–Chang–Pham [2] cumulative functions.

We give a software modules within the programming environment CAS Mathematica for illustrating the results.

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Definition 1. Pham [1] developed the following deterministic software reliability model:

$$M(t) = N\left(1 - \left(\frac{\beta}{\beta - 1 + a^{t^b}}\right)^{\alpha}\right). \tag{1}$$

Definition 2. Song, Chang and Pham [2] developed the following new software reliability model:

$$M_1(t) = N\left(1 - \left(\frac{\beta}{\beta + at - \ln(1 + at)}\right)^{\alpha}\right). \tag{2}$$

Definition 3. The shifted Heaviside function is defined as:

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0 \\ 1, & \text{if} \quad t > t_0 \end{cases} \tag{3}$$

We will note that the determination of compulsory in area of the Software Reliability Theory components, such as confidence intervals and confidence bounds, should also be accompanied by a serious analysis of the value of the best Hausdorff approximation of the function $h_{t_0}(t)$ by cumulative functions of type (1)–(2) - the subject of study in the present paper.

Definition 4. [5] The Hausdorff distance (the H–distance) $\rho(f,g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f,g) = \max\{\sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|,|x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A - B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

2. Main Results

2.1. A Note on the Pham's Model (1) [1]

Without loosing of generality we will look at the following "cumulative sigmoid":

$$M^*(t) = 1 - \left(\frac{\beta}{\beta - 1 + a^{t^b}}\right)^{\alpha}, \tag{4}$$

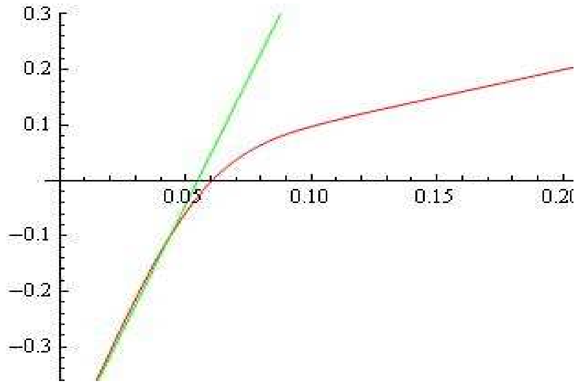

Figure 1: The functions $F(d)$ and $G(d)$.

with $N = 1$ (see (1)), and

$$t_0 = \left(\frac{\ln\left(1 - \beta\left(1 - \frac{1}{\left(\frac{1}{2}\right)^{\frac{1}{\alpha}}}\right)\right)}{\ln a} \right)^{\frac{1}{b}} ; \quad M^*(t_0) = \frac{1}{2}. \tag{5}$$

The one–sided Hausdorff distance d between the function $h_{t_0}(t)$ and the sigmoid ((4)–(5)) satisfies the relation

$$M^*(t_0 + d) = 1 - d. \tag{6}$$

The following theorem gives upper and lower bounds for d

Theorem. *Let*

$$p = -\frac{1}{2},$$

$q = 1+$

$$\frac{b\alpha \ln a}{\beta}\left(1 - \beta\left(1 - \frac{1}{\left(\frac{1}{2}\right)^{\frac{1}{\alpha}}}\right)\right)\left(\frac{\ln\left(1 - \beta\left(1 - \frac{1}{\left(\frac{1}{2}\right)^{\frac{1}{\alpha}}}\right)\right)}{\ln a} \right)^{\frac{b-1}{b}} \left(\frac{1}{2}\right)^{\frac{\alpha+1}{\alpha}}.$$

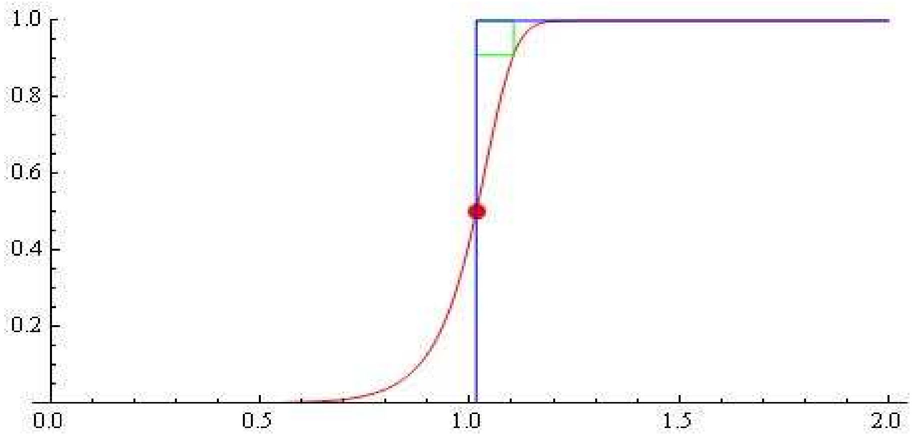

Figure 2: The model ((4)–(5)) for $\beta = 4$, $\alpha = 0.8$, $a = 5$, $b = 10$, $t_0 = 1.01533$; $H - -distance$d=0.0895807, $d_l = 0.0786509$.

For the one–sided Hausdorff distance d between $h_{t_0}(t)$ and the sigmoid ((4)–(5)) the following inequalities hold for:

$$2.1q > e^{1.05}$$

$$d_l = \frac{1}{2.1q} < d < \frac{\ln(2.1q)}{2.1q} = d_r. \tag{7}$$

Proof. Let us examine the function:

$$F(d) = M^*(t_0 + d) - 1 + d. \tag{8}$$

From $F'(d) > 0$ we conclude that function F is increasing.

Consider the function

$$G(d) = p + qd. \tag{9}$$

From Taylor expansion we obtain $G(d) - F(d) = O(d^2)$.

Hence $G(d)$ approximates $F(d)$ with $d \to 0$ as $O(d^2)$ (see Fig. 1).

In addition $G'(d) > 0$.

Further, for $2.1q > e^{1.05}$ we have $G(d_l) < 0$ and $G(d_r) > 0$.

This completes the proof of the theorem.

The model ((4)–(5)) for $\beta = 4$, $\alpha = 0.8$, $a = 5$, $b = 10$, $t_0 = 1.01533$ is visualized on Fig. 2.

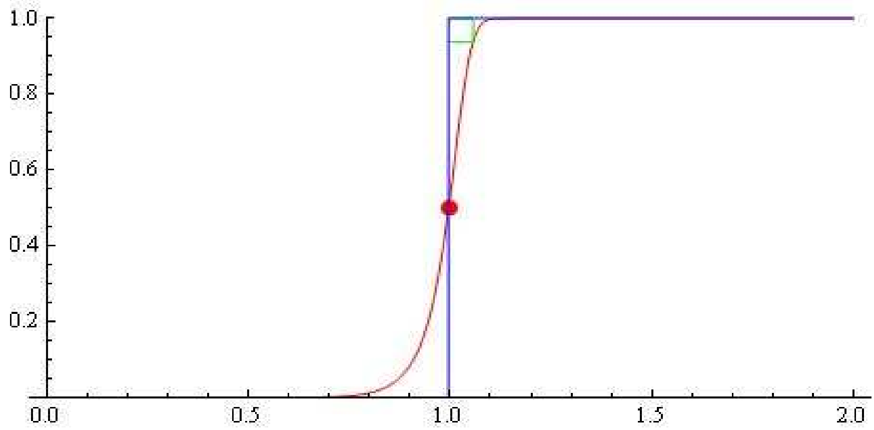

Figure 3: The model ((4)–(5)) for $\beta = 4$, $\alpha = 0.95$, $a = 5.9$, $b = 16$, $t_0 = 0.996093$; H–distance $d = 0.0606985$, $d_l = 0.0522077$.

From the nonlinear equation (6) and inequalities (7) we have: $d = 0.0895807$, $d_l = 0.0786509$.

The model ((4)–(5)) for $\beta = 4$, $\alpha = 0.95$, $a = 5.9$, $b = 16$, $t_0 = 0.996093$, $d = 0.0606985$, $d_l = 0.0522077$ is visualized on Fig. 3.

2.2. A Note on the Song–Chang–Pham Model (2) [2]

We consider the following "cumulative sigmoid":

$$M_1^*(t) = 1 - \left(\frac{\beta}{\beta + at - \ln(1+at)}\right)^{\alpha}, \tag{10}$$

with $N = 1$ (see (2)), and let t_0 is the unique positive root of the nonlinear equation

$$M_1^*(t_0) - \frac{1}{2} = 0. \tag{11}$$

The one–sided Hausdorff distance d_1 between the function $h_{t_0}(t)$ and the sigmoid ((10)–(11)) satisfies the relation

$$M_1^*(t_0 + d_1) = 1 - d_1. \tag{12}$$

Based on the methodology proposed in the present note, the reader may formulate the corresponding approximation problems on his/her own.

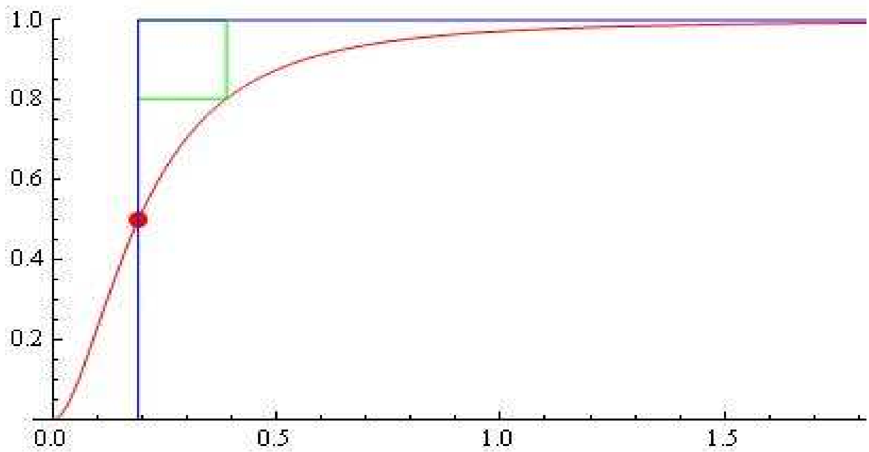

Figure 4: The model ((10)–(11)) for $\beta = 1.5$, $\alpha = 2.5$, $a = 6.9$, $t_0 = 0.191515$; H–distance $d_1 = 0.196555$.

The model ((10)–(11)) for $\beta = 1.5$, $\alpha = 2.5$, $a = 6.9$, $t_0 = 0.191515$ is visualized on Fig. 4.

3. Numerical Examples. Concluding Remarks

1. We examine the following data. (The small on–line data entry software package test data, available since 1980 in Japan [3], is shown in Table 1. For more details, see [4]).

Below, we will illustrate the fitting of this data, for example, with the $M_1(t)$ model.

The fitted model

$$M_1(t) = M_1(t) = N\left(1 - \left(\frac{\beta}{\beta + at - \ln(1 + at)}\right)^{\alpha}\right)$$

based on the data of Table 1 for the estimated parameters:

$$N = 46; \quad a = 2.96839 * 10^{-6}; \quad \beta = 3.81716 * 10^{-8}; \quad \alpha = 51.0754$$

is plotted on Fig. 5.

2. Dataset presented in Table 2, was proposed in [6]. The week index is from 1 week to 18 weeks, and there are 176 cumulative failures at 18 weeks in Dataset..

$Testing\, time\, (day)$	$Failures$	$Cumulative\, failures$
1	2	2
2	1	3
3	1	4
4	1	5
5	2	7
6	2	9
7	2	11
8	1	12
9	7	19
10	2	21
11	1	22
12	2	24
13	2	26
14	4	30
15	1	31
16	6	37
17	1	38
18	3	41
19	1	42
20	3	45
21	1	46

Table 1: On–line IBM entry software package [3]

The fitted model based on the data of Table 2 for the estimated parameters: $N = 176$; $a = 2.30793 * 10^{-6}$; $\beta = 2.19561 * 10^{-8}$; $\alpha = 98.0241$ is plotted on Fig. 6.

The example results show a good fit to the presented model.

Obviously, studying of phenomenon "super saturation" is mandatory element along with other important components - "confidence bounds" and "confidence intervals" when dealing with questions from Software Reliability Models domain.

For some software reliability models, see [7]–[54].

We hope that the results will be useful for specialists in this scientific area.

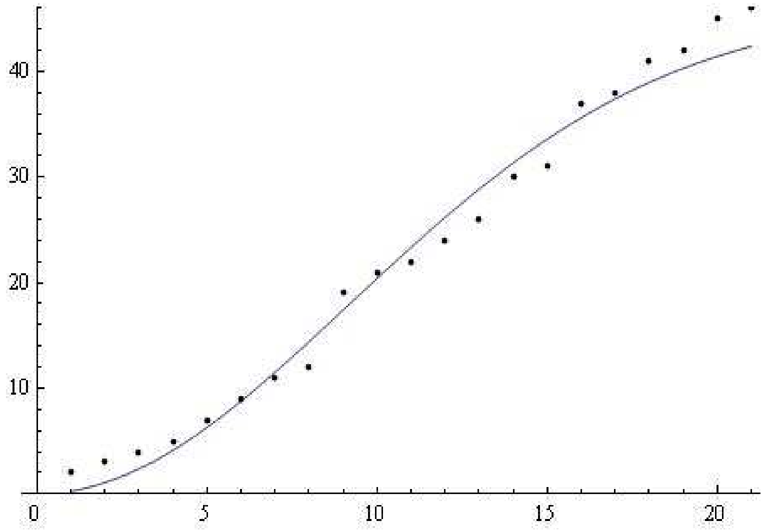

Figure 5: The model $M_1(t)$ with $N = 46$; $a = 2.96839 * 10^{-6}$; $\beta = 3.81716 * 10^{-8}$; $\alpha = 51.0754$.

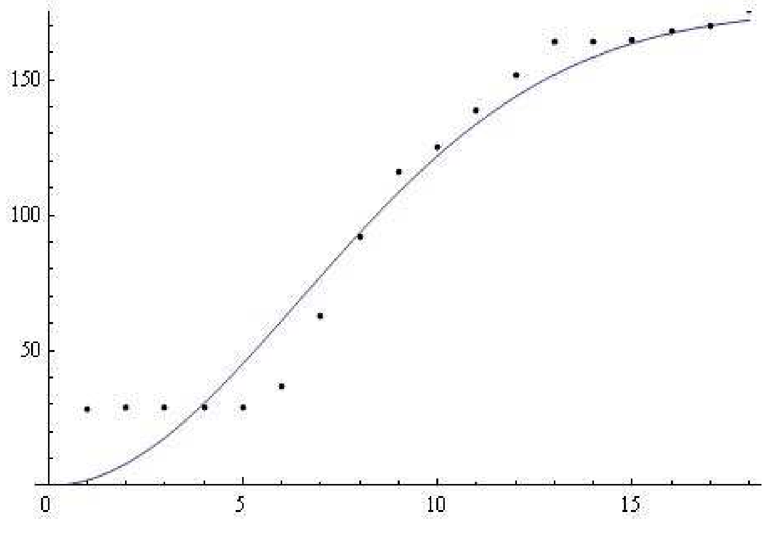

Figure 6: The fitted model $M_1(t)$ with $N = 176$; $a = 2.30793 * 10^{-6}$; $\beta = 2.19561 * 10^{-8}$; $\alpha = 98.0241$.

$Week\,Index$	$Failures$	$Cumulative\,failures$
1	28	28
2	1	29
3	0	29
4	0	29
5	0	29
6	8	37
7	26	63
8	29	92
9	24	116
10	9	125
11	14	139
12	13	152
13	12	164
14	0	164
15	1	165
16	3	168
17	2	170
18	6	176

Table 2: Dataset [6]

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