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ESTIMATION OF THE BASIC STOCK OPTION PARAMETERS IN THE SENSE OF ITO STOCHASTIC DYNAMICS

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Abstract: In this paper, stock price basic parameters: expected value and volatility are being estimated in the sense of Ito stochastic dynamics. For model efficiency, stock exchange data of DBS Group Holding Ltd (D05. SI) spanning between May 23, 2010 to May 15, 2016 (weekly data with 312 sample size) are considered. It is remarked that the data, and the proposed models have many applications in financial institutions, and other areas of applied sciences.

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1. Introduction

Decision analysis is a vital aspect of our daily activities either as separate individuals or entities. The state of decision making mostly occur under uncertainty which requires a set of quantitative decision making tools or techniques to help the decision maker under the probabilistic condition. Of the most universal or widely applied decision-making tools under risk or uncertainty is the expected value (EV). The EV calculated as a weighted average, denotes the return to be made if a decision is repeated again and again. The EV takes uncertainty into account by considering the probability of each possible outcome, it converts the information to a single number aiding easier decisions, and calculations involving EV are relatively simple. The stock price parameter is one of the highly volatile variables in a stock exchange market while the associated unstable property calls for concern on the part of investors, since the sudden change in share prices occurs randomly and frequently [1-10]. Therefore, expected value, and volatility functions of stock price process are derived using the stock exchange data of DBS Group Holding Ltd (D05. SI) spanning between May 23, 2010 to May 15, 2016. DBS Group Holding Ltd is an investment holding company with over 22, 194 full time employees in finance sector [11]. It renders a lot of financial and commercial banking services internationally. Its operation is via wealth management or consumer banking. Related works on volatility, stock markets and the likes can be linked to recent articles of [12-17] and the references therein.

2. The Operational Dynamics of Stock Price

Suppose $S(t)$ is the firm stock price of an asset at a specified time t. Then the evolutional dynamic of the stock price is a stochastic differential equation (SDE) of the form:

$$dS = \mu S dt + \sigma S dW,\ S = S\left(t\right),\ W = W\left(t\right) \tag{1}$$

where μ, σ, and $W\left(t\right)$ denote the drift parameter, volatility parameter, and Brownian motion (Wiener process) respectively.

In an integral form, (1) is expressed as:

$$\left\{ \begin{aligned} S\left(t\right) = S\left(t_0\right) &+ \textstyle\int_{t_0}^{t} \mu S\left(\tau\right) d\tau \\ &+ \textstyle\int_{t_0}^{t} \sigma S\left(\tau\right) dW\left(\tau\right),\ \forall t \geq 0. \end{aligned} \right. \tag{2}$$

In (2), the core problem is the calculation of the stochastic integral (the second integral in RHS). This can be resolved via Ito theorem.

2.1. One Dimensional Ito formula

Let $X = \{X_t, t \geq 0\}$ be an adapted stochastic process, then an Ito SDE associated to is defined as:

$$\begin{cases} dX = a\,(X,t)\,dt + b\,(X,t)\,dW\,(t)\,, \\ X\,(0) = X_0,\ t \in \mathrm{R}_+ \end{cases} \tag{3}$$

where $a\,(X,t)$ and $b\,(X,t)$ are average drift term, and diffusion term respectively, while $dW\,(t)$ is a Brownian noise. Suppose that X solves (3), and $h\,(X,t)$ is taken as a smooth function such that $h : \mathrm{R}^+ \to \mathrm{R}$, and define:

$$Z\,(t) = h\,(X,t)\,, \tag{4}$$

then, by Ito, solves the SDE:

$$\left(\begin{array}{c} dZ = \left(\begin{array}{c} \frac{\partial h(X,t)}{\partial t} + a\,(X,t)\,\frac{\partial h(X,t)}{\partial X} \\ +\frac{1}{2}b^2\,(X,t)\,\frac{\partial^2 h(X,t)}{\partial X^2} \end{array} \right) dt \\ +\,b\,(X,t)\,\frac{\partial h(X,t)}{\partial X} dW\,(t) \end{array} \right). \tag{5}$$

The proof of (4) can be found in [3]. So, the solution of (2) which is the stock price at time t is obtained as follows upon the application of the Ito formula:

$$S(t) = S\,(t_0)\,e^{\left(\mu - \frac{\sigma^2}{2}\right)(t-t_0)+\sigma W(t-t_0)}. \tag{6}$$

Therefore for $t_0 = 0$, (5) becomes:

$$S(t) = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t+\sigma W(t)},\ S\,(0) = S_0. \tag{7}$$

2.2. The Stock Price Expected Value

Recall for a GBM, $W = W\,(t)$, $W \sim N\,(0,t)$; showing that $W\,(t)$ has a moment generating function (MGF):

$$\overline{E}\left(e^{\mu W(t)}\right) = e^{\frac{\mu^2}{2}t},\ \mu \in \mathrm{R} \tag{8}$$

where the operator $\overline{E}\,(.)$ denotes mathematical expectation or expected value with respect to .

Now, for a GBM in (6), we take the mathematical expectation of both sides as follows:

$$\begin{aligned}\overline{E}\left(S(t)\right) &= \overline{E}\left(S_0 e^{\left(\mu-\frac{\sigma^2}{2}\right)t+\sigma W(t)}\right)\\ &= \overline{E}\left(S_0\right)\overline{E}\left(e^{\left(\mu-\frac{\sigma^2}{2}\right)t}\right)\overline{E}\left(e^{\sigma W(t)}\right) \\ &= S_0 e^{\left(\mu-\frac{\sigma^2}{2}\right)t}\overline{E}\left(e^{\sigma W(t)}\right)\end{aligned} \quad . \tag{9}$$

Hence, applying (7) in (8) yields:

$$\overline{E}\left(S(t)\right) = S_0 e^{\left(\mu-\frac{\sigma^2}{2}\right)t} e^{\frac{\sigma^2}{2}t}. \tag{10}$$

2.3. The Stock Price Variance

By definition, the variance of $S(t)$ is defined and denoted as:

$$\begin{aligned}Var\left(S\left(t\right)\right) &= \overline{E}\left[S(t)-\overline{E}\left(S(t)\right)\right]^2\\ &= \overline{E}\left[S^2(t)\right]-\left(\overline{E}\left[S(t)\right]\right)^2\end{aligned} \quad . \tag{11}$$

But

$$S^2(t) = S_0^2 e^{2\left(\mu-\frac{\sigma^2}{2}\right)t+2\sigma W(t)} \tag{12}$$

$$\overline{E}\left[S^2(t)\right] = \overline{E}\left[S_0^2 e^{2\left(\mu-\frac{\sigma^2}{2}\right)t+2\sigma W(t)}\right] \tag{13}$$

$$= S_0^2 e^{2\mu t+\sigma^2 t}. \tag{14}$$

Hence,

$$\begin{aligned}Var\left(S\left(t\right)\right) &= \overline{E}\left[S^2(t)\right]-\left(\overline{E}\left[S(t)\right]\right)^2\\ &= \left(S_0^2 e^{2\mu t+\sigma^2 t}\right)-\left(S_0 e^{\mu t}\right)^2\\ &= S_0^2 e^{2\mu t}\left(e^{\sigma^2 t}-1\right).\end{aligned} \tag{15}$$

Equations (9) and (14) thus represent the expected value and the variance of the stock price process, $S\left(t\right)$ respectively.

3. The Drift and Volatility Estimation

Here, the stock rentability is defined as:

$$R_{Si} = \begin{cases} R(t_j) = \frac{S(t_j)-S(t_{j-1})}{S(t_{j-1})}, j \geq 1, \text{ discrete time,} \\ \\ R(t) = \frac{dS(t)}{S(t)}, \text{ continuous time.} \end{cases} \tag{16}$$

Hence, the drift term μ , and the volatility parameter, σ of the rentability function are estimated using unbiased estimators which give: $\mu = \overline{E}[R_{Si}] = 6.211 \times 10^{-3}$, $\sigma = \sqrt{Var(R_{Si})} = 2.631 \times 10^{-2}$, with for data size of , $W(t) \sim N(0,t)$. As such:

$$\overline{E}[S(t)] = (13.62)\, e^{6.211\times 10^{-4} t}. \tag{17}$$

$$\begin{aligned} Var(S(t)) &= S_0^2 e^{2\mu t}\left(e^{\sigma^2 t} - 1\right) \\ &= (13.62)^2\, e^{2\left(6.211\times 10^{-4}\right)t}\left(e^{\left(2.631\times 10^{-2}\right)^2 t} - 1\right) \end{aligned}. \tag{18}$$

Hence,

$$Volatility(S(t)) = \sqrt{(13.62)^2\, e^{2(6.211\times 10^{-4})t}\left(e^{(2.631\times 10^{-2})^2 t} - 1\right)}. \tag{19}$$

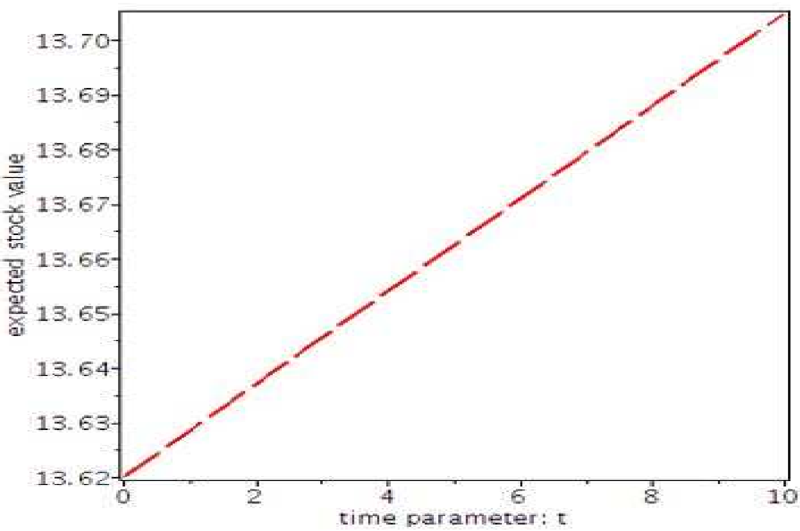

Fig. 3.1: Expected value of the stock price process

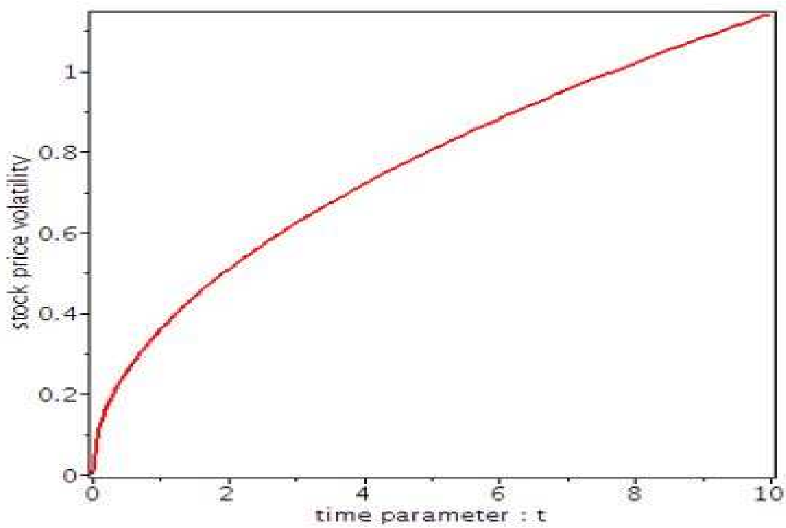

Fig. 3.2: Volatility of the stock price process

Fig. 3.1 and Fig. 3.2 show the expected value and the volatility (respectively) of the stock price process using the simulated data in [11].

4. Concluding Remarks

The result shows that expected value of the stock price is proportional to the time parameter. In a simple approach, we established volatility function with regard to decision making on stock trading. For stock option valuation, decision regarding uncertainty can easily be reached based on the expected value of the stock data. The approach can also be used to benchmark the result of Stratonovich integral in stock option valuation. This can also be used to test the robustness of other models.

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References

[1] L. Bachelier, Theorie de la Speculative, Paris: *Gauthier*- Villar. Translated in Cooler, (1964).

[2] F. Black F, P. Karasinki, Bond and option pricing when short rates lognormal, *Finan. Analysis J.*, **47** (4), (1991), 52-59.

[3] F. Black, M. Scholes, The pricing of options and corporate liability, *J. Political Economy*, **81**, (1973), 637-659.

[4] J. Hull , A. White, The pricing of options on assets with stochastic volatility, *J. Finance*, **42**, (1987), 271-301.

[5] S. Susumu, H. Mijuki, An improvement in the stochastic stock price differences Model, *Journal of Applied Operational Research.*, **4** (3), (2012), 125-138.

[6] J.P. McNicholas, J.L. Rzzo, Stochastic GBM Methods for Modelling Market Prices, *Casuality Actuarial Society E-Forum, Summer* (2012).

[7] S. O. Edeki, I. Adinya, O. O. Ugbebor, The Effect of Stochastic Capital Reserve on Actuarial Risk Analysis via an Integro-differential Equation, *IAENG International Journal of Applied Mathematics*, vol. **44** (2), (2014), 83-90.

[8] M. E. Adeosun, S. O. Edeki, O.O. Ugbebor, Stochastic Analysis of Stock Market Price Models: A Case Study of the Nigerian Stock Exchange (NSE), *WSEAS Transactions on Mathematics*, **14**, (2015), 353-363.

[9] R. S. Sorin, Stochastic modeling of stock prices, *Mongometry investment technology Inc.* 200 Federal Street Camden, NJ 08103, (1997).

[10] S. O. Edeki, M.E. Adeosun, E.A. Owoloko, G.O. Akinlabi, I. Adinya, Parameter Estimation of Local Volatility in Currency Option Valuation, *International Review on Modelling and Simulations* (IREMOS), **9** (2), (2016), 130-133. https://doi.org/10.15866/iremos.v9i2.8161.

[11] https://sg.finance.yahoo.com/quote/D05.SI/history?period1.

[12] S. O. Edeki, and O.O. Ugbebor, On a Generalized Squared Gaussian Diffusion Model for Option Valuation, *MATEC Web of Conferences*, **125**, 02015, (2017).

[13] S.O. Edeki, O.O. Ugbebor, E.A. Owoloko, On a Nonlinear Transaction-Cost Model for Stock Prices in an Illiquid Market Driven by a Relaxed Black-Scholes Model Assumptions, *Malaysian Journal of Mathematical Sciences*, **11** (1), (2017), 83 - 96.

[14] G. Wang, X. Wang . K. Zhoui, Pricing vulnerable options with stochastic volatility, *Physica A* **485**, (2017), 91–103.

[15] S. O. Edeki, O. O. Ugbebor, E.A. Owoloko, On a Dividend-Paying Stock Options Pricing Model (SOPM) Using Constant Elasticity of Variance Stochastic Dynamics, *International Journal of Pure and Applied Mathematics*, **106** (4), (2016), 1029-1036.

[16] L. Panzar, E.C. Cipu, Using of stochastic Ito and Stratonovich integrals derived security pricing, *Proceedings of The 3-rd International Colloquium. Mathematics in Engineering and Numerical Physics,* October 7-9, Bucharest, Romania, (2004), 222-229.Luigi T. De Luca, Propulsion physics (EDP Sciences, Les Ulis, 2009).

[17] S.O. Edeki, E.A. Owoloko, and O.O. Ugbebor, The Modified Black-Scholes Model via Constant Elasticity of Variance for Stock Options Valuation, *AIP Conference proceedings*, **1705**, 020041 (2016); doi: 10.1063/1.4940289.