

ON GEOMETRIC MOMENT INVARIANTS

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Abstract: In this paper we study the geometric moments invariants. We describe an image in terms of features which are invariant to some sort of transformations i.e mentioned translation, rotation and scaling change in exposure, brightness etc. Our aim is to check the performance of components for feature vectors.

AMS Subject Classification: 00A69

Key Words: geometric moments, scaling, translation, rotation

1. Introduction

It is not a secret that we perceive over 90% of information about the world visually. Thus analysis of images is of great importance for modern technology. The same object in different images could undergo different geometric transformations as well as light could be changed or parameters of camera. Hence there is a problem to describe an image or at least part of the image in terms of features which are invariant to some sort of transformations i.e. mentioned translation, rotation, scaling, change in exposure, brightness etc. One of the ways to construct such invariant descriptors is to use image moments (see [1], [2], [8] and [9]).

Received: November 18, 2017

Revised: September 7, 2018

Published: September 9, 2018

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url: www.acadpubl.eu

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2. Orthogonal Geometric Moments

We treat an intensity function of an image $f(x, y)$ as density. Analogy with probability theory on density is useful for image analysis. It is well known fact that it is possible to expand any distribution in Fourier series as [6]

$$\hat{f}(u, v) = \iint f(x, y) e^{i2\pi(ux+vy)} dx dy. \quad (1)$$

Expanding exponent in Taylor series we get

$$\hat{f}(u, v) = \iint f(x, y) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{(2\pi i)^{n+m}}{n!m!} (ux)^n (vy)^m dx dy \quad (2)$$

Taking into account the fact that exponential expansion is uniformly convergent we are allowed to exchange the order of integral and sum

$$\begin{aligned} \hat{f}(u, v) &= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{(2\pi i)^{n+m}}{n!m!} u^n v^m \iint f(x, y) x^n y^m dx dy \\ &= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{(2\pi i)^{n+m}}{n!m!} u^n v^m M_{nm} \end{aligned} \quad (3)$$

where we denoted M_{nm} are the moments of the distribution, i.e. image

$$M_{nm} = \iint x^n y^m f(x, y) dx dy \quad (4)$$

It's easy to that M_{00} is just an integral intensity of the image, M_{10} , M_{01} are the centroids.

Thus two dimensional image, in principle, can be represented as the sum of its moments. Inverse transformation can be performed in the same manner as

$$f(x, y) = \iint \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{(2\pi i)^{n+m}}{n!m!} u^n v^m M_{nm} e^{-i2\pi(ux+vy)} du dv \quad (5)$$

Here we cannot exchange the order of sum and integral in general. In case of discrete function $f(x, y)$ integrals have to be substituted by finite sums.

In theory, all polynomial bases of the same degree are equivalent because they generate the same space of functions. Any moment with respect to a certain basis can be expressed in terms of moments with respect to any other

basis. From this point of view, orthogonal geometric moments of any type are equivalent to geometric moments.

However, a significant difference appears when considering stability and computational issues in a discrete domain. Standard powers are nearly dependent both for small and large values of the exponent and increase rapidly in range as the order increases. This leads to correlated geometric moments and to the need for high computational precision. Using lower precision results in unreliable computation of geometric moments. Orthogonal geometric moments can capture the image features in an improved, nonredundant way. They also have the advantage of requiring lower computing precision because we can evaluate them using recurrent relations, without expressing them in terms of standard powers.

3. Construction of Invariant Moments

The simplest transformations are translation, rotation and scaling (TRS), it is a four parameter transform, which can be represented as [5].

$$x' = sR \cdot x + t \quad (6)$$

where s is a positive scaling coefficient, t is a translation vector and R is a rotation matrix of the convenient form

$$R = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad (7)$$

where θ is the angle of rotation.

Invariant moments regarding the translation, rotation and scaling is necessarily in almost all practical applications. Furthermore, the TRS model is a appropriate approximation of the existing image deformation if the sight is flat and perpendicular to the visual axis. Hence, more attention has been given to TRS invariants.

3.1. Moment Invariants to Translation

One way to build translation invariant is to use central geometric moments of the image.

$$\mu_{nm} = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} (x - \bar{x})^n (y - \bar{y})^m f(x, y), \quad (8)$$

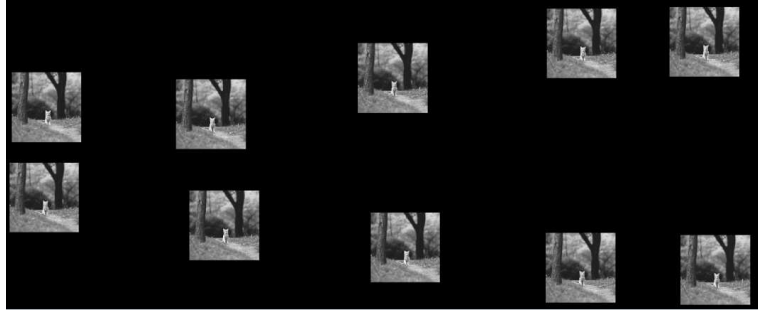


Figure 1: Translated image set

with $\bar{x} = \frac{M_{10}}{M_{00}}$ and $\bar{y} = \frac{M_{01}}{M_{00}}$ are coordinates of a center of mass of an image.

It is clear that if translation moves image by a vector t the centroid goes align and new coordinates will be $(\bar{x}', \bar{y}') = (\bar{x} + t_1, \bar{y} + t_2)$. Thus we eliminate this translation using moments.

Figure 1 shows the image set for different traslation. We apply image traslation in both of x and y axis with different size to test how to build traslation invariant feature with moments.

To estimate the variance caused by image translation, we introduce the relative difference defined as follows.

Definition 1. Relative difference of moment is defined as follows.

$$RD = \frac{M_1}{M_0} \times 100, \quad (9)$$

where M_0 is the original moment and M_1 is the moment of image after traslation.

Figure 2 displays the relative differences of images in Figure 1.

As it can be observed, the relative differences of moments are not trivial to be used as invariant features. It is expected that if the central moments would be much more stable to such transforms.

The central moments can be expressed in terms of geometric moments as

$$\mu_{nm} = \sum_{k=0}^n \sum_{j=0}^m \binom{n}{k} \binom{m}{j} (-1)^{k+j} \bar{x}^k \bar{y}^j M_{n-k, m-j} \quad (10)$$

This relation has importance for theoretic consideration, it is used to calculate the central moments. Figure 3 demonstrates the invariance of central moments.

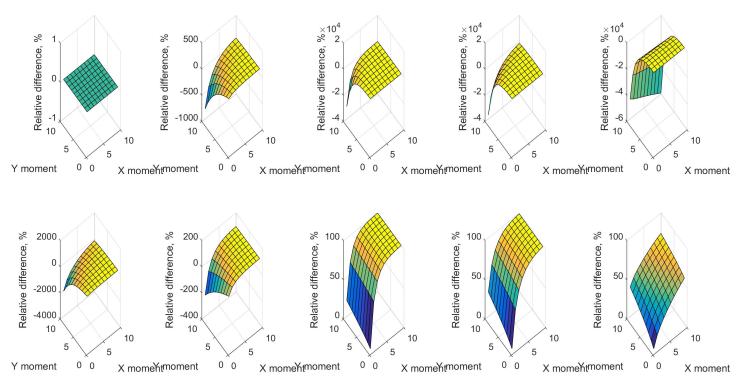


Figure 2: Changing of geometric moments under translations

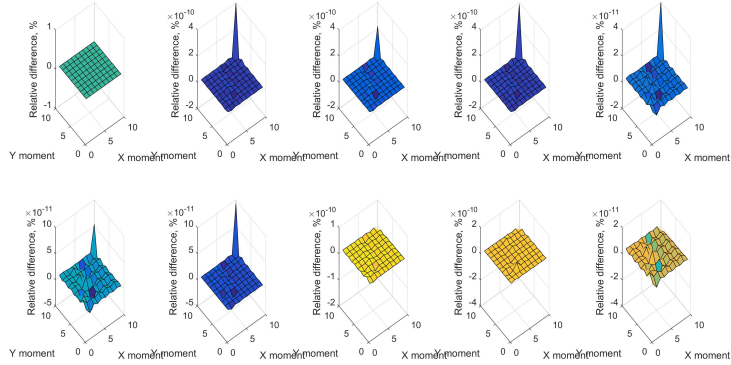


Figure 3: Relative difference of central moments under translation

As we can see in these figures, the relative differences of central moments are trivial to be ignored for consideration. From this, we can conclude that central moments can be used as invariant features against image translation transform.

3.2. Moment Invariants to Scaling

To make moment invariants to scaling, we apply different scaling transform on image set with different angles and estimate the relative difference of moments.

Scaled images are displayed in Figure 4



Figure 4: Scaled image set

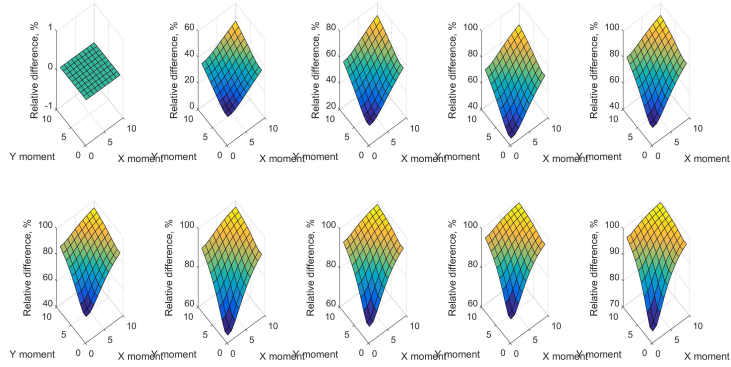


Figure 5: Changing of geometric moments under scale transformation

We calculate the relative difference of moments on this image set. They are shown in Figure 5 .

As we can observe in these figures, when applied on scaled image sets, relative differences of moments are not stable at all on the whole image region. From the result of Figure 4, we can get idea that some normalization might be improved the stability of moments under scaling. The following theorem reveals under which conditions the moment will be stabilized.

Theorem 2. *If we set $\lambda = M_{00}^{-\frac{1}{2}}$ and apply normalization with multiplier λ on central moments, the relative difference will be zero and the central moments will be one.*

Proof. Generally after scaling transform, the central moments can be ex-

pressed as follows.

$$M'_{00} = \lambda^2 M_{00}. \quad (11)$$

Since, any moment can be used as a normalizing factor provided that it is non-zero for all images in the experiment. we normalize most often by a proper power of M_{00} ,

$$M'_{nm} = \frac{M_{nm}}{M_{00}^w}.$$

To remove the scaling parameter λ , the power must be set as

$$w = \frac{n+m}{2} + 1.$$

The value of the corresponding normalized moment is always one (in the above normalization, $M'_{00} = 1$)

$$x' = \frac{1}{\lambda}x, y' = \frac{1}{\lambda}y \quad (12)$$

$$dx = \lambda dx', dy = \lambda dy' \quad (13)$$

thus (4) becomes

$$\begin{aligned} M'_{nm} &= \iint x^n y^m f\left(\frac{1}{\lambda}x, \frac{1}{\lambda}y\right) dx dy \\ &= \lambda^{n+m+2} \iint x'^n y'^m f(x', y') dx' dy'. \end{aligned} \quad (14)$$

Therefore

$$M'_{nm} = \lambda^{n+m+2} M_{nm}. \quad (15)$$

Now it's time to find λ which normalizes the image.

$$M'_{00} = \lambda^2 M_{00} = 1 \Rightarrow \lambda = \frac{1}{\sqrt{M_{00}}} \quad (16)$$

Combination of (16) and (15), we get $\lambda = \frac{1}{\sqrt{M_{00}}} = M M_{00}^{-\frac{1}{2}}$.

Substituting this into the initial equation (11), then we get

$$M'_{00} = M_{00} \cdot \left(M_{00}^{-\frac{1}{2}}\right)^2 = 1.$$

□

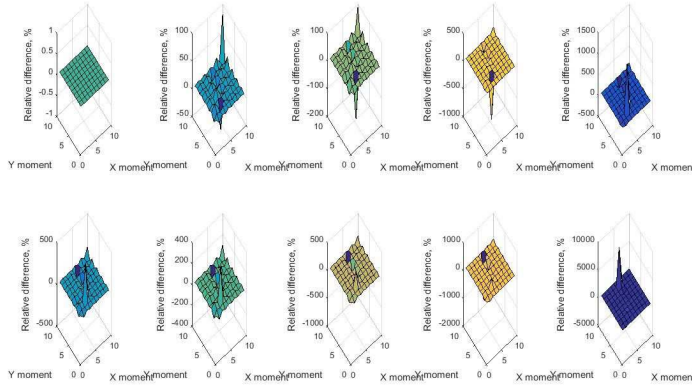


Figure 6: Relative difference of central geometric moments under scaling

Figure 4 and Figure 5 showing the image set and the changing of geometric moments under scale transformation respectively.

Figure 6 shows the effect of normalization with multiplier on central geometric moments.

After normalization, the influence of scaling transform on moments was greatly reduced and it is resulted that normalized central geometric moments can be used as stable features against scaling transform.

3.3. Moment Invariant to Rotation

The same approach can be used to get rotation invariance. Suppose we have a rotate image $f'(x, y)$ such that $f'(x, y) = f(x \cos \theta + y \sin \theta, -x \sin \theta + y \cos \theta)$ with variable transformation we get

$$\begin{aligned} x' &= x \cos \theta + y \sin \theta \\ y' &= -x \sin \theta + y \cos \theta \end{aligned}$$

By following $x' = R \cdot x$ that implies $x = R^{-1}x'$. It is easy to get x and y such that

$$\begin{aligned} x &= x' \cos \theta - y' \sin \theta & dx &= \cos \theta dx' \\ y &= -x' \sin \theta + y' \cos \theta & dy &= \cos \theta dy' \end{aligned}$$

Let us see how geometric moment invariants behave in the case of rotated

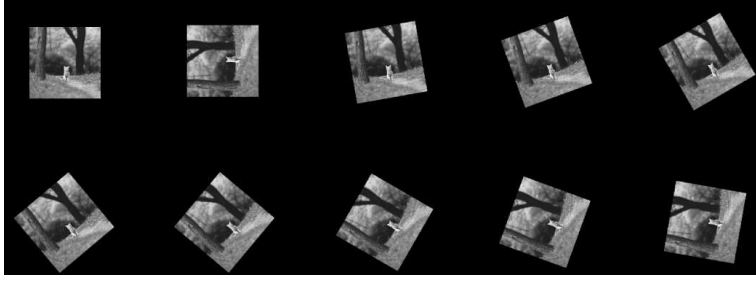


Figure 7: Rotated image set

image from the equation (4),

$$\begin{aligned}
 M_{nm} &= \iint x^n y^m f'(x, y) dx dy \\
 &= \iint x^n y^m f(x' \cos \theta - y' \sin \theta, -x' \sin \theta + y' \cos \theta) dx dy \\
 &= \iint (x' \cos \theta - y' \sin \theta)^n (x' \sin \theta + y' \cos \theta)^m f(x', y') \cos^2 \theta dx' dy'
 \end{aligned}$$

where the angle θ can be found as

$$\theta = \frac{1}{2} \tan^{-1} \frac{2\mu_{11}}{\mu_{20} - \mu_{02}} \quad (17)$$

which is derived as the angle between principal axes of covariance matrix

$$\begin{pmatrix} \mu'_{20} & \mu'_{11} \\ \mu'_{11} & \mu'_{02} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

Figure 7 shows the image set after rotation transform with different angles.

As in previous subsections, we can measure the influence of rotation transform on moments with relative difference. The results are shown in Figure 8

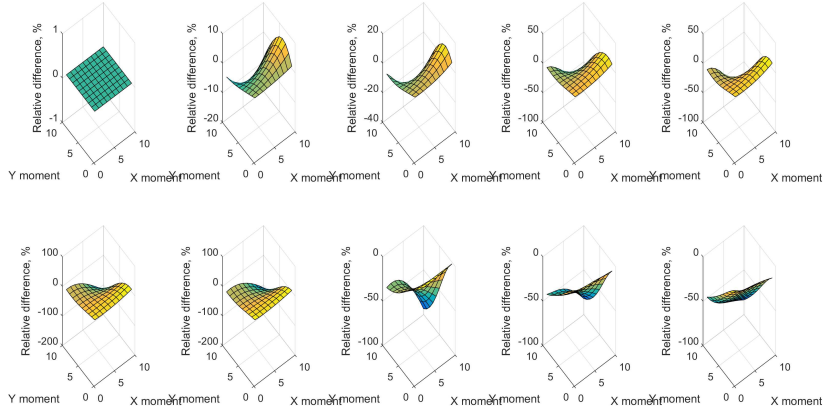


Figure 8: Changing of geometric moments under rotations

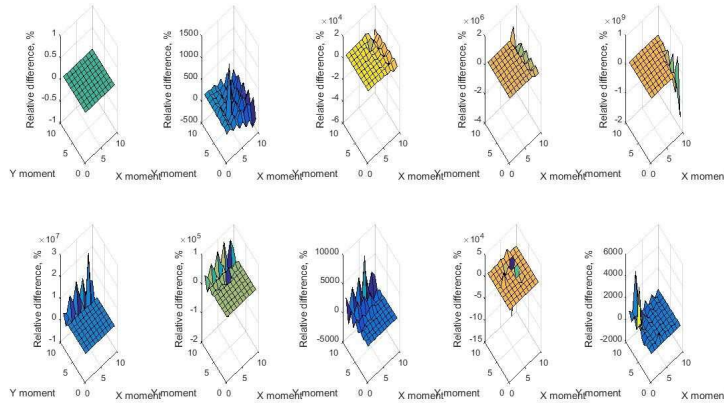


Figure 9: Relative difference of central geometric moments under rotations

We calculate the central geometric moments after transform with the angles founded above equation and display in Figure 9.

As we can see, the relative differences of central geometric moments are zero on almost whole region except for some relatively small regions. The results demonstrate the transformed central moments are stable enough for rotation

transform.

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