

LOCAL FRACTIONAL DERIVATIVE OPERATORS METHOD FOR SOLVING NONLINEAR GAS DYNAMIC EQUATION OF FRACTIONAL ORDER

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Abstract: In this article, local fractional decomposition method (LFDM) is applied to obtain an analytical approximate solution of nonlinear time-fractional Gas dynamics equation. A new application of local fractional decomposition method (LFDM) was extended to derive analytical solutions in the form of a convergent series with easily computable components, requiring no linearization or small perturbation for this equation. The solutions obtained by LFDM have been shown to be reliable, simple and the method is an effective technique for strong nonlinear partial equations.

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Key Words: local fractional decomposition method, fractional gas dynamics equation, Riemann-Liouville derivative

1. Introduction

Fractional calculus has created an alternative in understanding and modeling

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real life events through its “Memory Effect” property, and is currently a popular field in mathematics. Fractional Differentiation and Fractional Equations have been used in many areas for modeling natural phenomena and engineering problems like hydrodynamics [45], heat transfer, wave equations in fractal strings [46] and etc. Local Fractional Calculus has been implemented for solving non-differentiable equations in various physical events [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. Local fractional Fokker-Planck equation (LFFPE) [1], local fractional stress strain relations (LFSSR) [2], wave equation on Cantor sets (WECs) [14], Local fractional Laplace equation (LFLE) [15] and Newtonian mechanics on fractals subset of real-line [16] are only some of the models which include the application of this method.

Gas dynamics are the mathematical expressions of conservation laws that exist in engineering practices such as conservation of mass, momentum, energy and etc. Several different types of these equations have been solved in physics by Evans and Bulut [17], Kumar et al. [18], Jafari et al. [19], Steger et al. [20], Rasulov et al. [21], Jawad et al. [22], Aziz [23], Elizarova [24] via several analytical and numerical methods. Differential transform method [25, 26], Sumudu transform method [27], homotopy analysis method [28, 29] and fractional reduced differential transform method [30] were applied for solving nonlinear gas dynamics equations of fractional order.

Successful applications of the differential transformation method (DTM) [31], homotopy analysis method (HAM) [32, 33], Sumudu transform method [34] and the fractional reduced differential transform method [35] have been given for autonomous ordinary and partial differential equations and other areas. Jumarie has also presented a new modified Riemann-Liouville left derivative recently [5, 36, 37, 38, 39].

In this study, an application of LFDM is given to find the analytical approximate solution of the non-linear fractional gas dynamics problem

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} + u(x,t) \frac{\partial u(x,t)}{\partial x} - u(x,t)(1 - u(x,t)) = 0, \quad (1)$$

$$0 < x \leq 1, 0 < \alpha \leq 1, t > 0,$$

$$u(x, 0) = h(x), 0 < x \leq 1, \quad (2)$$

where α describes the order of the time-fractional derivative and the fractional derivative is considered in the modified Riemann-Liouville derivative. $u(x, t)$ vanishes for $t < 0$, meaning it is a causal function of time. The aim of this paper is to extend the application of the local fractional PDEs (LFPDEs) within local fractional derivative (LFD) for solving fractional nonlinear gas dynamics equations with modified Riemann-Liouville derivative.

This paper is organized as follows: Descriptions of local fractional calculus theory are given in section 2. Local fractional decomposition method resolution procedure is defined in order to display the inefficiency of this method in section 3 and the application of LFD is given in section 4 for fractional nonlinear gas dynamics equation with numerical results. The last section includes the conclusions.

2. Basic Definitions

Definitions and properties of local fractional continuity (LFC), local fractional derivative (LFD) and local fractional integral (LFI) of non-differentiable functions are given below [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 40, 41, 42, 43, 44].

Definition 1. Let the relation [9, 40, 41, 42, 43, 44]

$$|f(u) - f(u_0)| < \varepsilon^\alpha \quad (3)$$

exist with $|u - u_0| < \delta$, for $\varepsilon, \delta > 0$ and $\varepsilon, \delta \in \mathbf{R}$. $f(u)$ is called local fractional continuous at $u = u_0$ and denoted by $\lim_{u \rightarrow u_0} f(u) = f(u_0)$. Then, $f(u)$ is local fractional continuous on the interval (a, b) , denoted by

$$f(u) \in C_\alpha(a, b) \quad (4)$$

Definition 2. $f(u)$ is called a non-differentiable function of exponent, $0 < \alpha \leq 1$, which satisfies the Hölder function of exponent α , if the following is true for $u, v \in X$ [9, 40, 41, 42, 43, 44].

$$|f(u) - f(v)| < C|u - v|^\alpha \quad (5)$$

Definition 3. The function $f(x)$ is said to be continuous of order α , $0 < \alpha \leq 1$ (or α -continuous) if the following relation is satisfied [9, 10, 47]

$$|f(u) - f(u_0)| < \varepsilon^\alpha, f(u) - f(u_0) = o((u - u_0)^\alpha) \quad (6)$$

Compared with (6), (3) is standard definition of local fractional continuity (LFC). (5) is the unified local fractional continuity (ULFC).

Definition 4. For $f(u) \in C_\alpha(a, b)$, LFD of $f(x)$ of order α at $u = u_0$, is [9, 10, 47]:

$$f^{(\alpha)}(u_0) = \left. \frac{d^\alpha f(u)}{du^\alpha} \right|_{u=u_0} = \lim_{u \rightarrow u_0} \frac{\Delta^\alpha(f(u) - f(u_0))}{(u - u_0)^\alpha}, \quad 0 < \alpha \leq 1 \quad (7)$$

where $\Delta^\alpha(f(u) - f(u_0)) \cong \Gamma(1 + \alpha)\Delta(f(u) - f(u_0))$. For any $u \in (a, b)$, there exists $f^{(\alpha)}(u) = D_u^\alpha f(u)$ and is denoted by $f(u) \in D_u^\alpha(a, b)$. LFD of higher order can be expressed as

$$f^{(k\alpha)}(u) = \underbrace{D_u^\alpha \dots D_u^\alpha}_{k \text{ times}} f(u)$$

and local fractional partial derivative (LFPD) of higher order is denoted as:

$$\frac{\partial^{(k\alpha)}(u)}{\partial u^{k\alpha}} = \underbrace{\frac{\partial^\alpha}{\partial u^\alpha} \dots \frac{\partial^\alpha}{\partial u^\alpha}}_{k \text{ times}} f(u)$$

Definition 5. For $f(u) \in C_\alpha(a, b)$, the local fractional integral (LFI) of $f(u)$ of order α in the interval $[a, b]$ is defined by [40, 41, 42, 43, 44]:

$${}_a I_b^\alpha f(u) = \frac{1}{\Gamma(1 + \alpha)} \int_a^b f(t)(dt)^\alpha = \frac{1}{\Gamma(1 + \alpha)} \lim_{\Delta t \rightarrow 0} \sum_{j=0}^{N-1} f(t_j)(\Delta t_j)^\alpha, \quad (8)$$

for $0 < \alpha \leq 1$, where $\Delta t_j = t_{j+1} - t_j$, $\Delta t = \max(\Delta t_1, \Delta t_2, \Delta t_3, \dots)$ and $[\Delta t_j, \Delta t_{j+1}]$, $j = 0, 1, \dots, N - 1$, $t_0 = a$, $t_N = b$ is a partition of the interval $[a, b]$. For any $u \in (a, b)$, there exists ${}_a I_b^\alpha$ denoted by $f(u) \in I_u^{(\alpha)}(a, b)$. If $f(u) = D_u^\alpha f(a, b)$, or $I_u^{(\alpha)}(a, b)$, we have $f(u) \in C_\alpha(a, b)$.

For any $f(u) \in C_\alpha(a, b)$, $0 < \alpha \leq 1$, we have local fractional multiple integrals (LFMIs)

$${}_{u_0} I_u^{(k\alpha)} f(u) = \overbrace{{}_{u_0} I_u^{(\alpha)} \dots {}_{u_0} I_u^{(\alpha)}}^{k \text{ times}}.$$

For $0 < \alpha \leq 1$, $f^{(k\alpha)}(u) \in C_\alpha^k(a, b)$ we have $({}_{u_0} I_u^{(k\alpha)} f(u))^{(k\alpha)} = f(u)$, where, $f^{(k\alpha)}(u) = \underbrace{D_u^\alpha \dots D_u^\alpha}_{k \text{ times}} f(u)$, ${}_{u_0} I_u^{(k\alpha)} f(u) = \underbrace{{}_{u_0} I_u^{(\alpha)} \dots {}_{u_0} I_u^{(\alpha)}}_{k \text{ times}}$

Definition 6. The Mittag-Leffler function (MLF) in fractional space is defined by [9, 40, 41, 42, 43, 44]

$$E_\alpha(u^\alpha) = \sum_{n=0}^{\infty} \frac{u^{(n\alpha)}}{\Gamma(1 + n\alpha)}, \quad 0 < \alpha \leq 1. \quad (9)$$

Some useful formulas of LFD were summarized [9, 40] as follows:

$$\frac{d^\alpha u^{n\alpha}}{du^\alpha} = \frac{\Gamma(1 + n\alpha)u^{(n-1)\alpha}}{\Gamma(1 + (n-1)\alpha)}, \quad (10)$$

$$\frac{d^\alpha E_\alpha(u^\alpha)}{du^\alpha} = E_\alpha(u^\alpha), \quad (11)$$

$$\frac{d^\alpha E_\alpha(u^{n\alpha})}{du^\alpha} = nE_\alpha(u^\alpha), \quad (12)$$

$$\frac{1}{\Gamma(1+\alpha)} \int_a^b E_\alpha(u^\alpha)(du)^\alpha = E_\alpha(b^\alpha) - E_\alpha(a^\alpha), \quad (13)$$

$$\frac{1}{\Gamma(1+\alpha)} \int_a^b u^{n\alpha}(du)^\alpha = \frac{\Gamma(1+n\alpha)(b^{(n+1)\alpha} - a^{(n+1)\alpha})}{\Gamma(1+(n+1)\alpha)} \quad (14)$$

3. Local Fractional Decomposition Method

The fractional differential equation (15) is considered to present the solution procedure of local fractional decomposition method equation [18, 40, 41, 42, 43, 44]:

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} + u(x,t) \frac{\partial u(x,t)}{\partial x} - u(x,t)(1-u(x,t)) = 0 \quad (15)$$

$$0 < x \leq 1, 0 < \alpha \leq 1, t > 0$$

By LFDM, a local fractional differential operator form for Eq. (15) is given as follows

$$L_t^{(\alpha)} u(x,t) = -u(x,t) \frac{\partial u(x,t)}{\partial x} + u(x,t)(1-u(x,t)) \quad (16)$$

where $0 < \alpha \leq 1$ and $u(x,t)$ is a local fractional continuous function. Applying the inverse operator $L_t^{(-\alpha)}$ to both sides of (16) yields (for $n \geq 0$)

$$u_{n+1}(x,t) = L_t^{(-\alpha)} \left[-u_n(x,t) \frac{\partial u_n(x,t)}{\partial x} + u_n(x,t)(1-u_n(x,t)) \right], \quad (17)$$

$$u_0(x,t) = u(x,0).$$

Successive approximations $u_{n+1}(x,t), n \geq 0$ of $u(x,t)$ can be found with any function u_0 . Generally, u_0 is chosen as the initial values. Hence, the exact solution may be obtained through

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \quad (18)$$

4. Applications

An application of local fractional decomposition method (LFDM) is presented in this section for solving the non-linear fractional partial differential equation.

Example 4.1. The non-linear fractional gas dynamics equation

$$\frac{\partial^\alpha u(x, t)}{\partial t^\alpha} + u(x, t) \frac{\partial u(x, t)}{\partial x} - u(x, t)(1 - u(x, t)) = 0 \quad (19)$$

is considered for $0 < x \leq 1, 0 < \alpha \leq 1, t > 0$ along with the condition [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 40, 41, 42, 43, 44]

$$u(x, 0) = ae^{-x}. \quad (20)$$

Using (20), the recurrence relation becomes $u_0(x, t) = u(x, 0)$ and (for $n \geq 0$)

$$u_{n+1}(x, t) = L_t^{(-\alpha)} \left[-u_n(x, t) \frac{\partial u_n(x, t)}{\partial x} + u_n(x, t)(1 - u_n(x, t)) \right] \quad (21)$$

Applying the recursive relation (21) and the conditions in (21), we get the followings results:

$$u_0(x, t) = ae^{-x}, \quad (22)$$

$$\begin{aligned} u_1(x, t) &= L_t^{(-\alpha)} \left[-u_0(x, t) \frac{\partial u_0(x, t)}{\partial x} + u_0(x, t)(1 - u_0(x, t)) \right] \\ &= \frac{ae^{-x}t^\alpha}{\Gamma(1 + \alpha)}, \end{aligned} \quad (23)$$

$$\begin{aligned} u_2(x, t) &= L_t^{(-\alpha)} \left[-u_1(x, t) \frac{\partial u_1(x, t)}{\partial x} + u_1(x, t)(1 - u_1(x, t)) \right] \\ &= \frac{ae^{-x}t^{2\alpha}}{\Gamma(1 + 2\alpha)}, \end{aligned} \quad (24)$$

$$\begin{aligned} u_3(x, t) &= L_t^{(-\alpha)} \left[-u_2(x, t) \frac{\partial u_2(x, t)}{\partial x} + u_2(x, t)(1 - u_2(x, t)) \right] \\ &= \frac{ae^{-x}t^{3\alpha}}{\Gamma(1 + 3\alpha)}, \end{aligned} \quad (25)$$

...

$$\begin{aligned} u_n(x, t) &= L_t^{(-\alpha)} \left[-u_{n-1}(x, t) \frac{\partial u_{n-1}(x, t)}{\partial x} + u_{n-1}(x, t)(1 - u_{n-1}(x, t)) \right] \\ &= \frac{ae^{-x}t^{n\alpha}}{\Gamma(1 + n\alpha)}, \end{aligned} \quad (26)$$

Hence, the approximate solution becomes

$$\begin{aligned}
 u(x, t) &= \sum_{n=0}^{\infty} u_n(x, t) \\
 &= ae^{-x} \left(1 + \frac{t^{\alpha}}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + \dots \right) \\
 &= ae^{-x} \sum_{n=0}^{\infty} \frac{t^{(n\alpha)}}{\Gamma(1+n\alpha)}
 \end{aligned} \tag{27}$$

and the exact solution is

$$u(x, t) = ae^{-x} E_{\alpha}(t^{\alpha}) \tag{28}$$

For $\alpha = 1$ this can be given as

$$u(x, t) = ae^{-x} \sum_{n=0}^{\infty} \frac{t^n}{\Gamma(1+n)} = ae^{-x+t} \tag{29}$$

which is a complete solution of the nonlinear gas dynamics equation. Fig. 1 shows the approximate solutions for the nonlinear gas dynamics equation (19) obtained using local fractional decomposition method (LFDM). Figure 2 displays the approximate solution of Eq. (19) for $\alpha = 0.95, 0.85, 0.75, 0.65$. Figures 3 and 4 display the influence of α on the function $u(x, t)$.

The results in Figures 3 & 4 clearly show that the decrease of the fractional order α , for fixed $x = 0.5$ and $x = 0.1$, ends in an increase in the function. The values $\alpha = 1, 0.9, 0.8, 0.7, 0.6$ are shown in Figures 3 & 4.

Eq. (19) is solved in [30] using the fractional reduced differential transform method (FRDTM) [35], Homotopy analysis method HAM in [28, 29] and differential transformation method DTM in [26].

It is shown that the applied algorithm works efficiently and reliably for Local fractional decomposition method (LFDM). The results for LFDM are in absolute accordance with the results from modified variational iteration method (FRDTM), HAM and DTM.

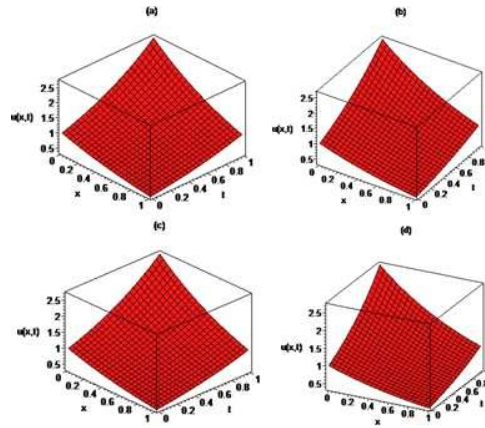


Figure 1: The graphs display the solution $u(x, t)$ of (19) for $\alpha = 1$ and $a = 1$. (a) Exact solution (b) 3-iterate LFD approximate solution, (c) 4-iterate LFD approximate solution and (d) 5-iterate LFD approximate solution.

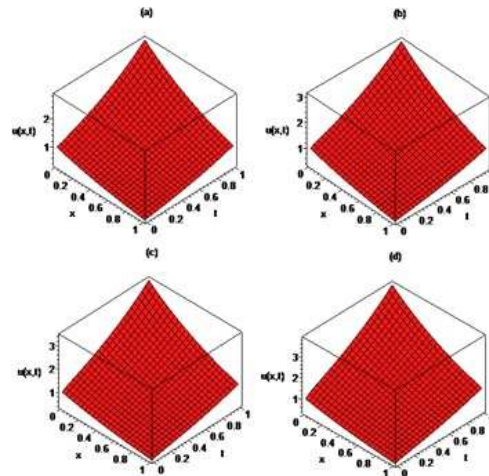


Figure 2: The graphs display the solution $u(x, t)$ of (19) for $a = 1$. (a) 5-iterate LFD approximate solution for $\alpha = 0.95$ (b) 5-iterate LFD approximate solution for $\alpha = 0.85$ (c) 5-iterate LFD approximate solution for $\alpha = 0.75$ and (d) 5-iterate LFD approximate solution for $\alpha = 0.65$.

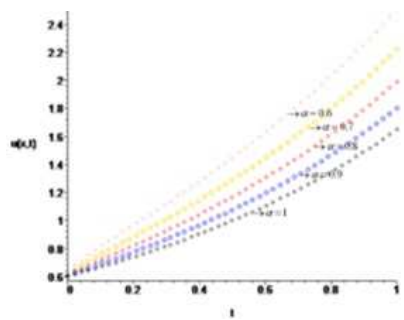


Figure 3: 5-iterate LFD approximate solution for $x = 0.5$ and $a = 1$.

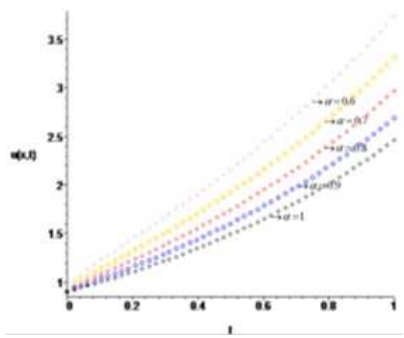


Figure 4: 5-iterate LFD approximate solution for $x = 0.1$ and $a = 1$.

Table 1: Numerical values for $\alpha = 0.5, 0.75, 1$ and $a = 1$ of $u(x, t)$ by LFD.

t	x	u_{5LFD}			u_{Exact}	u_{Error}
		$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$	$\alpha = 1$	
0.2	0	1.797292721	1.404661629	1.221402667	1.221402758	0.91×10^{-7}
	0.25	1.399732979	1.093951577	0.9512293535	0.9512294245	0.710×10^{-7}
	0.50	1.090113140	0.8519703445	0.7408181654	0.7408182207	0.553×10^{-7}
	0.75	0.8489809668	0.6635151714	0.5769497673	0.5769498104	0.431×10^{-7}
	1	0.6611870419	0.5167461352	0.4493289306	0.4493289641	0.335×10^{-7}
0.4	0	2.414405270	1.800095759	1.491818667	1.491824698	0.6031×10^{-5}
	0.25	1.880340715	1.401915987	1.161829546	1.161834243	0.4679×10^{-5}
	0.50	1.464410821	1.091813268	0.9048337602	0.9048374180	0.36578×10^{-5}
	0.75	1.140484294	0.8503050282	0.7046852410	0.7046880897	0.28487×10^{-5}
	1	0.8882100616	0.6622182219	0.5488094176	0.5488116361	0.22185×10^{-5}
0.6	0	3.087561961	2.260549371	1.822048000	1.822118800	0.000070800
	0.25	2.404595673	1.760517620	1.419012409	1.419067549	0.000055140
	0.50	1.872700993	1.371092501	1.105127975	1.105170918	0.000042943
	0.75	1.458461000	1.067807914	0.8606745326	0.8607079764	0.0000334438
	1	1.135850569	0.831609639	0.6702940001	0.6703200460	0.0000260459

Example 4.2. The non-linear and non-homogenous fractional gas dynamics equation

$$\frac{\partial^\alpha u(x, t)}{\partial t^\alpha} + u(x, t) \frac{\partial u(x, t)}{\partial x} - u(x, t)(1 - u(x, t)) \ln a = 0 \quad (30)$$

is considered for $0 < \alpha \leq 1, 0 < x \leq 1, t > 0$ with the conditions [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 40, 41, 42, 43, 44]

$$u(x, 0) = a^{-x}. \quad (31)$$

By LFDm, the recurrence relation becomes $u_0(x, t) = u(x, 0)$ and (for $n \geq 0$)

$$u_{n+1}(x, t) = L_t^{(-\alpha)} \left[-u_n(x, t) \frac{\partial u_n(x, t)}{\partial x} + u_n(x, t)(1 - u_n(x, t)) \ln a \right] \quad (32)$$

Using (32) and condition (31), we obtain:

$$u_0(x, t) = a^{-x}, \quad (33)$$

$$\begin{aligned} u_1(x, t) &= L_t^{(-\alpha)} \left[-u_0(x, t) \frac{\partial u_0(x, t)}{\partial x} + u_0(x, t)(1 - u_0(x, t)) \ln a \right] \\ &= \frac{a^{-x} \ln a t^\alpha}{\Gamma(1 + \alpha)}, \end{aligned} \quad (34)$$

$$\begin{aligned} u_2(x, t) &= L_t^{(-\alpha)} \left[-u_1(x, t) \frac{\partial u_1(x, t)}{\partial x} + u_1(x, t)(1 - u_1(x, t)) \ln a \right] \\ &= \frac{a^{-x} (\ln a)^2 t^{2\alpha}}{\Gamma(1 + 2\alpha)}, \end{aligned} \quad (35)$$

$$\begin{aligned} u_3(x, t) &= L_t^{(-\alpha)} \left[-u_2(x, t) \frac{\partial u_2(x, t)}{\partial x} + u_2(x, t)(1 - u_2(x, t)) \ln a \right] \\ &= \frac{a^{-x} (\ln a)^3 t^{3\alpha}}{\Gamma(1 + 3\alpha)}, \end{aligned} \quad (36)$$

...

$$\begin{aligned} u_n(x, t) &= L_t^{(-\alpha)} \left[-u_{n-1}(x, t) \frac{\partial u_{n-1}(x, t)}{\partial x} + u_{n-1}(x, t)(1 - u_{n-1}(x, t)) \ln a \right] \\ &= \frac{a^{-x} (\ln a)^n t^{n\alpha}}{\Gamma(1 + n\alpha)}, \end{aligned} \quad (37)$$

Hence, the approximate solutions is obtained as

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t)$$

$$\begin{aligned}
&= a^{-x} \left(1 + \frac{\ln at^\alpha}{\Gamma(1+\alpha)} + \frac{(\ln a)^2 t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{(\ln a)^3 t^{3\alpha}}{\Gamma(1+3\alpha)} + \dots \right) \\
&= a^{-x} \sum_{n=0}^{\infty} \frac{(\ln at^\alpha)^n}{\Gamma(1+n\alpha)}
\end{aligned} \tag{38}$$

and the exact solution is

$$u(x, t) = a^{-x} E_\alpha(\ln at^\alpha) \tag{39}$$

For $\alpha = 1$, we find

$$u(x, t) = a^{-x} \sum_{n=0}^{\infty} \frac{(\ln at)^n}{\Gamma(1+n)} = a^{-x+t} \tag{40}$$

for which the proof can be found in [44].

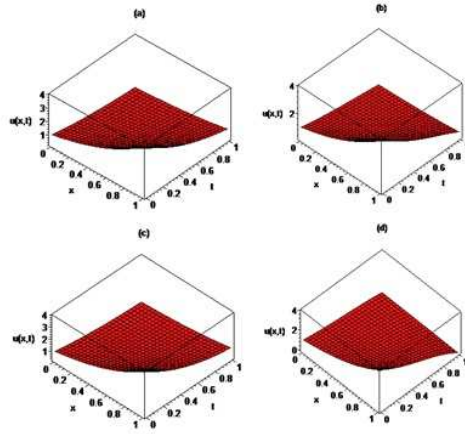


Figure 5: The graphs display the solution $u(x, t)$ of (30) for $\alpha = 1$ and $a = \frac{1}{4}$. (a) Exact solution (b) 3-iterate LFD approximate solution, (c) 4-iterate LFD approximate solution and (d) 5-iterate LFD approximate solution

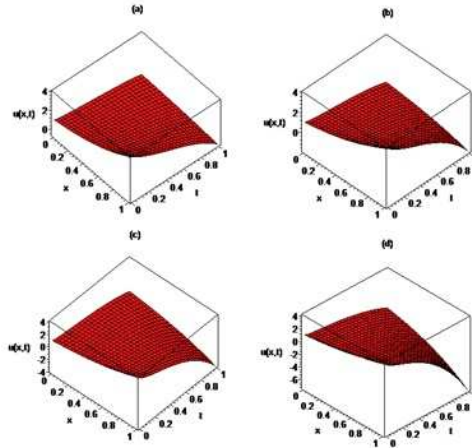


Figure 6: The graphs display the solution $u(x, t)$ of (30) for $a = \frac{1}{4}$, (a) 5-iterate LFD approximate solution for $\alpha = 0.93$ (b) 5-iterate LFD approximate solution for $\alpha = 0.83$ (c) 5-iterate LFD approximate solution for $\alpha = 0.73$ and (d) 5-iterate LFD approximate solution for $\alpha = 0.63$.

The solutions of the homogenous nonlinear fractional gas dynamics equation are displayed in Figure 5 for various α values. Figures 5 & 6 display the influence of α on

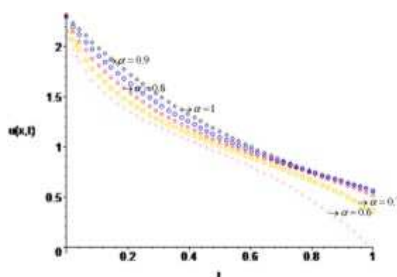


Figure 7: 5-iterate LFD approximate solution for $x = 0.6$ and $a = \frac{1}{4}$.

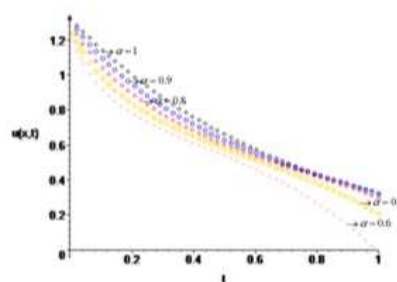


Figure 8: 5-iterate LFD approximate solution for $x = 0.2$ and $a = \frac{1}{4}$.

$u(x, t)$. Clearly, an increase in t triggers a decrease in $u(x, t)$ for $\alpha = 1, 0.9, 0.8, 0.7, 0.6$. (30) is solved with fractional reduced differential transform method (FRDTM) in [35], Homotopy analysis method (HAM) in [28, 29] and differential transform method (DTM) in [26]. Results in Figure 5 are in accordance with the results from FRDTM and DTM.

The numerical results in Figures 7 & 8 clearly show that the decrease of the fractional order α , for fixed $x = 0.6$ and $x = 0.2$, ends in an increase in the function. Five values of $\alpha = 1, 0.9, 0.8, 0.7, 0.6$ are shown in Figures 7 & 8.

5. Conclusions

In this paper, Local fractional decomposition method (LFD) has been successfully applied to obtain the approximate analytical solution of the time-fractional Gas Dynamics given with initial conditions. We have taken Local fractional decomposition method using integral w. r. t. $(d\tau)^\alpha$ which has been presented by Jumarie. The analytical results are given in power series form, for which the conditions can be easily calculated. Results show that the method is a powerful tool for the solutions of non-linear fractional differential equations. Solutions of the examples show that the results from local fractional decomposition method using integral w. r. t. $(d\tau)^\alpha$ are in perfect

accordance with those from classical DTM, HAM and FRDTM methods, which can be found in the literature

Table 2: Numerical values when $\alpha = 0.5, 0.75, 1$ and $a = \frac{1}{4}$ for $u(x, t)$ obtained using the LFDm.

t	x	u_{5LFDm}			u_{Exact}	u_{Error}
		$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$	$\alpha = 1$	
0.2	0	0.5518523610	0.6544260680	0.7578576764	0.7578582833	0.6069×10^{-6}
	0.25	0.7804370933	0.9254982207	1.071772604	1.071773463	0.859×10^{-6}
	0.5	1.103704722	1.308852135	1.515715353	1.515716567	0.1214×10^{-5}
	0.75	1.560874188	1.850996443	2.143545209	2.143546925	0.1716×10^{-5}
	1	2.207409444	2.617704272	3.031430706	3.031433133	0.2427×10^{-5}
0.4	0	0.4119492547	0.5035375612	0.5743117881	0.5743491775	0.000299116
	0.25	0.5825842223	0.7121096483	0.8121995190	0.8122523964	0.000299116
	0.5	0.8238985085	1.007075122	1.148623576	1.148698355	0.000423014
	0.75	1.165168445	1.424219297	1.624399039	1.624504793	0.000598232
	1	1.647797020	2.014150245	2.297247153	2.297396710	0.00084603
0.6	0	0.2451579318	0.3999643401	0.4348647902	0.4352752816	0.002322091
	0.25	0.3467056723	0.5656349953	0.6149916840	0.6155722067	0.003283932
	0.5	0.4903158622	0.7999286804	0.8697295805	0.8705505633	0.004644180
	0.75	0.693411344	1.131269989	1.229983368	1.231144413	0.006567866
	1	0.980631725	1.599857362	1.739459160	1.741101127	0.009288366

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