International Journal of Pure and Applied Mathematics
Volume 120 No. 3 2018, 471-486
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v120i3.15

ijpam.eu

A NOTE ON THE ZUBAIR–G FAMILY WITH BASELINE LOMAX CUMULATIVE DISTRIBUTION FUNCTION. SOME APPLICATIONS

Nikolay Pavlov[1], Nikolay Kyurkchiev[2], Anton Iliev[3], Asen Rahnev[4]
[1,2,3,4]Faculty of Mathematics and Informatics
University of Plovdiv Paisii Hilendarski
24 Tzar Asen Str., 4000 Plovdiv, BULGARIA

Abstract: In this paper we study the one–sided Hausdorff approximation of the shifted Heaviside step function by a class of the Zubair–G family of cumulative lifetime distribution with baseline Lomax c.d.f. The estimates of the value of the best Hausdorff approximation obtained in this article can be used in practice as one possible additional criterion in "saturation" study.

As an illustrative example we consider the modelling of the growth of red abalone (*Haliotis Rufescens*) in Northern California.

We also look at a possible extension, which we call α–Zubair–G Family with baseline Lomax (cdf).

Finally, the potentiality of the new software reliability model analyzed by means of real dataset. Some comparisons are made. Numerical examples, illustrating our results are presented using programming environment *CAS Mathematica*.

AMS Subject Classification: 68N30, 41A46

Key Words: Zubair–G family of cumulative lifetime distribution, Zubair–G Family with baseline Lomax (cdf), α–Zubair–G Family with baseline Lomax (cdf), Heaviside function, Hausdorff approximation, upper and lower bounds

1. Introduction

In [1], a new family of lifetime distributions, called the Zubair–G family of distributions is introduces.

Received: April 2, 2018
Revised: December 12, 2018
Published: January 14, 2018

The new family is defined by the following cumulative distribution function (cdf)

$$F(t;\lambda) = \frac{e^{\lambda G^2(t)} - 1}{e^{\lambda} - 1}, \tag{1}$$

where $\lambda > 0$.

If, G is the (cdf) of the baseline model, then the distribution function (1) will be the (cdf) of the Zubair–G family.

Some comments on a Zubair-G Family of cumulative lifetime distributions with baseline Weibull (cdf) can be found in [2].

We also look at a possible extension, which we call α–Zubair–G Family.

For example, if $G(t)$ be (cdf) of the Lomax distribution [10] given by

$$G(t) = 1 - \frac{1}{1 + \left(\frac{t}{b}\right)^a} \tag{2}$$

then the (cdf) of the Z–Lomax distribution has the form

$$F(t) = \frac{e^{\lambda\left(1 - \frac{1}{1+\left(\frac{t}{b}\right)^a}\right)^2} - 1}{e^{\lambda} - 1} \tag{3}$$

where $a > 0$ and $b > 0$.

We consider the following class of this family with application to the population dynamics and debugging theory:

$$M(t) = \frac{e^{\lambda\left(1 - \frac{1}{1+\left(\frac{t}{b}\right)^a}\right)^2} - 1}{e^{\lambda} - 1}, \tag{4}$$

with

$$t_0 = b\left(\frac{\sqrt{\frac{1}{\lambda}\ln\frac{e^{\lambda}+1}{2}}}{1 - \sqrt{\frac{1}{\lambda}\ln\frac{e^{\lambda}+1}{2}}}\right)^{\frac{1}{a}}; \qquad M(t_0) = \frac{1}{2}. \tag{5}$$

Some extensions of the well-known Poisson, Poisson–exponential, Chen, Exponentiated Chen, modified Weibull and Burr distributions can be found in: [3]–[14].

For some approximation, computational and modelling aspects, see [15]–[31].

Some software reliability models, can be found in [32]–[51], [58]–[59].

In this note we study the Hausdorff approximation of the *shifted Heaviside step function*

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0 \end{cases}$$

by this family.

Definition 1. [52] The Hausdorff distance (the H–distance) $\rho(f,g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$.

More precisely,

$$\rho(f,g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|, |x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A - B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

We recall that completed graph of f is the closure of the graph of f as a subset of $\Omega \times \mathbb{R}$. If the graph of an interval function f equals $\mathcal{F}(f)$, then the f is called S-continuous.

The Hausdorff distance $\rho(f,g) = \max\{\overrightarrow{\rho}(f,g), \overrightarrow{\rho}(g,f)\}$ defines a metric in the set of the S-continuous interval functions [53]–[56].

We propose a software modules (intellectual properties) within the programming environment CAS Mathematica for the analysis.

The models have been tested with real data (the growth of red abalone (*Haliotis Rufescens*) in Northern California).

Finally, the potentiality of the new software reliability model analyzed by means of real dataset.

Some comparisons are made.

2. Main Results

The one–sided Hausdorff distance d between the function $h_{t_0}(t)$ and the sigmoidal function - ((4)–(5)) satisfies the relation

$$M(t_0 + d) = 1 - d. \tag{6}$$

The following theorem gives upper and lower bounds for d

Theorem 1. Let

$$\begin{aligned} &p = -\tfrac{1}{2}, \\ &q = 1 + \frac{a\lambda(1+e^{\lambda})\sqrt{\frac{1}{\lambda}\ln\frac{e^{\lambda}+1}{2}}\left(1-\sqrt{\frac{1}{\lambda}\ln\frac{e^{\lambda}+1}{2}}\right)^2}{b(e^{\lambda}-1)} \times \\ &\quad \left(\frac{\sqrt{\frac{1}{\lambda}\ln\frac{e^{\lambda}+1}{2}}}{1-\sqrt{\frac{1}{\lambda}\ln\frac{e^{\lambda}+1}{2}}}\right)^{\frac{a-1}{a}}, \\ &r = 2.1q. \end{aligned} \tag{7}$$

For the one–sided Hausdorff distance d between $h_{t_0}(t)$ and the sigmoid ((4)–(5)) the following inequalities hold for: $q > \dfrac{e^{1.05}}{2.1}$

$$d_l = \frac{1}{r} < d < \frac{\ln r}{r} = d_r. \tag{8}$$

Proof. Let us examine the function:

$$F(d) = M(t_0 + d) - 1 + d. \tag{9}$$

From $F'(d) > 0$ we conclude that function F is increasing.

Consider the function

$$G(d) = p + qd. \tag{10}$$

From Taylor expansion we obtain $G(d) - F(d) = O(d^2)$.

Hence $G(d)$ approximates $F(d)$ with $d \to 0$ as $O(d^2)$ (see Fig. 1).
In addition $G'(d) > 0$.

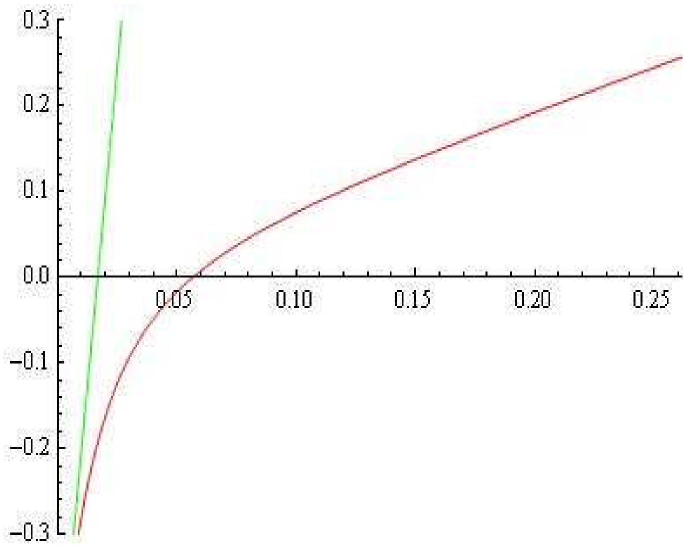

Figure 1: The functions $F(d)$ and $G(d)$ for $b = 0.01$; $a = 1.95\ \lambda = 0.95$.

Further, for $q > \dfrac{e^{1.05}}{2.1}$ we have $G(d_l) < 0$ and $G(d_r) > 0$.

This completes the proof of the theorem.

3. Numerical examples.

The model ((4)–(5)) for $b = 0.05$; $a = 1.9$; $\lambda = 0.9$, $t_0 = 0.0974305$ is visualized on Fig. 2.

From the nonlinear equation (6) and inequalities (8) we have: $d = 0.139818$, $d_l = 0.0725397$, $d_r = 0.190317$.

The model ((4)–(5)) for $b = 0.01$; $a = 1.95\ \lambda = 0.95$, $t_0 = 0.0193646$ is visualized on Fig. 3.

From the nonlinear equation (6) and inequalities (8) we have: $d = 0.0566589$, $d_l = 0.01602$, $d_r = 0.0662253$.

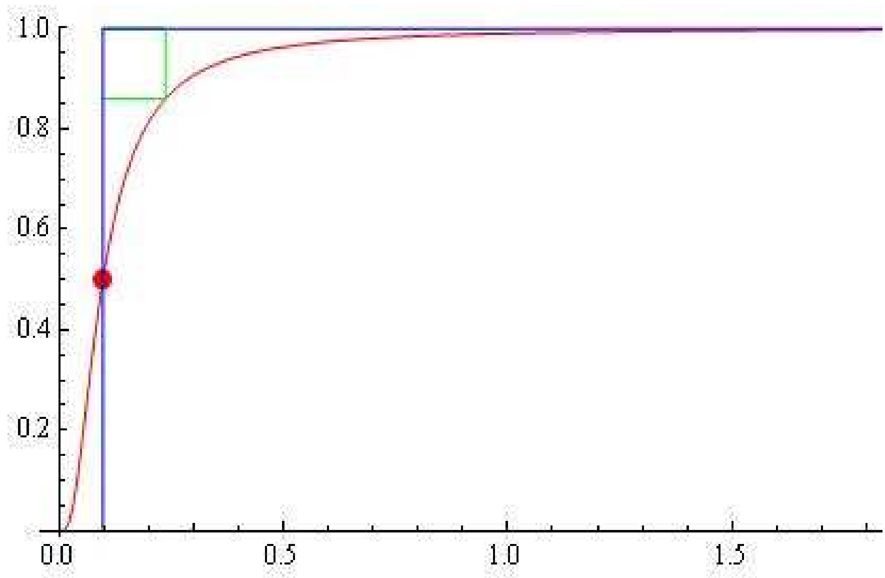

Figure 2: The model ((4)–(5)) for $b = 0.05$; $a = 1.9$; $\lambda = 0.9$, $t_0 = 0.0974305$; H–distance $d = 0.139818$, $d_l = 0.0725397$, $d_r = 0.190317$.

From the above examples, it can be seen that the proven estimates (see Theorem 1) for the value of the Hausdorff approximation is reliable when assessing the important characteristic - "saturation".

This characteristic (as we have already shown in our previous publications) has its equal participation together with the other two characteristics - "confidence intervals" and "confidence bounds" in the area of the Software Reliability Theory.

We propose a software module (intellectual properties) within the programming environment *CAS Mathematica* for the analysis of the considered family $M(t)$.

The module offers the following possibilities:

- generation of the function under user defined values of the parameters a, λ, and b;

- calculation of the H-distance d between the function $h_{t_0}(t)$ and the function $M(t)$;

- software tools for animation and visualization.

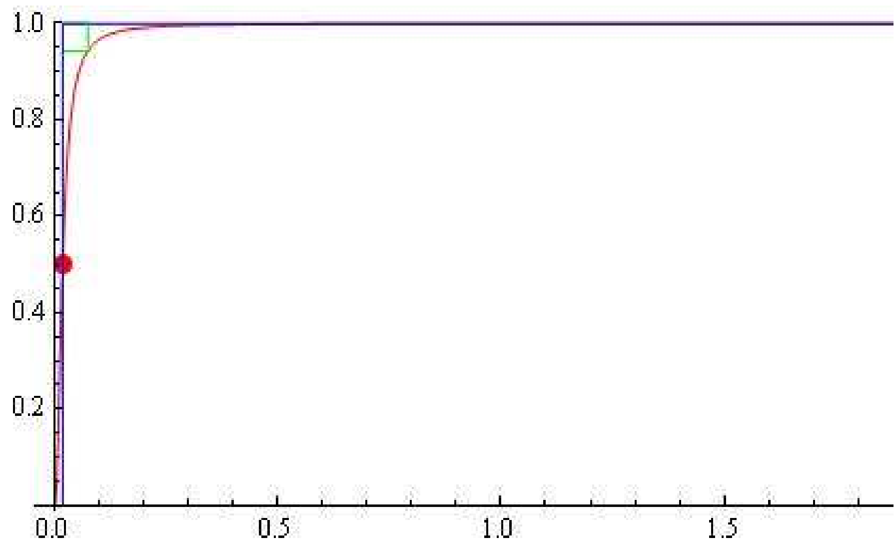

Figure 3: The model ((4)–(5)) for $b = 0.01$; $a = 1.95$ $\lambda = 0.95$, $t_0 = 0.0193646$; H–distance $d = 0.0566589$, $d_l = 0.01602$, $d_r = 0.0662253$.

4. Some applications.

4.1. Application to the Population Dynamics

We examine the following data. (The data for modeling the growth of red abalone is shown in Table 1. For more details, see [57]).

The model

$$M(t) = \omega \left(\frac{e^{\lambda\left(1-\frac{1}{1+\left(\frac{t}{b}\right)^a}\right)^2} - 1}{e^{\lambda} - 1} \right)$$

based on the data of Table 1 for the estimated parameters:

$$\omega = 242.5; \;\; b = 1.7205; \;\; \lambda = 2.17481; \;\; a = 1.39039$$

is plotted on Fig. 4.

Age	*Length(mm)*
1	16.1
2	33.9
3	54.3
4	76.2
5	97.8
6	117.1
7	133.3
8	146.5
9	157.2
10	166
11	173.3
12	179.6
13	185
14	189.5
15	194

Table 1: Data for modeling the growth of red abalone *Haliotis Rufescens* in Northern California [57]

For the predictive power (PP) criterion:

$$PP = \sum_{i=1}^{n} \left(\frac{M(t_i) - y_i}{y_i} \right)^2$$

measures the distance of model actual data from the estimates against the actual data, we find $PP = 0.29094$.

We hope that the results will be useful for specialists in this scientific area.

4.2. Application to the Software Reliability Growth Theory

We examine the following data.

The fitted model $M(t)$ based on the data of Table 2 for the estimated parameters:

$$\omega = 6543; \;\; b = 4.07363; \;\; \lambda = 2.33458; \;\; a = 1.16652$$

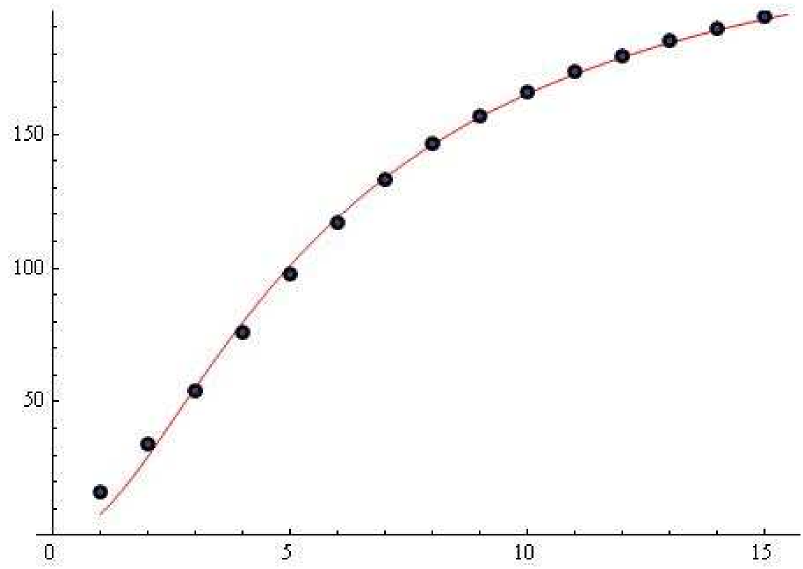

Figure 4: The model $M(t)$ based on the data of Table 1

is plotted on Fig. 5.

We will explicitly note that in some cases the presented software reliability model provides better results than other much more sophisticated models.

Comparison between the models $M(t)$ and the Gompertz software reliability model show a good fit by the presented new model (see, Fig. 6).

5. Concluding remarks.

Following the ideas given in [2] we consider the *new α–Zubair–G Family* with baseline Lomax (cdf):

$$M_\alpha(t) = \omega \left(\frac{e^{\lambda\left(1-\frac{1}{1+\left(\frac{t}{b}\right)^a}\right)^2} - 1}{e^\lambda - 1} \right)^\alpha, \tag{11}$$

where $\alpha > 0$.

$Week$	$Cumulative\,number\,of\,software\,faults$
1	248
5	742
10	1936
15	2567
20	3205
25	3750
30	4301
35	4548
40	4749
45	4901
50	5024
55	5110

Table 2: The actual data (week; cumulative number of software faults) [61], [60]

The model $M_\alpha(t)$ based on the data of Table 1 for the estimated parameters: $\omega = 194$; $b = 7.88871$; $\lambda = 2.2408$; $a = 4.71474$; $\alpha = 0.110994$ is plotted on Fig. 7.

For the predictive power (PP) criterion:

$$PP = \sum_{i=1}^{n} \left(\frac{M_\alpha(t_i) - y_i}{y_i} \right)^2$$

we find $PP = 0.0035919$.

Comparison between the models $M(t)$ and $M_\alpha(t)$ show a good fit by the presented new model $M_\alpha(t)$ (see, Fig. 7).

Based on the methodology proposed in the present note, the reader may formulate the corresponding approximation problems for the general model $M_\alpha(t)$ (11) on his/her own.

```
Manipulate[Dynamic@Show[Plot[f[t], {t, 1, 55}, LabelStyle → Directive[Green, Bold],
    PlotLabel → 6543 * (Exp[λ * (1 - 1 / (1 + (t / b) ^a)) ^2] - 1) / (Exp[λ] - 1)],
   ListPlot[{{1, 248}, {5, 742}, {10, 1936}, {15, 2567}, {20, 3205}, {25, 3750},
     {30, 4301}, {35, 4548}, {40, 4749}, {45, 4901}, {50, 5024}, {55, 5110}},
    PlotStyle → Red, PlotMarkers → {Automatic, Small}],
   PlotRange → {Automatic, {0, 5100}}, AxesOrigin → {0, 0}],
 {{b, 0}, 0.1, 10, Appearance → "Open"}, {{a, 0}, 0.01, 10, Appearance → "Open"},
 {{λ, 0}, 0.01, 10, Appearance → "Open"},
 Initialization ⧴ (f[t_] := 6543 * (Exp[λ * (1 - 1 / (1 + (t / b) ^a)) ^2] - 1) / (Exp[λ] - 1))]
```

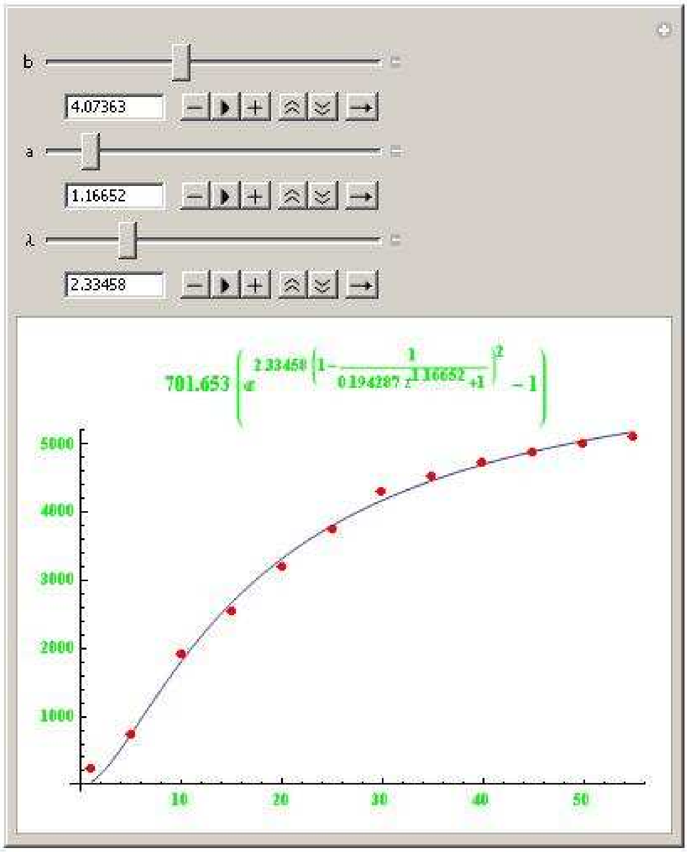

Figure 5: The model $M(t)$ based on the data of Table 2.

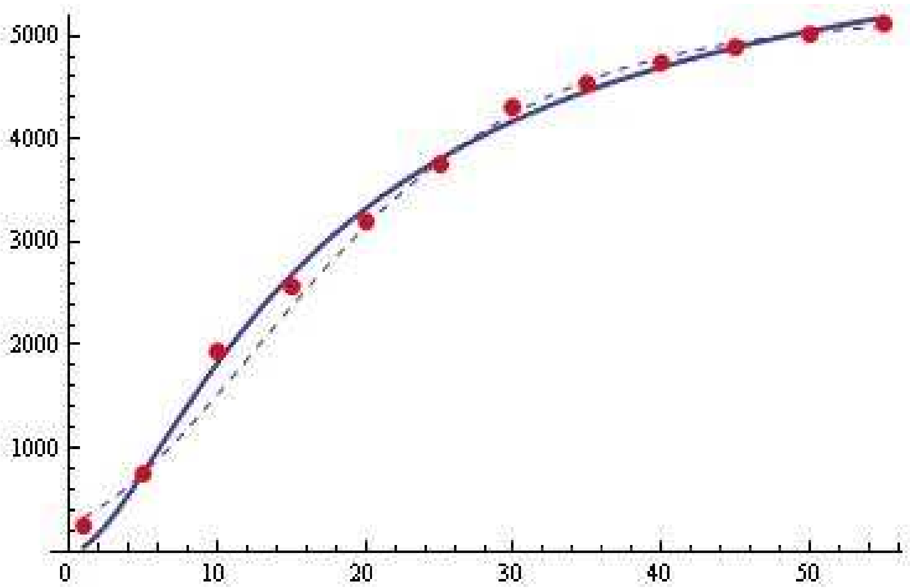

Figure 6: The models: $M(t)$ (thick) and Gompertz software reliability model (dashed) based on the data of Table 2.

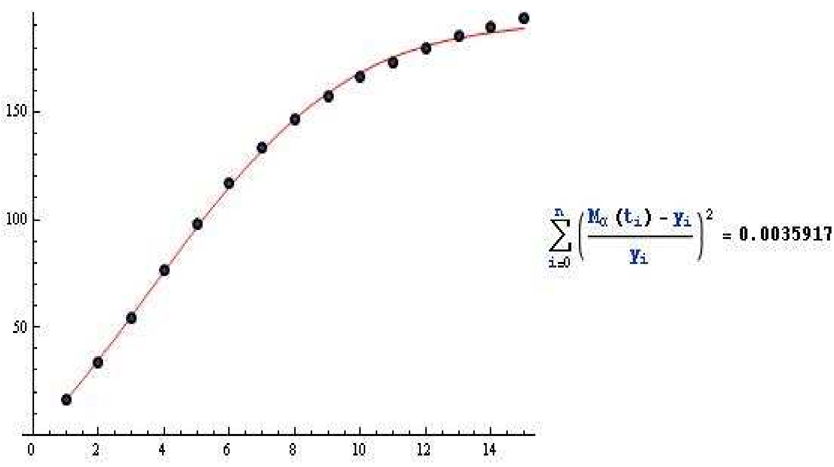

Figure 7: The model $M_\alpha(t)$ based on the data of Table 1.

Acknowledgments

This work has been supported by D01-205/23.11.2018 National Scientific Program "Information and Communication Technologies for a Single Digital Market in Science, Education and Security (ICTinSES)", financed by the Ministry of Education and Science, Bulgaria.

References

[1] Z. Ahmad, The Zubair–G Family of Distributions: Properties and Applications, *Annals of Data Science*, (2018), doi: 10.1007/s40745-018-0169-9.

[2] N. Kyurkchiev, A. Iliev, A. Rahnev, Comments on a Zubair-G Family of Cumulative Lifetime Distributions. Some Extensions, *Communications in Applied Analysis*, **23**, (1), (2019), 1–20.

[3] Z. Chen, A new two–parameter lifetime distribution with bathtub shape or increasing failure rate function, *Stat. and Prob. Letters*, **49** (2000), 155–161.

[4] M. Xie, Y. Tang, T. Goh, A modified Weibull extension with bathtub–shaped failure rate function, *Reliability Eng. and System Safety*, **76** (2002), 279–285.

[5] M. Khan, A. Sharma, Generalized order statistics from Chen distribution and its characterization, *J. of Stat. Appl. and Prob.*, **5** (2016), 123–128.

[6] S. Dey, D. Kumar, P. Ramos, F. Louzada, Exponentiated Chen distribution: Properties and Estimations, *Comm. in Stat.–Simulation and Computation*, **46**, No. 10 (2017), 8118–8139.

[7] Y. Chaubey, R. Zhang, An extension of Chen's family of survival distributions with bathtub shape or increasing hazard rate function, *Comm. in Stat.–Theory and Methods*, **44** (2015), 4049–4069.

[8] D. Kumar, M. Kumar, A new generalization of the extended exponential distribution with application, *Ann. Data Sci.* (2018), 22 pp.

[9] M. Khan, Transmuted generalized power Weibull distribution, *Thailand Statistician*, **16**, No. 2 (2018), 156–172.

[10] G. Cordeiro, A. Afify, E. Ortega, A. Suzuki, M. Mead, The odd Lomax generator of distributions, *J. Comp. Appl. Math.*, **377** (2019), 222–237.

[11] V. Cancho, F. Louzada, G. Barriga, The Poisson–exponential lifetime distribution, *Comp. Stat. Data Anal.*, **55** (2011), 677–686.

[12] G. Rodrigues, F. Louzada, P. Ramos, Poisson–exponential distribution: different methods of estimation, *J. of Appl. Stat.*, **45**, No. 1 (2018), 128–144.

[13] F. Louzada, P. Ramos, P. Ferreira, Exponential–Poisson distribution: estimation and applications to rainfall and aircraft data with zero occurrence, *Communication in Statistics–Simulation and Computation*, (2018), doi: 10.1080/03610918.2018.1491988.

[14] P. Ramos, D. Dey, F. Louzada, V. Lachos, An extended Poisson family of lifetime distribution: A unified approach in competitive and Complementary risk, *arXiv*: submit/2267507 [stat.AP], (2018).

[15] N. Kyurkchiev, S. Markov, On the Hausdorff distance between the Heaviside step function and Verhulst logistic function, *J. Math. Chem.*, **54**, No. 1 (2016), 109–119.

[16] N. Kyurkchiev, S. Markov, *Sigmoid functions: Some Approximation and Modelling Aspects*, LAP LAMBERT Academic Publishing, Saarbrucken (2015), ISBN 978-3-659-76045-7.

[17] N. Kyurkchiev, A. Iliev, S. Markov, *Some Techniques for Recurrence Generating of Activation Functions: Some Modeling and Approximation Aspects*, LAP LAMBERT Academic Publishing (2017), ISBN: 978-3-330-33143-3.

[18] R. Anguelov, M. Borisov, A. Iliev, N. Kyurkchiev, S. Markov, On the chemical meaning of some growth models possessing Gompertzian-type property, *Math. Meth. Appl. Sci.*, (2017), 1–12, doi: 10.1002/mma.4539.

[19] R. Anguelov, N. Kyurkchiev, S. Markov, Some properties of the Blumberg's hyper-log-logistic curve, *BIOMATH*, **7**, No. 1 (2018), 8 pp.

[20] A. Iliev, N. Kyurkchiev, S. Markov, On the Approximation of the step function by some sigmoid functions, *Mathematics and Computers in Simulation*, **133** (2017), 223–234.

[21] A. Iliev, N. Kyurkchiev, S. Markov, Approximation of the cut function by Stannard and Richards sigmoid functions, *IJPAM*, **109**, No. 1 (2016), 119–128.

[22] S. Markov, A. Iliev, A. Rahnev, N. Kyurkchiev, A note on the Log-logistic and transmuted Log-logistic models. Some applications, *Dynamic Systems and Applications*, **27**, No. 3 (2018), 593–607.

[23] S. Markov, N. Kyurkchiev, A. Iliev, A. Rahnev, On the approximation of the cut functions by hyper-log-logistic function, *Neural, Parallel and Scientific Computations*, **26**, No. 2 (2018), 169–182.

[24] N. Kyurkchiev, A. Iliev, S. Markov, Families of recurrence generated three and four parametric activation functions, *Int. J. Sci. Res. and Development*, **4**, No. 12 (2017), 746–750.

[25] N. Kyurkchiev, A note on the new geometric representation for the parameters in the fibril elongation process, *C. R. Acad. Bulg. Sci.*, **69**, No. 8 (2016), 963–972.

[26] N. Kyurkchiev, On the numerical solution of the general "ligand-gated neuroreceptors model' via CAS Mathematica, *Pliska Stud. Math. Bulgar.*, **26** (2016), 133–142.

[27] N. Kyurkchiev, S. Markov, On the numerical solution of the general kinetic "K-angle" reaction system, *Journal of Mathematical Chemistry*, **54**, No. 3 (2016), 792–805.

[28] S. Markov, N. Kyurkchiev, A. Iliev, A. Rahnev, On the approximation of the generalized cut functions of degree $p+1$ by smooth hyper-log-logistic function, *Dynamic Systems and Applications*, **27**, No. 4 (2018), 715–728.

[29] S. Markov, A. Iliev, A. Rahnev, N. Kyurkchiev, A Note On the Three–stage Growth Model, *Dynamic Systems and Applications*, **28**, No. 1 (2019), 63–72.

[30] S. Markov, A. Iliev, A. Rahnev, N. Kyurkchiev, A Note On the n–stage Growth Model. Overview, *Biomath Communications*, **5**, No. 2 (2018).

[31] O. Rahneva, H. Kiskinov, I. Dimitrov, V. Matanski, Application of a Weibull Cumulative Distribution Function Based on m Existing Ones to Population Dynamics, *International Electronic Journal of Pure and Applied Mathematics*, **12**, No. 1 (2018), 111–121.

[32] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, *Some software reliability models: Approximation and modeling aspects*, LAP LAMBERT Academic Publishing (2018), ISBN: 978-613-9-82805-0.

[33] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, *Nontrivial Models in Debugging Theory (Part 2)*, LAP LAMBERT Academic Publishing (2018), ISBN: 978-613-9-87794-2.

[34] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, On the extended Chen's and Pham's software reliability models. Some applications, *Int. J. of Pure and Appl. Math.*, **118**, No. 4 (2018), 1053–1067.

[35] N. Pavlov, G. Spasov, A. Rahnev, N. Kyurkchiev, A new class of Gompertz–type software reliability models, *International Electronic Journal of Pure and Applied Mathematics*, **12**, No. 1 (2018), 43–57.

[36] N. Pavlov, G. Spasov, A. Rahnev, N. Kyurkchiev, Some deterministic reliability growth curves for software error detection: Approximation and modeling aspects, *International Journal of Pure and Applied Mathematics*, **118**, No. 3 (2018), 599–611.

[37] N. Pavlov, A. Golev, A. Rahnev, N. Kyurkchiev, A note on the Yamada–exponential software reliability model, *International Journal of Pure and Applied Mathematics*, **118**, No. 4 (2018), 871–882.

[38] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, A Note on The "Mean Value" Software Reliability Model, *International Journal of Pure and Applied Mathematics*, **118**, No. 4 (2018), 949–956.

[39] D. R. Jeske, X. Zhang, Some successful approaches to software reliability modeling in industry, *J. Syst. Softw.*, **74** (2005), 85–99.

[40] K. Song, H. Pham, A Software Reliability Model with a Weibull Fault Detection Rate Function Subject to Operating Environments, *Appl. Sci.*, **7** (2017), 16 pp., doi: 10.3390/app7100983.

[41] N. Pavlov, A. Golev, A. Rahnev, N. Kyurkchiev, A note on the generalized inverted exponential software reliability model, *International Journal of Advanced Research in Computer and Communication Engineering*, **7**, No. 3 (2018), 484–487.

[42] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Transmuted inverse exponential software reliability model, *Int. J. of Latest Research in Engineering and Technology*, **4**, No. 5 (2018), 1–6.

[43] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Analysis of the Chen's and Pham's Software Reliability Models, *Cybernetics and Information Technologies*, **18**, No. 3 (2018), 37–47.

[44] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, On Some Nonstandard Software Reliability Models, *Dynamic Systems and Applications*, **27**, No. 4 (2018), 757–771.

[45] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Some deterministic growth curves with applications to software reliability analysis, *Int. J. of Pure and Appl. Math.*, **119**, No. 2 (2018), 357–368.

[46] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Investigations of the K-stage Erlangian software reliability growth model, *Int. J. of Pure and Appl. Math.*, **119**, No. 3 (2018), 441–449.

[47] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Some Transmuted Software Reliability Models, *Journal of Mathematical Sciences and Modelling*, **1**, No. 2 (2018). (to appear)

[48] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, A Note on Ohbas Inflexion S–shaped Software Reliability Growth Model, *Collection of scientific works from conference Mathematics. Informatics. Information Technologies. Application in Education*, Pamporovo, Bulgaria, October 10-12, (2018). (to appear)

[49] V. Kyurkchiev, A. Malinova, O. Rahneva, P. Kyurkchiev, On the Burr XII-Weibull Software Reliability Model, *Int. J. of Pure and Appl. Math.*, **119**, No. 4 (2018), 639–650.

[50] V. Kyurkchiev, A. Malinova, O. Rahneva, P. Kyurkchiev, Some Notes on the Extended Burr XII Software Reliability Model, *Int. J. of Pure and Appl. Math.*, **120** No. 1 (2018), 127–136.

[51] N. Kyurkchiev, A. Iliev, *Extension of Gompertz-type Equation in Modern Science: 240 Anniversary of the birth of B. Gompertz*, LAP LAMBERT Academic Publishing (2018), ISBN: 978-613-9-90569-0.

[52] F. Hausdorff, *Set Theory* (2 ed.) (Chelsea Publ., New York, (1962 [1957]) (Republished by AMS-Chelsea 2005), ISBN: 978-0-821-83835-8.

[53] R. Anguelov, S. Markov, Hausdorff Continuous Interval Functions and Approximations, In: *SCAN 2014 Proceedings, LNCS*, ed. by J.W.von Gudenberg, Springer, Berlin (2015).

[54] R. Anguelov, S. Markov, B. Sendov, On the Normed Linear Space of Hausdorff Continuous Functions. In: Lirkov, I., et al. (Eds.): *Lecture Notes in Computer Science* **3743**, Springer (2006), 281–288.

[55] R. Anguelov, S. Markov, B. Sendov, Algebraic Operations on the Space of Hausdorff Continuous Functions. In: Bojanov, B. (Ed.): *Constructive Theory of Functions*, Prof. M. Drinov Academic Publ. House, Sofia (2006), 35–44.

[56] R. Anguelov, S. Markov, B. Sendov, The Set of Hausdorff Continuous Functions - the Largest Linear Space of Interval Functions, *Reliable Computing*, **12** (2006), 337–363.

[57] L. Rogers–Bennett, D. W. Rogers, S. A. Schultz, Modeling growth and mortality of red abalone *Haliotis Rufescens* in Northern California, *J. of Shellfish Research*, **26**, No. 3 (2007), 719–727.

[58] H. Pham, *System Software Reliability*, In: Springer Series in Reliability Engineering, Springer–Verlag London Limited (2006).

[59] M. Ohba, Software reliability analysis models, *IBM J. Research and Development*, **21**, No. 4 (1984).

[60] D. Satoh, A discrete Gompertz equation and a software reliability growth model, *IEICE Trans. Inform. Syst.*, **E83-D** No. 7 (2000), 1508–1513.

[61] T. Mitsuhashi, *A method of software quality evaluation*, JUSE Press, Tokyo (1981). (in Japanese)