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COMMON FIXED POINT THEOREM FOR THREE SELFMAPS OF A COMPLETE G-METRIC SPACE

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Abstract: In this paper we prove a common fixed point theorem for three weakly compatible self maps of a complete G-metric space.As an illustration we provide an example to support our theorem.

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Key Words: G-Metric space, fixed point, weakly compatible selfmaps, associated sequence for three selfmaps, implicit relation

1. Introduction

Sessa[7] generalized the notion of commuting maps by introducing weakly commuting mapps.As a further generalizations of this G.Jungck[4, 5] initiated compatibility,later Jungck and Rhoades[6] introduced the concept of weakly compatible mappings.Gahler[2, 3]introduced the notion of 2-metric spaces, Dhage[1] initiated the notion of D- metric spaces. Subsequently several researchers have proved that most of their claims made are not valid.Later Shaban Sedghi,Nabi Shobe and Haiyun Zhou[8] introduced D^* metric spaces.In 2006,Zead Mustafa

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and Brailey Sims[10, 11] initiated G- metric spaces.Among these generalizations,G -metric spaces are noteworthy, as several researchers have established many results on these.

The purpose of this paper is to prove a common fixed point theorem for three weakly compatible self maps of a complete G -metric space.

2. Preliminaries

Definition 1. [10] Let X be a non empty set and
$G : X^3 \to [0, \infty)$ be a function satisfying
$(G1)$ $G(x, y, z) = 0$ if $x = y = z$
$(G2)$ $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$
$(G3)$ $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$
$(G4)$ $G(x, y, z) = G(\sigma(x, y, z))$ for all $x, y, z \in X$ where $\sigma(x, y, z)$ is a permutation of the set $\{x, y, z\}$ and
$(G5)$ $G(x, y, z) \leq G(x, w, w) + G(w, y, z)$ for all $x, y, z, w \in X$
Then G is called a G-metric on X and the pair (X, G) is called a G- metric space.

Example: Let(X, d) be a metric space.Define $G_s^d : X^3 \to [0, \infty)$ by

$$G_s^d(x, y, z) = \frac{1}{3}[d(x, y) + d(y, z) + d(z, x)],$$

for $x, y, z \in X$, then (X, G_s^d) is a G-metric space

Lemma 2. *[10] Let (X, G) be a G-metric space then*
$G(x, y, y) \leq 2G(y, x, x)$ *for all* $x, y \in X$

Definition 3. [10] Let (X, G) be a G-metric Space. A sequence $\{x_n\}$ in X is said to be G-convergent if there is a $x_0 \in X$ such that to each $\varepsilon > 0$ there is a natural number N for which $G(x_n, x_n, x_0) < \varepsilon$ for all $n \geq N$.

Definition 4. [10] Let (X, G) be a G-metric Space. A sequence $\{x_n\}$ in X is said to be G-Cauchy if for each $\varepsilon > 0$ there exists is a natural number N such that $G(x_n, x_m, x_l) < \varepsilon$ for all $n, m, l \geq N$.

Note that every G-convergent sequence in a G-metric space (X, G) is G-Cauchy.

Definition 5. [10] A G-metric space (X, G) is said to be
G-complete if every G-Cauchy sequence in (X, G) is G-convergent in (X, G)

Lemma 6. *[10] Let (X, G) be a G-metric space.Then for any $x, y, z, a \in X$ it follows that:*
(1) $G(x, y, z) = 0$ if $x = y = z$
(2) $G(x, y, z) \leq G(x, x, y) + G(x, x, z)$
(3) $G(x, y, y) \leq 2G(y, x, x))$
(4) $G(x, y, z) \leq G(x, a, z) + G(a, y, z)$
(5) $G(x, y, z) \leq \frac{2}{3}G(x, a, a) + G(y, a, a) + G(z, a, a)$

Definition 7. Let f and g be two self maps of a G-metric space (X, G) such that $\lim_{n\to\infty} G(fgx_n, gfx_n, gfx_n) = 0$ for every sequence $\{x_n\}$ in X with $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in X$, then the functions f and g are said to be compatible.

Definition 8. Let f, g be two self maps mappings of a metric space $\big(X, G\big)$. The pair (f, g) is said to be weakly compatible, if $G(fgx, gfx, gfx) = 0$ whenever $G(fx, gx, gx) = 0$.That is the mappings f and g are said to be weakly compatible if they commute at their coincident points.

Definition 9. A function $\phi : (\mathbb{R}^+)^5 \to \mathbb{R}^+$ which is continuous and increasing in each co-ordinate with $\phi(t, t, a_1t, a_2t, t) < t$ for every $t \in \mathbb{R}^+$ where $a_1 + a_2 = 3$ is called an Implicit relation
The set of all implicit relations is denoted by Φ

Lemma 10. *[9] Let $\gamma : \mathbb{R}^+ \to \mathbb{R}^+$ be a mapping such that*
$\gamma(t) = \phi(t, t, a_1t, a_2t, t) < t$ where $a_1 + a_2 = 3$.
For every $t > 0$, $\gamma(t) <$ t if and only if $\lim_{n\to\infty} \gamma^n(t) = 0$ where γ^n denotes the composition of γ with itself n times.

Definition 11. Let (X, G) be a G-metric space and f, g and p be self maps of a G-metric space(X, G) such that $f(X) \subseteq g(X)$, $f(X) \subseteq p(X)$. For any $x_0 \in X$,we can find a sequence $\{x_n\}$ in X such that $fx_{2n} = gx_{2n+1}, fx_{2n+1} = px_{2n+2}$ for $n \geq 0$ then $\{x_n\}$ is called an associated sequence of x_0 relative to the self maps f, g and p.

3. Main Result

Theorem 12. *Let $A, S,$ and T be selfmaps of a complete G-metric space(X, G) satisfying the following conditions*

(i) $A(X) \subseteq T(X)$ and $A(X) \subseteq S(X)$

(ii) one of $S(X)$ and $T(X)$ is closed subset of X.

(iii) $G(Ax, Ay, Ay) \leq \phi \begin{pmatrix} G(Sx, Ty, Ty), & G(Ax, Ty, Ty), \\ G(Sx, Ay, Ay), & G(Sx, Ax, Ax), \\ G(Ty, Ay, Ay) & \end{pmatrix}$
for every $x, y \in X$ *and* $\phi \in \Phi$

(iv) The pairs (A, T)*and* (A, S) *are weakly compatible*

Then A, S *and* T *have a unique common fixed point in* X

Proof. let $x_0 \in X$ be an arbitrary point.Then by definition 11 there exists an associated sequence $\{x_n\}$ in X such that
$y_{2n} = Ax_{2n} = Tx_{2n+1}, y_{2n+1} = Ax_{2n+1} = Sx_{2n+2}$ for $n = 0, 1, 2 \cdots$
Let $G_n = G(y_n, y_{n+1}, y_{n+1})$.
first we prove $G_{2n} \leq G_{2n+1}$.
If $G_{2n} > G_{2n+1}$ for some $n \in \mathbb{N}$,
then from (iii)of the Theorem 12,we have

$$\begin{aligned}
G_{2n} &= G(y_{2n}, y_{2n+1}, y_{2n+1}) \\
&= G(Ax_{2n}, Ax_{2n+1}, Ax_{2n+1}) \\
&\leq \phi \begin{pmatrix} G(Sx_{2n}, Tx_{2n+1}, Tx_{2n+1}), & G(Ax_{2n}, Tx_{2n+1}, Tx_{2n+1}), \\ G(Sx_{2n}, Ax_{2n+1}, Ax_{2n+1}), & G(Sx_{2n}, Ax_{2n}, Ax_{2n}), \\ G(Tx_{2n+1}, Ax_{2n+1}, Ax_{2n+1}) & \end{pmatrix} \\
&= \phi \begin{pmatrix} G(y_{2n-1}, y_{2n}, y_{2n}), & G(y_{2n}, y_{2n}, y_{2n}), \\ G(y_{2n-1}, y_{2n+1}, y_{2n+1}), & G(y_{2n-1}, y_{2n}, y_{2n}), \\ G(y_{2n}, y_{2n+1}, y_{2n+1}) & \end{pmatrix} \\
&\leq \phi\Big(G_{2n-1}, 0, G_{2n-1} + G_{2n}, G_{2n-1}, G_{2n} \Big) \\
&\leq \phi\Big(G_{2n-1}, 0, 2G_{2n}, G_{2n-1}, G_{2n} \Big) \\
&\leq \phi\Big(G_{2n}, G_{2n}, 2G_{2n}, G_{2n}, G_{2n} \Big) \\
&< G_{2n}
\end{aligned}$$

which is a contradiction since $G_{2n+1} \leq G_{2n}$
similarly we can prove that $G_{2n+1} \leq G_{2n}$
Hence we have $G_n \leq G_{n-1}$ for all $n \geq 1$.Therefore $\{G_n\}$ is a non increasing sequence of non-negative real numbers. Now

$$G_1 = G(y_1, y_2, y_2)$$

$$
\begin{aligned}
&= G(Ax_1, Ax_2, Ax_2) \\
&\leq \phi\begin{pmatrix}(G(Sx_1, Tx_2, Tx_2), & G(Ax_1, Tx_2, Tx_2), \\ G(Sx_1, Ax_2, Ax_2), & G(Sx_1, Ax_1, Ax_1), \\ G(Tx_2, Ax_2, Ax_2, & \end{pmatrix} \\
&= \phi\begin{pmatrix}G(y_0, y_1, y_1), & G(y_1, y_1, y_1), & G(y_0, y_2, y_2), \\ G(y_0, y_1, y_1), & G(y_1, y_2, y_2) & \end{pmatrix} \\
&\leq \phi\Big(G_0, 0, G_0 + G_1, G_0, G_1\Big) \\
&\leq \phi\Big(G_0, G_0, 2G_0, G_0, G_0\Big) \\
&= \gamma(G_0)
\end{aligned}
$$

In general,we have $G_n \leq \gamma^n(G_0)$. So $G_0 > 0$, then by Lemma 10 $\lim_{n\to\infty} G_n = 0$. For $G_0 = 0$ we have $\lim_{n\to\infty} G_n = 0$. Therefore we get $\lim_{n\to\infty} G_n = 0$, for every n. we now claim that $\{y_n\}$ is a Cauchy sequence.
It is sufficient to show that $\{y_{2n}\}$ is a Cauchy.Suppose that $\{y_{2n}\}$ is not a Cauchy sequence,then there is an$\epsilon > 0$ such that for each integer 2k, for $k = 0, 1, 2\cdots$,there exists even integers $2n_k, 2m_k$ with $2k < 2n_k < 2m_k$ such that $G(y_{2n_k}, y_{2m_k}, y_{2m_k}) > \epsilon$ and
$G(y_{2n_k}, y_{2m_k-2}, y_{2m_k-2}) \leq \epsilon$
Now

$$
\begin{aligned}
\epsilon < G(y_{2n_k}, y_{2m_k}, y_{2m_k}) \\
&\leq G(y_{2n_k}, y_{2m_k-2}, y_{2m_k-2}) + G(y_{2m_k-2}, y_{2m_k-1}, y_{2m_k-1}) \\
&\quad + G(y_{2m_k-1}, y_{2m_k}, y_{2m_k})
\end{aligned}
$$

on letting $k \to \infty$, we obtain $G(y_{2n_k}, y_{2m_k}, y_{2m_k}) = \epsilon$.
Moreover,we have
$|G(y_{2n_k-1}, y_{2m_k}, y_{2m_k}) - G(y_{2n_k}, y_{2m_k}, y_{2m_k})| \leq 2G(y_{2n_k-1}, y_{2n_k}, y_{2n_k})$

$$
\begin{aligned}
&|G(y_{2n_k}, y_{2m_k+1}, y_{2m_k+1}) - G(y_{2n_k}, y_{2m_k}, y_{2m_k})| \\
&\qquad \leq 2G(y_{2m_k}, y_{2m_k+1}, y_{2m_k+1})
\end{aligned}
$$

$$
\begin{aligned}
&|G(y_{2n_k-1}, y_{2m_k+1}, y_{2m_k+1}) - G(y_{2n_k-1}, y_{2m_k}, y_{2m_k})| \\
&\qquad \leq 2G(y_{2m_k}, y_{2m_k+1}, y_{2m_k+1})
\end{aligned}
$$

on letting $k \to \infty$ we get
$\lim_{n\to\infty} G(y_{2n_k-1}, y_{2m_k}, y_{2m_k}) = \epsilon$

$\lim_{n\to\infty} G(y_{2n_k}, y_{2m_k+1}, y_{2m_k+1}) = \epsilon$
$\lim_{n\to\infty} G(y_{2n_k-1}, y_{2m_k+1}, y_{2m_k+1}) = \epsilon$
Now by (iii) of the Theorem 12,we have

$$\begin{aligned}
&G(y_{2n_k}, y_{2m_k+1}, y_{2m_k+1})\\
&= G(Ax_{2n_k}, Ax_{2m_k+1}, Ax_{2m_k+1})\\
&\leq \phi\begin{pmatrix} G(Sx_{2n_k}, Tx_{2m_k+1}, Tx_{2m_k+1}), & G(Ax_{2n_k}, Tx_{2m_k+1}, Tx_{2m_k+1}),\\ G(Sx_{2n_k}, Ax_{2m_k+1}, Ax_{2m_k+1}), & G(Sx_{2n_k}, Ax_{2n_k}, Ax_{2n_k}),\\ G(Tx_{2m_k+1}, Ax_{2m_k+1}, Ax_{2m_k+1}) & \end{pmatrix}\\
&\leq \phi\begin{pmatrix} G(y_{2n_k-1}, y_{2m_k}, y_{2m_k}), & G(y_{2n_k}, y_{2m_k}, y_{2m_k}),\\ G(y_{2n_k-1}, y_{2m_k+1}, y_{2m_k+1}), & G(y_{2n_k-1}, y_{2n_k}, y_{2n_k}),\\ G(y_{2m_k}, y_{2m_k+1}, y_{2m_k+1}) & \end{pmatrix}
\end{aligned}$$

on letting $k \to \infty$, we get $\epsilon \leq \phi(\epsilon, \epsilon, \epsilon, 0, 0) < \epsilon$ which is a contradiction,hence the sequence $\{y_{2n}\}$ is a Cauchy.
Since X is complete G-metric space,it converges to a point z in X such that $\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} Ax_{2n} = \lim_{n\to\infty} Sx_{2n+1} = \lim_{n\to\infty} Ax_{2n+1}$
$= \lim_{n\to\infty} Tx_{2n+2} = z$.
Suppose $S(X)$ is closed subset of X then there exists a point $u \in X$ such that $Su = z$. we now show that $Au = z$.If $Au \neq z$,then $G(Au, z, z) > 0$.
From(iii)of the Theorem 12

$$\begin{aligned}
&G(Au, y_{2n+1}, y_{2n+1})\\
&= G(Au, Ax_{2n+1}, Ax_{2n+1})\\
&\leq \phi\begin{pmatrix} G(Su, Tx_{2n+1}, Tx_{2n+1}), & G(Au, Tx_{2n+1}, Tx_{2n+1}),\\ G(Su, Ax_{2n+1}, Ax_{2n+1}), & G(Su, Au, Au),\\ G(Tx_{2n+1}, Ax_{2n+1}, Ax_{2n+1}) & \end{pmatrix}
\end{aligned}$$

on letting $n \to \infty$ we obtain

$$\begin{aligned}
G(Au, z, z) &\leq \phi\begin{pmatrix} G(z, z, z), & G(Au, z, z), & G(z, z, z),\\ G(z, Au, Au), & G(z, z, z) & \end{pmatrix}\\
&= \phi\Big(0, G(Au, z, z), 0, G(z, Au, Au), 0\Big)\\
&\leq \phi\begin{pmatrix} G(Au, z, z), & G(Au, z, z), & G(Au, z, z),\\ 2G(Au, z, z), & G(Au, z, z) & \end{pmatrix}\\
&< G(Au, z, z)
\end{aligned}$$

which is a contradiction,hence $Au = z$,thus $Au = Su = z$.
Since the pair (A, S) is weakly compatible then $ASu = SAu$ implies $Az = Sz$.We now show that$Az = z$. If $Az \neq z$ then $G(Az, z, z) > 0$.
From(iii) of the Theorem 12, we have

$$\begin{aligned} &G(Az, y_{2n+1}, y_{2n+1}) \\ &= G(Az, Ax_{2n+1}, Ax_{2n+1}) \\ &\leq \phi\begin{pmatrix} G(Sz, Tx_{2n+1}, Tx_{2n+1}), & G(Az, Tx_{2n+1}, Tx_{2n+1}), \\ G(Sz, Ax_{2n+1}, Ax_{2n+1}), & G(Sz, Az, Az), \\ G(Tx_{2n+1}, Ax_{2n+1}, Ax_{2n+1}) & \end{pmatrix} \end{aligned}$$

on letting $n \to \infty$ we obtain

$$\begin{aligned} G(Az, z, z) &\leq \phi\begin{pmatrix} G(Sz, z, z), & G(Az, z, z), & G(Sz, z, z), \\ G(Az, Az, Az), & G(z, z, z) & \end{pmatrix} \\ &= \phi\Big(G(Az, z, z), G(Az, z, z), G(Az, z, z), 0, 0\Big) \\ &\leq \phi\begin{pmatrix} G(Az, z, z), & G(Az, z, z), & G(Az, z, z), \\ 2G(Az, z, z), & G(Az, z, z) & \end{pmatrix} \\ &< G(Az, z, z) \end{aligned}$$

which is a contradiction,hence $Az = z$,thus $Az = Sz = z$.
Proving that z is common fixed point of A and S.
Since $A(X) \subseteq T(X)$,then there exists a point $v \in X$ such that $Tv = Az = z$.We now show that $Av = z$. If $Av \neq z$,then $G(z, Av, Av) > 0$.From (iii)of the Theorem 12 ,we have

$$\begin{aligned} &G(z, Av, Av) \\ &= G(Az, Av, Av) \\ &\leq \phi\begin{pmatrix} G(Sz, Tv, Tv), & G(Az, Tv, Tv), & G(Sz, Av, Av), \\ G(Sz, Az, Az), & G(Tv, Av, Av) & \end{pmatrix} \\ &= \phi\Big(G(z, z, z), G(z, z, z), G(z, Av, Av), G(z, z, z), G(z, Av, Av)\Big) \\ &= \phi\Big(0, 0, G(z, Av, Av), 0, G(z, Av, Av)\Big) \\ &\leq \phi\begin{pmatrix} G(z, Av, Av), & G(z, Av, Av), & G(z, Av, Av), \\ 2G(z, Av, Av), & G(z, Av, Av) & \end{pmatrix} \\ &< G(z, Av, Av) \end{aligned}$$

which is a contradiction,hence $Av = z$,thus $Av = Tv = z$.
Since the pair (A, T) is weakly compatible then $ATu = TAu$ implies $Az = Tz = z$.
Showing that z is a common fixed point of A, S and T.
Uniqueness: Let w be the another common fixed point of A, S and T.If $z \neq w$ then From (iii)of the Theorem 12,we have

$$\begin{aligned}
&G(z, w, w)\\
&= G(Az, Aw, Aw)\\
&\leq \phi \begin{pmatrix} G(Sz, Tw, Tw), & G(Az, Tw, Tw), & G(Sz, Aw, Aw),\\ G(Sz, Az, Az), & G(Tw, Aw, Aw) & \end{pmatrix}\\
&= \phi\Big(G(z, w, w), G(z, w, w), G(z, w, w), G(z, z, z), G(w, w, w)\Big)\\
&= \phi\Big((G(z, w, w), G(z, w, w), G(z, w, w), 0, 0\Big)\\
&\leq \phi\Big((G(z, w, w), G(z, w, w), G(z, w, w), 2G(z, w, w), G(z, w, w)\Big)\\
&< G(z, w, w)
\end{aligned}$$

which is a contradiction,hence $w = z$. Therefore z is the the unique common fixed point of A, S and T.
As an example of the above theorem we have the following
Example: Let $X = [0, 1]$ with $G(x, y, z) = |x - y| + |y - z| + |z - x|$ for $x, y, z \in X$ then G is a G-metric on X.
Define $A : X \to X, S : X \to X, T : X \to X$, by
$Ax = \frac{1}{2}$,$Sx = \frac{x+1}{3}$, $Tx = \frac{x+2}{5}$ for all $x \in X$
$AX = \{\frac{1}{2}\}, SX = \left[\frac{1}{3}, \frac{2}{3}\right], TX = \left[\frac{2}{5}, \frac{3}{5}\right]$
Clearly $A(X) \subseteq S(X)$ and $A(X) \subseteq T(X)$.
Also $S(x)$ and $T(X)$ are closed subsets of X.
Note that $\frac{1}{2}$ is a coincident point of the pairs (A, S) and (A, T) also we have $AS(\frac{1}{2}) = SA(\frac{1}{2})$ and $AT(\frac{1}{2}) = TA(\frac{1}{2})$, so that the pairs (A, S) and (A, T) are weakly compatible.
And $G(Ax, Ay, Ay) = 2|Ax - Ay| = 0$ for all $x, y \in X$

$$G(Ax, Ay, Ay) = 0 \leq \phi \begin{pmatrix} G(Sx, Ty, Ty), & G(Ax, Ty, Ty),\\ G(Sx, Ay, Ay), & G(Sx, Ax, Ax),\\ G(Ty, Ay, Ay) & \end{pmatrix}$$

Therefore all the conditions of the Theorem 12 hold. Moreover $\frac{1}{2}$ is a unique common fixed of A, S and T. □

Corollary 13. *Let A, S, and T be selfmaps of a complete G-metric space(X, G) satisfying the following conditions*

(i) $A(X) \subseteq S(X)$ *and* $A(X) \subseteq T(X)$

(ii) one of $S(X)$ *and* $T(X)$ *is closed subset of* X.

(iii) $G(Ax, Ay, Ay) \leq \phi \begin{pmatrix} G(Sx, Ty, Ty), & G(Ax, Ty, Ty), \\ G(Sx, Ay, Ay), & G(Sx, Ax, Ax), \\ G(Ty, Ay, Ay) & \end{pmatrix}$
for every $x, y \in X$ *and* $\phi \in \Phi$

(iv) The pairs (A, T)*and* (A, S) *are commuting*

Then A, S *and* T *have a unique common fixed point in* X

Proof. From the fact commutativity implies weakly compatibility, The proof of the corollary follows from the Theorem 12 □

Corollary 14. *Let A and T be selfmaps of a complete G-metric space(X, G) satisfying the following conditions*

(i) $A(X) \subseteq T(X)$

(ii) $T(X)$ *is a closed subset of* X.

(iii) $G(Ax, Ay, Ay) \leq \phi \begin{pmatrix} G(Tx, Ty, Ty), & G(Ax, Ty, Ty), \\ G(Tx, Ay, Ay), & G(Tx, Ax, Ax), \\ G(Ty, Ay, Ay) & \end{pmatrix}$
for every $x, y \in X$ *and* $\phi \in \Phi$

(iv) The pair (A, T) *is weakly compatible*

Then A *and* T *have a unique common fixed point in* X

Proof. By taking $T = S$ in the Theorem 12 □

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