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CONTINUITY ASPECTS ON λ_g^δ-CLOSED SETS

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Abstract: This article aims at developing and characterizing various continuities concerned with λ_g^δ-closed sets in topological spaces, followed by the notion of λ_g^δ-irresoluteness, λ_g^δ-closed and λ_g^δ-open maps.

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1. Introduction

Velicko[12] modelled δ-closed sets in 1968, using the concept of regular closed sets defined by Stone[9] in 1937. The concept of δ-closed sets are stronger than that of closed sets. Following this notion, several generalizations of δ-closed sets came into existence. Among the various generalizations, λ_g^δ-closed sets[10] is the nearest weaker form of δ-closed sets and more interestingly, the family of λ_g^δ-open sets form an Alexandorv topology[11]. As continuity

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plays a phenomenal role in the study of topological spaces, few interesting type of continuities namely quasi λ_g^δ-continuity, perfectly λ_g^δ-continuity, totally λ_g^δ-continuity, strongly λ_g^δ-continuity, contra λ_g^δ-continuity are introduced and their properties are obtained. The nature of composition of mappings amongst the above mentioned types of continuity is brought into the lime light. The study of λ_g^δ-irresoluteness is initiated to answer the question, "Does λ_g^δ-irresoluteness preserve the nature of the domain?". The concepts of λ_g^δ-open map and λ_g^δ-closed map are studied to find "Under what conditions these maps coincide?".

2. Prerequisites

Definition 2.1. A subset A of a topological space (X, τ) is called

1. *regular closed* [9] if $A = cl(int(A))$.
2. *δ-open* [12] if A is the union of regular open sets, the complement of δ-open is called δ-closed.
3. *Λ_δ-set* [3] if $\Lambda_\delta(A) = A$, where $\Lambda_\delta(A) = \cap O \in \delta O(X, \tau) \mid A \subseteq O$.
4. *(Λ, δ)-closed* [3] if $A = T \cap C$, where T is a Λ_δ-set and C is a δ-closed set.
5. *λ_g^δ-closed* [10] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is (Λ, δ)-open in X.

The complement of the above mentioned closed sets are called their respective open sets.

Definition 2.2. A map $f : (X, \tau) \to (Y, \sigma)$ is called

1. *strongly continuous* [4] if $f^{-1}(V)$ is clopen in (X, τ) for every subset V in (Y, σ).
2. *super continuous* [6] if $f^{-1}(V)$ is a δ-closed set in (X, τ) for every closed set V in (Y, σ).
3. *irresolute* [1](resp. *δ-continuous*[8]) if $f^{-1}(V)$ is semi-open(resp. δ-open) in (X, τ) for every semi-open(resp. δ-open) V in (Y, σ).

3. λ_g^δ-continuity

Definition 3.1. A map $f : (X, \tau) \to (Y, \sigma)$ is said to be λ_g^δ-*continuous* [11] if the inverse image of every closed set in (Y, σ) is λ_g^δ-closed in (X, τ).

Definition 3.2. For a subset A of a topological space (X, τ),

(i) The λ_g^δ- *closure* of A (briefly $\lambda_g^\delta cl(A)$) in a topological space (X, τ) is defined to be the intersection of all λ_g^δ-closed sets containing A.

(ii) The λ_g^δ- *interior* of A (briefly $\lambda_g^\delta int(A)$) in a topological space (X, τ) is defined to be the union of all λ_g^δ-open sets contained in A.

Theorem 3.3. *For a map $f : (X, \tau) \to (Y, \sigma)$, the following are equivalent.*

(i) f is λ_g^δ-continuous;

(ii) $\lambda_g^\delta cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$, for each $B \in Y$;

(iii) $f(\lambda_g^\delta cl(A)) \subseteq cl(f(A))$, for each $A \in X$;

(iv) $f^{-1}(int(B)) \subseteq \lambda_g^\delta int(f^{-1}(B))$, for each $B \in Y$.

Proof. (i) $\Rightarrow$ (ii) Let $B \in Y$. $cl(B)$ then is closed in $Y \Rightarrow f^{-1}(cl(B))$ is λ_g^δ-closed in X. Therefore $f^{-1}(cl(B)) = \lambda_g^\delta cl(f^{-1}(cl(B))) \supseteq \lambda_g^\delta cl(f^{-1}(B))$.
(ii) $\Rightarrow$ (iii) Let $A \in X$ then $f(A) \in Y$. By (ii), $f^{-1}(cl(f(A))) \supseteq \lambda_g^\delta cl(f^{-1}(f(A))) \supseteq \lambda_g^\delta cl(A)$ and hence $f(\lambda_g^\delta cl(A)) \subseteq cl(f(A))$.
(iii) $\Rightarrow$ (iv) Let $B \in Y$. By (iii), $f(\lambda_g^\delta cl(X \backslash f^{-1}(B))) \subseteq cl(f(X \backslash f^{-1}(B))) \Rightarrow f(X \backslash \lambda_g^\delta int(f^{-1}(B))) \subseteq cl(Y \backslash B) = Y \backslash int(B) \Rightarrow X \backslash \lambda_g^\delta int(f^{-1}(B)) \subseteq f^{-1}(Y \backslash int(B)) \Rightarrow f^{-1}(int(B)) \subseteq \lambda_g^\delta int(f^{-1}(B))$.
(iv) $\Rightarrow$ (i) Let B be an open subset of Y. By (iv), $f^{-1}(int(B)) \subseteq \lambda_g^\delta int(f^{-1}(B)) \Rightarrow f^{-1}(B) \subseteq \lambda_g^\delta int(f^{-1}(B))$. But $\lambda_g^\delta int(f^{-1}(B)) \subseteq f^{-1}(B) \Rightarrow f^{-1}(B)$ is λ_g^δ-open in X. Hence f is λ_g^δ-continuous. $\square$

Theorem 3.4. *If $f : (X, \tau) \to (Y, \sigma)$ is λ_g^δ-continuous with (X, τ) being an almost weakly Hausdorff space then f is continuous.*

4. Other continuities on λ_g^δ-closed sets

Definition 4.1. A map $f : (X, \tau) \to (Y, \sigma)$ is called

1. *quasi λ_g^δ -continuous* if the inverse image of every λ_g^δ-open set in (Y, σ) is open in (X, τ).
2. *perfectly λ_g^δ-continuous* if the inverse image of every λ_g^δ-open set in (Y, σ) is clopen in (X, τ).

Theorem 4.2. *For a map $f : (X, \tau) \to (Y, \sigma)$, the following are equivalent.*

(i) quasi λ_g^δ-continuous;

(ii) $f^{-1}(B)$ is closed in (X, τ), for every λ_g^δ-closed B in (Y, σ);

(iii) For each $x \in X$ and each λ_g^δ-open set B containing $f(x)$, there exists an open set A containing x such that $f(A) \subseteq B$.

Proof. (i) $\Rightarrow$ (ii) is obvious.
(i) $\Rightarrow$ (iii) Let $x \in X$ and let B be an open set containing $f(x)$ then by hypothesis $f^{-1}(B)$ is a λ_g^δ-open set containing x. Let $A = f^{-1}(B)$ then $f(A) = f(f^{-1}(B)) \subseteq B$.
(iii) $\Rightarrow$ (i) Let B be λ_g^δ-open in Y with $x \in f^{-1}(B) \Rightarrow f(x) \in B$ then by hypothesis there exists an open set A containing x such that $f(A) \subseteq B \Rightarrow A \subseteq f^{-1}(B)$. That is, $x \in A \subseteq f^{-1}(B) \Rightarrow f^{-1}(B)$ is open in X. □

Theorem 4.3. *For a map $f : (X, \tau) \to (Y, \sigma)$, the following statements are true.*

(i) Every strongly continuous function is a quasi λ_g^δ-continuous function but not conversely.

(ii) Every perfectly λ_g^δ-continuous function is a quasi λ_g^δ-continuous function but not conversely.

(iii) Every strongly continuous function is a perfectly λ_g^δ-continuous function.

Example 4.4. Let $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be an identity mapping. Take $A = \{a, b\}$ in Y then f is quasi λ_g^δ-continuous but not strongly continuous since $A = \{a, b\}$ is λ_g^δ-open in (Y, σ) whereas $f^{-1}(A) = \{a, b\}$ is open but not closed in (X, τ).

Example 4.5. Let X, Y, τ, σ, f and A be defined as in Example 4.4. Then f is quasi λ_g^δ-continuous but not perfectly λ_g^δ-continuous since A is λ_g^δ-open in (Y, σ) whereas $f^{-1}(A)$ is open but not closed in (X, τ).

Theorem 4.6. *Let $(X, \tau$ be a partition space[7], (Y, σ) be a topological space and $f : (X, \tau) \to (Y, \sigma)$ be any map. Then the following are equivalent.*

(i) f is perfectly λ_g^δ-continuous;

(ii) f is quasi λ_g^δ-continuous.

Definition 4.7. A map $f : (X, \tau) \to (Y, \sigma)$ is called

1. *totally λ_g^δ-continuous* if the inverse image of every open subset of (Y, σ) is λ_g^δ-clopen in (X, τ).
2. *strongly λ_g^δ-continuous* if the inverse image of every subset of (Y, σ) is λ_g^δ-clopen in (X, τ).
3. *contra λ_g^δ-continuous* if the inverse image of every closed set of (Y, σ) is λ_g^δ-open in (X, τ).

Theorem 4.8. *If (X, τ) and (Y, σ) are any two topological spaces with a map $f : (X, \tau) \to (Y, \sigma)$ then the following statements are true.*

(i) Every strongly λ_g^δ-continuous function is a totally λ_g^δ-continuous function but not conversely.

(ii) Every totally λ_g^δ-continuous function is a contra λ_g^δ-continuous function but not conversely.

(iii) Every strongly λ_g^δ-continuous function is a contra λ_g^δ-continuous function but not conversely.

Example 4.9. Let $X = Y = \{a, b, c, d\}, \tau = \{X, \phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$ and $\sigma = \{X, \phi, \{a, b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be an identity mapping. Then f is totally λ_g^δ-continuous but not strongly λ_g^δ-continuous since the inverse image of the closed set $\{d\}$ is λ_g^δ-clopen in (X, τ) but the inverse image of $\{a\}$ is not λ_g^δ-clopen in (X, τ).

Example 4.10. Let X, Y, τ and σ be defined as in Example 4.9. Let $f : (X, \tau) \to (Y, \sigma)$ be defined as $f(a) = d, f(b) = b, f(c) = c$ and $f(d) = a$. Then f is contra λ_g^δ-continuous but not totally λ_g^δ-continuous as the inverse image of $\{d\}$ is $\{a\}$ and $\{a\}$ is λ_g^δ-open but not λ_g^δ-closed in (X, τ).

Example 4.11. Let X, Y, τ, σ and f be defined as in Example 4.10. Then f is contra λ_g^δ-continuous but not strongly λ_g^δ-continuous as the inverse image of $\{b\}$ is $\{b\}$ itself and $\{b\}$ is not λ_g^δ-clopen in (X, τ).

Theorem 4.12. *Let $f : (X, \tau) \to (Y, \sigma)$ be a totally λ_g^δ-continuous function, where (Y, σ) is a partition space. Then f is a strongly λ_g^δ-continuous function.*

Remark 4.13. The composition of two contra λ_g^δ-continuous functions need not be a contra λ_g^δ-continuous function.

Example 4.14. Let $X = Y = Z = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}, \sigma = \{X, \phi, \{a, b\}\}$and $\eta = \{X, \phi, \{c\}\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c, f(b) = a, f(c) = b$ and $g : (Y, \sigma) \to (Z, \eta)$ as an identity mapping. Then f and g are contra λ_g^δ-continuous functions but $g \circ f$ is not λ_g^δ-continuous as $f^{-1}(g^{-1}\{c\}) = \{c\}$ is not λ_g^δ-open in (X, τ).

Remark 4.15. A significant result on the composition of various continuities is established in the following theorem.

Theorem 4.16. *For topological spaces $(X, \tau), (Y, \sigma), (Z, \eta)$, $f : (X, \tau) \to (Y, \sigma), g : (Y, \sigma) \to (Z, \eta)$ and $g \circ f : (X, \tau) \to (Z, \eta)$ the following results are true.*

(i) If f is quasi λ_g^δ-continuous and g is super continuous then $g \circ f$ is continuous.

(ii) If f is a quasi λ_g^δ-continuous and g is a totally λ_g^δ-continuous then $g \circ f$ is a continuous.

(iii) If f is perfectly λ_g^δ-continuous and g is super continuous then $g \circ f$ is perfectly continuous.

(iv) If f is continuous and g is perfectly λ_g^δ-continuous(resp. quasi λ_g^δ-continuous) then $g \circ f$ is perfectly λ_g^δ-continuous(resp. quasi λ_g^δ-continuous).

(v) If f is super continuous and g is perfectly λ_g^δ-continuous(resp. quasi λ_g^δ-continuous) then $g \circ f$ is perfectly λ_g^δ-continuous(resp. quasi λ_g^δ-continuous)

(vi) If f is a perfectly λ_g^δ-continuous and g is a strongly λ_g^δ-continuous then $g \circ f$ is a strongly continuous.

(vii) If f is a strongly λ_g^δ-continuous and g is any map then $g \circ f$ is a strongly λ_g^δ-continuous.

(viii) *If f is a quasi λ_g^δ-continuous and g is a contra λ_g^δ-continuous then $g \circ f$ is a contra continuous function.*

(ix) *If f is a contra λ_g^δ-continuous and g is a continuous(resp. super continuous) then $g \circ f$ is a contra λ_g^δ-continuous.*

(x) *If f is a totally λ_g^δ-continuous and g is a continuous(resp. super continuous) then $g \circ f$ is a totally λ_g^δ-continuous.*

Definition 4.17. A map $f : (X, \tau) \to (Y, \sigma)$ is called

1. λ_g^δ-irresolute if $f^{-1}(B)$ is λ_g^δ-closed in (X, τ) for every λ_g^δ-closed set B in (Y, σ).
2. contra λ_g^δ-irresolute if $f^{-1}(B)$ is λ_g^δ-closed in (X, τ) for every λ_g^δ-open set B in (Y, σ).

Remark 4.18. If (X, τ) is almost weakly Hausdorff[2] then the g-closed sets of (X, τ_s) are δ-closed and hence λ_g^δ-closed.

Theorem 4.19. *For a map $f : (X, \tau) \to (Y, \sigma)$ with (Y, σ) being an almost weakly Hausdorff space, every λ_g^δ-continuous function is λ_g^δ-irresolute.*

Remark 4.20. λ_g^δ-irresolute maps and irresolute maps are independent of each other.

Example 4.21. Let $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c, f(b) = b$ and $f(c) = a$. Then f is λ_g^δ-irresolute but not irresolute since the inverse image of $\{a\}$ is $\{c\}$ and $\{c\}$ is not semi-open in (X, τ).

Example 4.22. Let $X = Y = \{a, b, c, d\}, \tau = \{X, \phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$ and $\sigma = \{X, \phi, \{a\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be an identity mapping. Then f is irresolute but not λ_g^δ-irresolute since the inverse image of $\{a, b, d\}$ is $\{a, b, d\}$ which is not λ_g^δ-closed in (X, τ).

Theorem 4.23. *Composition of two λ_g^δ-irresolute maps is a λ_g^δ-irresolute map.*

Theorem 4.24. *For topological spaces $(X, \tau), (Y, \sigma), (Z, \eta)$, $f : (X, \tau) \to (Y, \sigma), g : (Y, \sigma) \to (Z, \eta)$ and $g \circ f : (X, \tau) \to (Z, \eta)$ the following results are true.*

(i) *If f is a λ_g^δ-irresolute map and g is a λ_g^δ-continuous(resp. totally λ_g^δ-continuous, strongly λ_g^δ-continuous, contra λ_g^δ-continuous, contra λ_g^δ-irresolute map) function then $g \circ f$ is a λ_g^δ-continuous (resp. totally λ_g^δ-continuous, strongly λ_g^δ-continuous, contra λ_g^δ-continuous, contra λ_g^δ-irresolute map) function.*

(ii) *If f is a contra λ_g^δ-irresolute map(resp. strongly continuous, quasi λ_g^δ-continuous, perfectly λ_g^δ-continuous) and g is a λ_g^δ-irresolute map then $g \circ f$ is a contra λ_g^δ-irresolute map(resp. strongly continuous, quasi λ_g^δ-continuous, perfectly λ_g^δ-continuous).*

From the Theorem 4.23, we observe that λ_g^δ-irresoluteness acts as a mirror in reflecting the type of continuity in its composition.

Theorem 4.25. *If $f : (X, \tau) \to (Y, \sigma)$ is λ_g^δ-irresolute then*

(i) *For $A \subseteq X, f(\lambda_g^\delta cl(A)) \subseteq cl_\delta(f(A))$.*

(ii) *For $B \subseteq Y, \lambda_g^\delta cl(f^{-1}(B)) \subseteq f^{-1}(cl_\delta(A))$.*

Proof. (i) Let $A \subseteq X$ then $cl_\delta(f(A))$ is δ-closed in Y and thus λ_g^δ-closed in Y. Since f is λ_g^δ-irresolute, $f^{-1}(cl_\delta(f(A)))$ is λ_g^δ-closed in X. $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(cl_\delta(f(A))) \Rightarrow \lambda_g^\delta cl(A) \subseteq \lambda_g^\delta cl(f^{-1}(cl_\delta(f(A)))) = f^{-1}(cl_\delta(f(A))) \Rightarrow f(\lambda_g^\delta cl(A)) \subseteq cl_\delta(f(A))$.

(ii) Let $B \subseteq Y$ then $cl_\delta(B)$ is δ-closed in Y and thus λ_g^δ-closed in Y. By hypothesis, $f^{-1}(cl_\delta(B))$ is λ_g^δ-closed in X. Since $B \subseteq cl_\delta(B), f^{-1}(B) \subseteq f^{-1}(cl_\delta(B)) \Rightarrow \lambda_g^\delta cl(f^{-1}(B)) \subseteq \lambda_g^\delta cl(f^{-1}(cl_\delta(B))) = f^{-1}(cl_\delta(B))$.

□

Theorem 4.26. *If $f : (X, \tau) \to (Y, \sigma)$ is bijective, δ-open and λ_g^δ-continuous then f is λ_g^δ-irresolute.*

Proof. Let B be λ_g^δ-closed in Y with $f^{-1}(B) \subseteq A$, where A is δ-open in X giving $B \subseteq f(A)$. Since f is δ-open, $f(A)$ is δ-open in Y. Also B is λ_g^δ-closed in Y giving $cl_\delta(B) \subseteq f(A) \Rightarrow f^{-1}(cl_\delta(B)) \subseteq A$. Since $cl_\delta(B)$ is

δ-closed in Y, it is closed in Y. Since f is λ_g^δ-continuous, $f^{-1}(cl_\delta(B))$ is λ_g^δ-closed in X. Thus $cl_\delta(f^{-1}(cl_\delta(B))) \supseteq A$, where A is (Λ, δ)-open as A is δ-open $\Rightarrow cl_\delta(f^{-1}(B)) \subseteq A$, as $cl_\delta(B)$ is δ-closed. Therefore $f^{-1}(B)$ is λ_g^δ-closed in X proving f is λ_g^δ-irresolute. □

Theorem 4.27. *If $f : (X, \tau) \to (Y, \sigma)$ is δ-continuous and closed then λ_g^δ-closedness is preserved under f (i.e.,) for every λ_g^δ-closed subset A of X, $f(A)$ is λ_g^δ-closed in Y.*

Proof. Let $f(A) \subseteq G$, where G is δ-open in Y. Then $A \subseteq f^{-1}(G)$, where $f^{-1}(G)$ is δ-open in X. Since A is λ_g^δ-closed, $cl_\delta(A) \subseteq f^{-1}(G) \Rightarrow f(cl_\delta(A)) \subseteq G$. Since f is δ-continuous, $f(cl_\delta(A))$ is δ-closed in Y and thus λ_g^δ-closed in Y. Thus $cl_\delta(f(cl_\delta(A))) \subseteq G \Rightarrow cl_\delta(f(A)) \subseteq cl_\delta(f(cl_\delta(A))) = f(cl_\delta(A)) \subseteq G$. Thus $f(A)$ is λ_g^δ-closed in Y. □

Lemma 4.28. *If a topological space (X, τ) is almost weakly Hausdorff then every λ_g^δ-closed subset of X is closed.*

Theorem 4.29. *If $f : (X, \tau) \to (Y, \sigma)$ is surjective, λ_g^δ-irresolute and closed with (X, τ) being an almost weakly Hausdorff space then every λ_g^δ-closed set is closed in (Y, σ).*

Proof. Let B be λ_g^δ-closed in Y. By hypothesis, $f^{-1}(B)$ is λ_g^δ-closed in X. Since X is almost weakly Hausdorff, $f^{-1}(B)$ is closed in X. Once again by hypothesis, B is closed in Y. Since f is surjective, this is true for any arbitrary B in Y. Therefore every λ_g^δ-closed set is closed in Y. □

Theorem 4.28 answers the question, "Does λ_g^δ-irresoluteness preserve the nature of the domain?". One can easily observe that the property of almost weakly Hausdorffness is not inherited through λ_g^δ-irresolute maps even after including additional conditions.

5. λ_g^δ-open and λ_g^δ-closed maps

Definition 5.1. A map $f : (X, \tau) \to (Y, \sigma)$ is

1. λ_g^δ-*open* if $f(A)$ is λ_g^δ-open for every open set A in X.
2. λ_g^δ-*closed* if $f(A)$ is λ_g^δ-closed for every closed set A in X.

Theorem 5.2. *Let $f : (X, \tau) \to (Y, \sigma)$ be a map with $f(int(A)) \subseteq \lambda_g^\delta int(f(A))$, for every $A \subseteq X$. Then f is λ_g^δ-open.*

Proof. Let A be an open subset of X. By hypothesis, $f(int(A)) \subseteq \lambda_g^\delta int(f(A))$. Since A is open, $f(A) \subseteq \lambda_g^\delta int(f(A))$. Thus $f(A)$ is λ_g^δ-open and hence f is λ_g^δ-open. □

Theorem 5.3. *Let $f : (X, \tau) \to (Y, \sigma)$ be λ_g^δ-closed then $\lambda_g^\delta cl(f(A)) \subseteq f(cl(A))$, for each $A \subseteq X$.*

Proof. Similar to Theorem 5.2. □

Theorem 5.4. *For topological spaces $(X, \tau), (Y, \sigma), (Z, \eta), f : (X, \tau) \to (Y, \sigma), g : (Y, \sigma) \to (Z, \eta)$ and $g \circ f : (X, \tau) \to (Z, \eta)$ the following results are true.*

(i) If $g \circ f$ is λ_g^δ-open and f is continuous, surjective then g is λ_g^δ-open.

(ii) If $g \circ f$ is open and g is λ_g^δ-continuous, injective then f is λ_g^δ-open.

Proof. (i) Let B be open in Y. Then $f^{-1}(B)$ is open in X. By hypothesis, $(g \circ f)(f^{-1}(B)) = g(f(f^{-1}(B))) = g(B)$ is λ_g^δ-open in Z. Thus g is λ_g^δ-open.

(ii) Let A be open in X. By hypothesis, $(g \circ f)(A) = g(f(A))$ is open in Z. Therefore $g^{-1}(g(f(A))) = f(A)$ is λ_g^δ-open, as g is an injective λ_g^δ-continuous function. Hence f is λ_g^δ-open.

□

Theorem 5.5. *A map $f : (X, \tau) \to (Y, \sigma)$ is λ_g^δ-closed iff for each $B \subseteq Y$ and each open set U containing $f^{-1}(B)$, there exists a λ_g^δ-open set $V \subseteq Y$ such that $B \subseteq V$ and $f^{-1}(V) \subseteq U$.*

Proof. Let $B \subseteq Y$. Take an open set U in X such that $f^{-1}(B) \subseteq U$. Then $X \backslash U$ is closed in X. By hypothesis, $f(X \backslash U)$ is λ_g^δ-closed. Define $V = Y \backslash (f(X \backslash U))$ then V is λ_g^δ-open in Y. Further we prove, $B \subseteq V$ and $f^{-1}(V) \subseteq U$. Suppose that $v \notin V$ then $v \in f(X \backslash U) \Rightarrow f^{-1}(v) \in X \backslash U \Rightarrow f^{-1}(v) \notin U$ then $f^{-1}(v) \notin f^{-1}(B) \Rightarrow v \notin B$. Hence $B \subseteq V$. Now $v \in V \Rightarrow v \notin f(X \backslash U) \Rightarrow f^{-1}(v) \notin X \backslash U$ then $f^{-1}(v) \in U$. Hence $f^{-1}(V) \subseteq U$. □

Theorem 5.6. *For a bijective map $f : (X, \tau) \to (Y, \sigma)$, the following are equivalent:*

(i) f is λ_g^δ-open.

(ii) f is λ_g^δ-closed.

(iii) f^{-1} *is* λ_g^δ*-continuous.*

Proof. (i) $\Rightarrow$ (ii) Let $A \subseteq X$ be closed. Then $X \backslash A$ is open in X. By (i), $f(X\backslash A)$ is λ_g^δ-open in Y. Since f is bijective, $f(X\backslash A) = Y\backslash f(A)$ which implies $f(A)$ is λ_g^δ-closed in Y. Hence f ia λ_g^δ-closed.
(ii) $\Rightarrow$ (iii) Let $A \subseteq X$ be closed. Since f is bijective, $(f^{-1})^{-1}(A) = f(A)$ is λ_g^δ-closed in Y. Therefore f^{-1} is λ_g^δ-continuous.
(iii) $\Rightarrow$ (i) Let $A \subseteq X$ be open. Since f^{-1} is λ_g^δ-continuous and f is bijective, $(f^{-1})^{-1}(A) = f(A)$ is λ_g^δ-open. Hence f is λ_g^δ-open. □

Theorem 5.6 gives the conditions under which λ_g^δ-open and λ_g^δ-closed maps coincide.

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