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SOME NOTES ON THE EXTENDED BURR XII SOFTWARE RELIABILITY MODEL

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Abstract: In this paper we study the Hausdorff approximation of the Heaviside step function by extended Burr XII cumulative distribution function.

The results have independent significance in the study of issues related to debugging theory. Numerical examples, illustrating our results are presented using programming environment Mathematica.

We give also real example with data provided in Yamada and Tamura [8] for testing Apache HTTP Server Project which is developed and maintained an open-source Apache HTTP server for modern operating systems including UNIX and Windows.

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Key Words: extended Burr XII cumulative function (Bcdf), Hausdorff approximation, upper and lower bounds

1. Introduction

Based on the general Type I half–logistic–family of distributions [1], Ghosh and

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Bourguignon [2] consider the following cumulative distribution function

$$M(t;\lambda;c) = 1 - \frac{2}{1+(1+t^c)^\lambda} \tag{1}$$

(so-called extended Burr XII cdf(where $c > 0$; $\lambda > 0$.

In this note we study the Hausdorff approximation of the Heaviside step function (see definitions 1–2) by the family (1).

Definition 1. The (basic) step function is:

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0, \end{cases}$$

usually known as *shifted Heaviside step function.*

Definition 2. [3] The Hausdorff distance (the H–distance) $\rho(f,g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f,g) = \max\{ \sup_{A\in F(f)} \inf_{B\in F(g)} ||A-B||, \sup_{B\in F(g)} \inf_{A\in F(f)} ||A-B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|,|x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A-B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

We propose a software modules (intellectual properties) within the programming environment CAS Mathematica for the analysis. The models have been tested with real-world data.

For other extensions of the Burr and Weibull (c.d.f.), see [4] – [7].

2. Main Results

Without loosing of generality we will look at the following extended Burr XII (c.d.f.):

$$M^*(t) = 1 - \frac{2}{1+(1+t^c)^\lambda}, \tag{2}$$

with

$$t_0 = \left(3^{\frac{1}{\lambda}} - 1\right)^{\frac{1}{c}}; \qquad M^*(t_0) = \frac{1}{2}. \tag{3}$$

The one–sided Hausdorff distance d between the Heaviside step function function and the c.d.f. ((2)–(3)) satisfies the relation

$$M^*(t_0 + d) = 1 - d. \tag{4}$$

The following theorem gives upper and lower bounds for d

Theorem 1. Let

$$p = -\frac{1}{2},$$

$$q = 1 + \frac{1}{8}c\lambda\left(3^{\frac{1}{\lambda}} - 1\right)^{\frac{c-1}{c}} 3^{\frac{\lambda-1}{\lambda}}.$$

For the one–sided Hausdorff distance d between $h_{t_0}(t)$ and function ((2)–(3)) the following inequalities hold for:

$$2.1q > e^{1.05} \approx 1.36079$$

$$d_l = \frac{1}{2.1q} < d < \frac{\ln(2.1q)}{2.1q} = d_r. \tag{5}$$

Proof. Let us examine the function:

$$F(d) = M^*(t_0 + d) - 1 + d. \tag{6}$$

From $F'(d) > 0$ we conclude that function F is increasing.

Consider the function

$$G(d) = p + qd. \tag{7}$$

From Taylor expansion we obtain $G(d) - F(d) = O(d^2)$.

Hence $G(d)$ approximates $F(d)$ with $d \to 0$ as $O(d^2)$ (see Fig. 1).

In addition $G'(d) > 0$.

Further, for $2.1q > e^{1.05}$ we have $G(d_l) < 0$ and $G(d_r) > 0$.

This completes the proof of the theorem.

The model ((2)–(3)) for $c = 25$, $\lambda = 0.6$, $t_0 = 1.0685$ is visualized on Fig. 2. From the nonlinear equation (4) and inequalities (5) we have: $d = 0.127277$, $d_l = 0.0878454$, $d_r = 0.213656$.

The model ((2)–(3)) for $c = 35$, $\lambda = 0.8$, $t_0 = 1.03137$ is visualized on Fig. 3. From the nonlinear equation (4) and inequalities (5) we have: $d = 0.0854269$, $d_l = 0.0553577$, $d_r = 0.160202$.

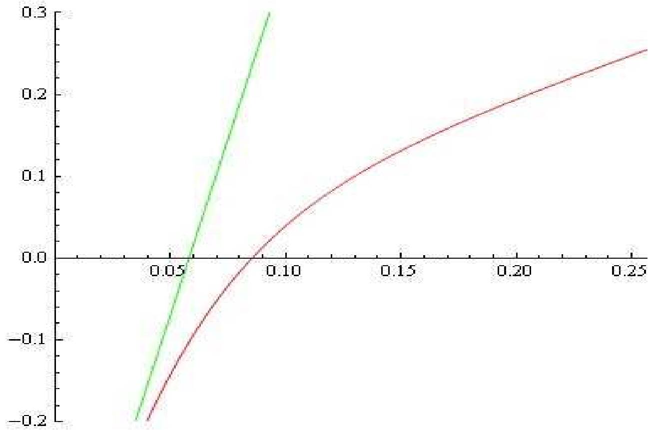

Figure 1: The functions $F(d)$ and $G(d)$.

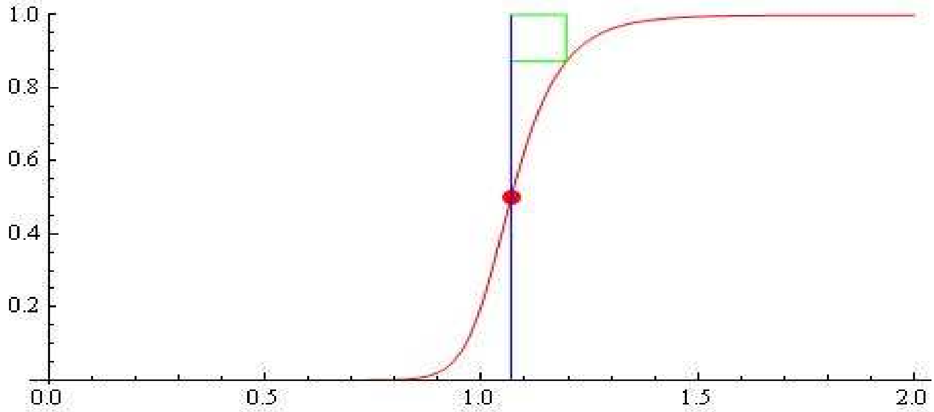

Figure 2: The model ((2)–(3)) for $c = 25$, $\lambda = 0.6$, $t_0 = 1.0685$; H–distance $d = 0.127277$, $d_l = 0.0878454$, $d_r = 0.213656$.

3. Application in the Field of Debugging and Test Theory

We will illustrate what we have said by approximating the data from the Yamada and Tamura [8] for testing Apache HTTP Server Project which is developed and maintained an open-source Apache HTTP server for modern operating systems including UNIX and Windows.

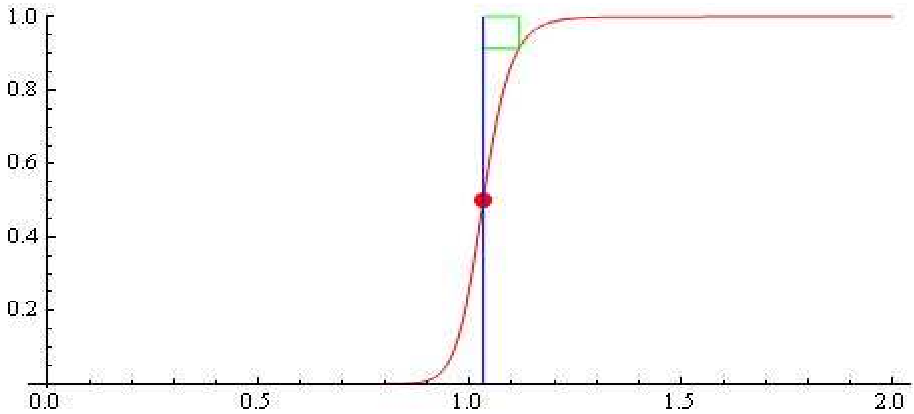

Figure 3: The model ((2)–(3)) for $c = 35$, $\lambda = 0.8$, $t_0 = 1.03137$; H–distance $d = 0.0854269$, $d_l = 0.0553577$, $d_r = 0.160202$.

Unit Time	Cumulative number of detected faults	Unit Time	Cumulative number of detected faults
1	11	18	28
2	11	19	31
3	12	20	31
4	15	21	31
5	17	22	32
6	18	23	32
7	18	24	32
8	18	25	32
9	20	26	33
10	21	27	35
11	24	28	35
12	24	29	37
13	25	30	38
14	26	31	39
15	26	32	39
16	26	33	39
17	27		

Table 1. The actual data in Apache HTTP Server [8].

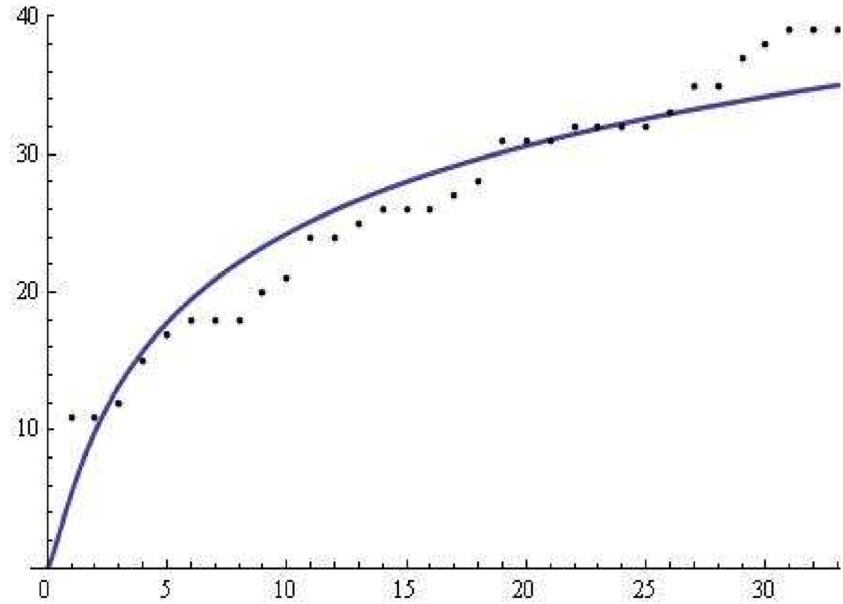

Figure 4: The fitted model $M^*(t)$.

The fitted model $M^*(t)$ based on the data of Table 1 for the estimated parameters: $c = 1.38133$; $\lambda = 0.199886$ is plotted on Fig. 4.

From the above examples, it can be seen that the proven bottom estimate (see Theorem 1) for the value of the Hausdorff approximation is reliable when assessing the important characteristic - "saturation".

This characteristic as it is already known has its equal participation together with the other two characteristics - "confidence intervals" and "confidence bounds" in the area of the Software Reliability Theory.

Some software reliability models, can be found in [9]–[60].

Other important in practice activation functions and possibility of their recurrent generations are explored in [61] and [62].

We hope that the results will be useful for specialists in this scientific area.

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