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OPTIMAL SOLUTION OF BALANCED AND UNBALANCED FUZZY TRANSPORTATION PROBLEM BY USING OCTAGONAL FUZZY NUMBERS

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Abstract: The transportation problem is one of the oldest applications of linear programming problem. In this present article Fuzzy Transportation problem has been taken to know that the values are Fuzzy i.e. cost, supply, demand and so on. Octagonal Fuzzy Numbers are used and showed membership function with normal graphical representation. By using BFTP and UFTP we have solved numerical examples with the help of such method for optimal solutions.

AMS Subject Classification: 49K, 90B, 94D
Key Words: fuzzy transportation problem, octagonal fuzzy number, Vogel's approximation method, Best candidate method, fuzzy ranking technique

1. Introduction

Many methods are available to solve the mathematical problems in operation Research, so these methods are useful to clear the complexity. Linear programming problem is the most prominent technique in operation research. Taha [6] introduced transportation model and different methods to understand transport

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route. The transportation model is one of the techniques of linear programs that deal with shipping schedule which can satisfy supply and demand of goods. Transportation problem is more convenient in supply chain management to reduce time and better effect.

Klir [5] constructed fuzzy models to clear the complexity, credibility and uncertainty. Fuzzy transportation problem of transportation problem is more accurate to solve such problems which cant come by the used methods. It means that FTP is leading method in transportation problem. It gives ranking techniques for numbers. Dhanalakshmi and feblin [18] are used ranking method with fuzzy numbers. Fuzzy transportation problem is a transportation problem that all parameters must be fuzzy numbers i.e. supply and demand quantities are fuzzy quantities. Fegade and Jaddhav [8] used zero suffix method to find FTP with triangular fuzzy numbers. Annie and Malini [9] proposed centroid ranking method to solve FTP by using BCM method. They used hexagonal fuzzy numbers. Mohideen and Devi [15] deal with alpha cut and fuzzy octagonal numbers with the help of ranking method. There are many methods to calculate the fuzzy transportation problem. Pathinathan and ponnivalam [17] used pentagonal fuzzy numbers. Also Anandhi and Ramesh [13] solved pentagonal transportation problem using fuzzy numbers and gave optimum solution. Thamarasailvi and Santhi [3] used fuzzy numbers and calculate the fuzzy transportation problem.Solving transportation problem supply and demand must be balanced but some time problem occurring with unbalanced numbers. Finally, with the help of dummy variables we solved fuzzy transportation problem. Ghadle and Pathade[7] compare balanced and unbalanced fuzzy transportation problem by using hexagonal fuzzy numbers and robust ranking technique. Cheng [4] used a fuzzy approach to solve fuzzy transportation problem. Rajarajeshwari and Sangeeta [11] used hexagonal fuzzy transportation problem. For nearest optimal solution BCM is best option. Ahmed [2] and Ahmed and Mohammad [1] solved FTP by using Best Candidate method.

2. Preliminary

In this section, we collect some basic definitions that will be important to us in the sequel.

Definition 2.1. A fuzzy set is characterized by a membership function mapping element of a domain, space, or the universe of discourse X to the unit interval $[0,1]$ i.e $A = \{(\mu_A(x); x \in X)\}$. Here $\mu_A(x) : X \longrightarrow [0,1]$ is a mapping called the degree of membership value of $x\ inX$ in the fuzzy set A.

These membership grades are often represented by real numbers ranking from $[0, 1]$ [14, 12].

Definition 2.2. An octagonal fuzzy number denoted by A_ω is defined to be the ordered quadruple $A_\omega = (l_1(r), s_1(t), s_2(t), l_2(r))$, for $r \in [0, k]$ and $t \in [k, w]$, where,

1. $l_1(r)$ is a bounded left continuous non decreasing function over $[0, w_1]$, $[0 \leq w_1 \leq k]$.
2. $s_1(t)$ is a bounded left continuous non decreasing function over $[k, w_2]$, $[k \leq w_2 \leq w]$.
3. $s_2(t)$ is a bounded left continuous non increasing function over $[k, w_2]$,$[k \leq w_2 \leq w]$.
4. $l_2(r)$ is a bounded left continuous non increasing function over $[0, w_1]$, $[0 \leq w_1 \leq k]$.

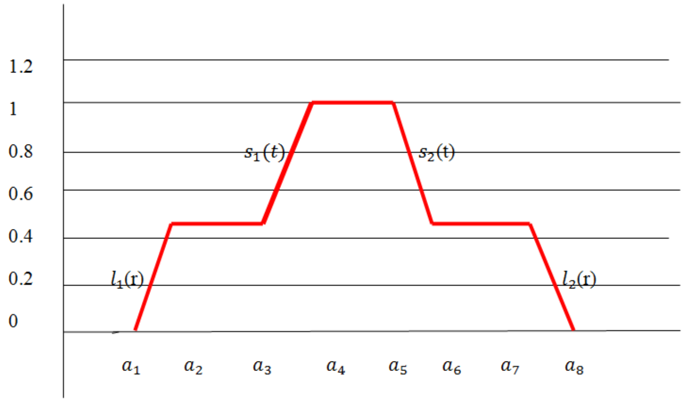
Remark 2.1. If w = 1, then the above defined number is called a normal octagonal fuzzy number.

Definition 2.3. Membership Function

$$\mu_A(x) = \begin{cases} 0, & x < a_1, \\ k\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \leq x \leq a_2 \\ k, & a_2 \leq x \leq a_3 \\ k + (1-k)\left(\frac{x-a_3}{a_4-a_3}\right), & a_3 \leq x \leq a_4 \\ 1, & a_4 \leq x \leq a_5 \\ k + (1-k)\left(\frac{a_6-x}{a_6-a_5}\right), & a_5 \leq x \leq a_6 \\ k, & a_6 \leq x \leq a_7 \\ k\left(\frac{a_8-x}{a_8-a_7}\right), & a_7 \leq x \leq a_8 \\ 0, & x \geq a_8. \end{cases} \tag{2.1}$$

Remark 2.2. If $k = 0$, the octagonal fuzzy number reduces to the trapezoidal number (a_3, a_4, a_5, a_6) and if $k = 1$ it reduces to the trapezoidal number (a_1, a_4, a_5, a_8).

Remark 2.1. Membership functions μ_A are continuous functions.



Remark 2.2. Here A_ω represents a fuzzy number in which "ω" is the maximum membership value that a fuzzy number takes on. Whenever a normal fuzzy number is meant, the fuzzy number is shown by A, for convenience.

Definition 2.4. If A_ω be an octagonal fuzzy number, then the α-cut of A_ω is

$$[A_\omega]\alpha = X/A_\omega(X) \geq \alpha = \begin{cases} [(\alpha), l_2(\alpha)] \quad for\ \alpha \in [0,k), \\ [s_1(\alpha), s_2(\alpha)] for\ \alpha \in [k,\omega) \end{cases} \tag{2.2}$$

Remark 2.3. The octagonal fuzzy number is convex as their α- cuts are convex sets in the classical sense.

Remark 2.4. The collection of all octagonal fuzzy numbers from R to I is denoted as $R_\omega(I)$ and if $\omega = 1$, then the collection of normal octagonal fuzzy numbers is denoted by $R(I)$.

Graphical representation of octagonal fuzzy number

Definition 2.5. **[16]** A measure of fuzzy number A_ω is a function M_α : $R_\omega(I) \to R^+$ which assigns a non negative real number M_α (A_ω) that expresses the measure of A_ω.

$$\begin{aligned} M_\alpha^{oct}(A_\omega) =& \frac{1}{2}\int_\alpha^k (l_1(r) + l_2(r))dr \\ &+ \frac{1}{2}\int_k^\omega (s_1(t) + s_2(t))dt \\ where& \qquad 0 \leq \alpha \leq 1 \end{aligned}$$

The measure of an octagonal fuzzy number is obtained by the average of the two fuzzy side areas, left side area and right side area, from membership function α

axis. Let A be a normal octagonal fuzzy number. The value $M_{\alpha}^{oct}(A_{\omega})$, called the measure of A is calculated as follows:

$$
\begin{aligned}
M_{\alpha}^{oct}(A_{\omega}) =& \frac{1}{2}\int_{\alpha}^{k}(l_1(r)+l_2(r))dr \\
&+\frac{1}{2}\int_{k}^{\omega}(s_1(t)+s_2(t))dt \\
&where \qquad 0 \le \alpha \le 1 \\
&=\frac{1}{4}[(a_1+a_2+a_7+a_8)k]+(a_3+a_4+a_5+a_6)(1-k]
\end{aligned}
$$

3. Best Candidate Method

step 1: Must check the matrix balance, if the total supply equal to the total demand then the matrix is balanced and also apply step 2. If the total supply is not equal to the total demand, then we add a dummy row or column as needed to make supply is equal to the demand. So the transportation cost to this row or column will be assigned to zero.
step 2: Applying the BCM to determine the best combination that is to produce the lowest total weight of the costs, where is one candidate for each row and column.
step 3: Identify the row with the smallest cost candidate from the chosen combination. Then allocate the demand and the supply as much as possible to the variable with the least unit cost in the selected row or column. Also, we should adjust the supply and demand by crossing out the row/column to be then assigned to zero. If the row or column is not assigned to zero, then we check the selected row if it has an element with lowest cost comparing to the determined element in the chosen combination, then we elect it.
step 4: Elect the next least cost from the chosen combination and repeat step 3 until all columns and rows is exhausted.

Numerical Example:1 Consider the following octagonal fuzzy transportation problem.

Solution: The given octagonal fuzzy transportation problem is a Balanced octagonal fuzzy transportation problem (BFTP).Problem has solved by using above ranking technique and the value of k has 0.4 and complete as follows:

	d_1	d_2	d_3	**supply**
s_1	(0,1,2,3,4,5,6,7)	(8,9,10,11,12,13,14,15)	(4,5,6,7,8,9,10,11)	(3,4,5,7,8,9,10,12)
s_2	(2,4,5,6,7,8,9,11)	(5,6,8,9,10,11,12,15)	(0,1,2,3,4,5,6,7)	(2,3,4,5,6,7,8,9)
s_3	(2,3,4,5,6,7,8,9)	(3,6,7,8,9,10,12,13)	(2,4,5,6,7,8,9,11)	(0,1,2,3,4,5,6,7)
demand	(4,5,6,7,8,9,10,11)	(1,2,3,5,6,7,8,10)	(0,1,2,3,4,5,6,7)	

	d_1	d_2	d_3	**supply**
s_1	3.5	11.5	7.5	7.25
s_2	6.5	9.5	3.5	5.5
s_3	5.5	8.5	6.5	3.5
demand	7.5	5.25	3.5	

By using Vogels approximation method, we can get optimal solution,

	d_1	d_2	d_3	**supply**
s_1	$^{7.25}3.5$	11.5	7.5	7.25
s_2	6.5	$^{2}9.5$	$^{3.5}3.5$	5.5
s_3	$^{0.25}5.5$	$^{3.25}8.5$	6.5	3.5
demand	7.5	5.25	3.5	

The result is as per using VAM,
=(3.5)(7.25)+ (9.5)(2)+ (3.5)(3.5)+ (5.5)(0.25)(8.5)(3.25)
= 85.61
Applying Best Candidate method (BCM)[9] which can also give optimal solution.

	d_1	d_2	d_3	**supply**
s_1	(3.5)	11.5	7.5	7.25
s_2	6.5	9.5	(3.5)	5.5
s_3	5.5	(8.5)	6.5	3.5
demand	7.5	5.25	3.5	

	d_1	d_2	d_3	**supply**
s_1	$^{7.25}$(3.5)	11.5	7.5	7.25
s_2	6.5	$^{2}9.5$	$^{3.5}$(3.5)	5.5
s_3	$^{0.25}5.5$	$^{3.25}$(8.5)	6.5	3.5
demand	7.5	5.25	3.5	

The optimal solution obtained by BCM given as follows:
= (3.5)(7.25)+(9.5)(2)+(3.5)(3.5)+(5.5)(0.25)+(8.5)(3.25)

= 85

Numerical Example:2 Consider the following octagonal fuzzy transportation problem:

	d_1	d_2	d_3	d_4	**supply**
s_1	(-1,0,1,2,3,4,5,6)	(0,1,2,3,4,5,6,7)	(8,9,10,11,12,13,14,15)	(4,5,6,7,8,9,10,11)	(2,4,5,7,7,8,9,10)
s_2	(-2,-1,0,1,2,3,4,5)	(-3,-2,1,0,1,2,3,4)	(2,4,5,6,7,8,9,11)	(-3,-1,0,1,2,4,5,6)	(0,1,1,1,2,3,4,8)
s_3	(2,3,4,5,6,7,8,9)	(3,6,7,8,9,10,12,13)	(11,12,14,15,16,17,18,21)	(5,6,8,9,10,11,12,15)	(1,2,2,2,4,5,6,6)
demand	(4,5,6,7,8,9,10,11)	(1,2,3,5,6,7,8,10)	(0,1,2,3,4,5,6,7)	(-1,0,1,2,3,4,5,6)	

Solution: The given octagonal fuzzy transportation problem is unbalanced fuzzy transportation problem(UBFP). So we add dummy row with zero cost and apply ranking technique.

	d_1	d_2	d_3	d_4	**supply**
s_1	(-1,0,1,2,3,4,5,6)	(0,1,2,3,4,5,6,7)	(8,9,10,11,12,13,14,15)	(4,5,6,7,8,9,10,11)	(2,4,5,7,7,8,9,10)
s_2	(-2,-1,0,1,2,3,4,5)	(-3,-2,1,0,1,2,3,4)	(2,4,5,6,7,8,9,11)	(-3,-1,0,1,2,4,5,6)	(0,1,1,1,2,3,4,8)
s_3	(2,3,4,5,6,7,8,9)	(3,6,7,8,9,10,12,13)	(11,12,14,15,16,17,18,21)	(5,6,8,9,10,11,12,15)	(1,2,2,2,4,5,6,6)
s_4	(0)	(0)	(0)	(0)	(1,1,4,7,8,9,10,10)
demand	(4,5,6,7,8,9,10,11)	(1,2,3,5,6,7,8,10)	(0,1,2,3,4,5,6,7)	(-1,0,1,2,3,4,5,6)	

By using ranking method,

	d_1	d_2	d_3	d_4	**supply**
s_1	2.5	3.5	11.5	7.5	6.5
s_2	1.5	0.5	6.5	1.75	2.5
s_3	5.5	8.5	15.5	9.5	3.5
s_4	0	0	0	0	6.25
demand	7.5	5.25	3.5	2.5	

By vogel's approximation method,

	d_1	d_2	d_3	d_4	**supply**
s_1	2.5	$^{4}3.5$	$^{2.5}11.5$	7.5	6.5
s_2	1.5	0.5	$^{1}6.5$	$^{2.5}1.75$	2.5
s_3	$^{1.25}5.5$	$^{1.25}8.5$	15.5	9.5	3.5
s_4	$^{6.25}0$	0	0	0	6.25
demand	7.5	5.25	3.5	2.5	

The optimal solution obtained by Vogel's approximation method is given as follows:

=(3.5)(4)+(11.5)(2.5)+(6.5)(1)+(1.75)(2.5)

(5.5)(1.25)+(8.5)(1.25)+(0)(6.25)

= 71.11

By Best Candidate Method

	d_1	d_2	d_3	d_4	**supply**
s_1	$^{6.5}$(2.5)	3.5	11.5	7.5	6.5
s_2	1.5	(0.5)	$^{2.5}$6.5	1.75	2.5
s_3	^{1}5.5	8.5	15.5	$^{2.5}$(9.5)	3.5
s_4	0	$^{5.25}$0	^{1}0	0	6.25
demand	7.5	5.25	3.5	2.5	

The optimal solution obtained by BCM is given as follows:

=(2.5)(6.5)+(6.5)(2.5)+(5.5)(1)+(9.5)(2.5)+(0)(5.25)+(0)(1)

=61.75

4. Conclusion

All used values in FTP are fuzzy values. These values are octagonal fuzzy numbers. We have applied BFTP and UFTP in VAM method which gives optimal solution. It is also found that by using BCM we got nearest optimal solution.It means that by using different new methods we get nearest and approximate optimal solution.

References

[1] A. Hlayel, A. Mohammad, Solving transportation problems using the best candidate method, *An International Journal (CSEJI)*, **2**, No. 5, (2012), 23-30.

[2] A. Hlayel, The best candidate method for solving optimization problems, *Journal of Computer Science*, **8**, No. 5, (2012), 711-715.

[3] A. Thamaraiselvi, R. Santhi, Optimal solution of transportation problem using hexagonal fuzzy numbers, *International Journal of Scientific and Engineering Research*, **6**, No. 3, (2015), 40-45.

[4] C. Cheng, A new approach for ranking fuzzy numbers by distance method, *fuzzy sets and systems*, **95**, (1998), 307-317.

[5] G. Klir, *Fuzzy Set Theory Foundations and Applications*, Prentice hall, Inc, 1997.

[6] H. Taha, *Operations Research an Introduction*, Prentice hall, Pearson Education Inc, 2007.

[7] K. Ghadle, P. Pathade, Optimal solution of balanced and unbalanced fuzzy transportation problem using hexagonal fuzzy numbers, *International Journal of Mathematical Research (Pakinsight)*, **5**, No. 2, (2016), 131-137.

[8] M. Fegade, V. Jadhav, A. Muley, Solving fuzzy transportation problem using zero suffix and robust ranking methodology, *IOSR Journal of Engineering*, **2**, No. 7, (2012), 36-39.

[9] M. Annie, D. Malini, An approach to solve transportation problems with octagonal fuzzy numbers with BCM and different ranking techniques, *International Journal of Computer Applications*, **6**, No. 1, (2016), 71-85.

[10] M. Annie, D. Malini, Solving transportation problems with hexagonal fuzzy numbers using best candidate method and different ranking techniques, *Journal of Engineering Research and Applications*, **6**, No. 2, (2016), 76-82.

[11] P. Rajarajeshwari, M. Sangeeta, An effect for solving Fuzzy Transportation Problem using hexagonal fuzzy numbers, *International Journal of Research in Information Technology*, **3**, No. 6, (2015), 295-307.

[12] A. Hamoud and K. Ghadle, Existence and uniqueness theorems for fractional Volterra-Fredholm integro-differential equations, *International Journal of Applied Mathematics*, **31**, No. 3, (2018), 333-348.

[13] R. Anandhi, D. Ramesh, An optimum solution for solving fuzzy pentagonal transportation problem, *International Journal for Scientific Research and Development*, **3**, No. 11, (2016), 292-293.

[14] A. Hamoud and K. Ghadle, Modified Adomian decomposition method for solving fuzzy Volterra-Fredholm integral equations, *J. Indian Math. Soc.*, **85**, No. (1-2), (2018), 52–69.

[15] S. Mohideen, K. Devi, M. Durga, Fuzzy transportation problem of octagon fuzzy numbers with alpha cut and ranking technique, *Journal of Computer*, **1**, No. 2, (2016), 60-67.

[16] S. Malini, F. Kennedy, An approach for solving fuzzy transportation using octagonal fuzzy numbers, *Applied Mathematical Science*, **7**, No. 54, (2013), 2662-2673.

[17] T. Pathinathan, K. Ponnivalam, Pentagonal fuzzy number, *International Journal of Computing Algorithm*, **3**, (2014), 1003-1005.

[18] V. Dhanalakhsmi, F. Kennedy, Some ranking methods for octagonal fuzzy numbers, *International Journal of Mathematical Archive*, **5**, No. 6, (2014), 177-188.