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EDGE TRIMAGIC GRACEFUL LABELING OF GRAPHS

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Abstract: A (p, q) graph G is called edge trimagic total if there exists a bijection $f : V(G) \cup E(G) \to \{1, 2, 3, ..., p + q\}$ such that for each edge xy in $E(G)$ the value of $f(x) + f(xy) + f(y) = K_1$ or K_2 or K_3. G is called edge trimagic graceful if there exists a bijection $f : V(G) \cup E(G) \to \{1, 2, 3, ..., p+q\}$ such that for each edge xy in $E(G)$, $|f(x) - f(xy) + f(y)|$ $= c_1$ or c_2 or c_3, where c_1, c_2, and c_3 are constants. In this paper, we introduce edge trimagic graceful labeling of some graphs and proved that the square graph P_n^2, $(P_n; S_1)$ and the comp $P_n \odot K_1$ are edge trimagic graceful graphs.

AMS Subject Classification: 05C78
Key Words: graph, labeling, magic, trimagic

1. Introduction

Let G be a simple, undirected graph with n vertices. Let $V(G)$ and $E(G)$ denote the vertex set and the edge set of the graph G, respectively. Labeling of

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a graph G is an assignment f of labels to either the vertices or the edges or both subject to certain conditions. Graph labeling is an increasingly useful and important method of Mathematical models from a broad range of applications such as coding theory, X-ray, crystallography, radar, astronomy, circuit design, communication networks and data base management etc. Graph labeling was first introduced in 1960's. In 1970, Kotzig and Rosa[1] defined, a magic labeling of graph G is a bijection $f : V \cup E \to \{1, 2, 3, ..., p + q\}$ such that for each edge $uv \in E(G)$, $f(u) + f(uv) + f(v)$ is a magic constant.

Rosa [1] introduced the β - valuations of a graph G with q edges is an injection f from the vertices of G to the set $\{0, 1, 2, ..., q\}$ such that, when each edge xy is assigned the label $|f(x) - f(y)|$, the resulting edge labels are distinct. Golomb [4] called such labeling as graceful. G. Marimuthu and M. Balakrishnan [3] introduced, super edge magic graceful labeling of graphs. In 2013, C. Jayasekaran, M. Regees and C. Davidraj introduced the edge trimagic total labeling of graphs [2]. A (p, q) graph G is called an edge magic graceful if there exists a bijection $f : V(G) \cup E(G) \to \{1, 2, 3, ..., p + q\}$ such that for each edge xy in E(G) the value of $|f(x) + f(y) - f(xy)| = k$, a constant. The graph G is said to be super edge magic graceful if $V(G) = \{1, 2, ..., p\}$. An edge trimagic total labeling of a (p, q) graph G is a bijective function $f : V(G) \cup E(G) \to \{1, 2, 3, ..., p + q\}$ such that for each edge $xy \in E(G)$, the value of $f(x) + f(xy) + f(y)$ is equal to any of the distinct constants k_1 or k_2 or k_3. A graph G is said to be edge trimagic total if it admits an edge trimagic total labeling [2, 5]. An edge trimagic total labeling is called a super edge trimagic total labeling if G has the additional property that the vertices are labeled with smallest positive integers [6, 7]. The useful survey on graph labeling by J. A. Gallian (2017) can be found in [4]. Square of a graph G denoted by G^2 has the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance *1* or *2* apart in G. A square graph P_n^2 has n vertices and $2n - 3$ edges. $(P_n; S_m)$ is a graph obtained from n copies of S_m and the path P_n by joining u_i with the central vertex $v_1^{(i)}$ of the i^{th} copy of S_m by means of an edge for $1 \leq i \leq n$ [5], $(P_n; S_1)$ has 3n vertices and $3n - 1$ edges. The corona product $G \odot H$ of two graphs G and H is obtained by taking one copy of G and $| V(G) |$ copies of H; and by joining each vertex of the i^{th} copy of H to the i^{th} vertex of G, where $1 \leq i \leq| V(G) |$ [8]. The corona graph $P_n \odot K_1$ is known as comb, and it has 2n vertices and 2n $-$ 1 edges.

In this paper, we introduce the concept of edge trimagic graceful labeling of graphs and proved that the square graph P_n^2, $(P_n; S_1)$ and the comp $P_n \odot K_1$ admit edge trimagic graceful labeling.

2. MAIN RESULTS:

Theorem 2.1. *The square graph P_n^2 admits an edge trimagic graceful labeling.*

Proof. Let $V(P_n^2) = \{u_1, u_2, ..., u_n\}$ be the vertex set and $E(P_n^2)$ =$\{u_iu_{i+1}/1 \leq i \leq n-1\} \cup \{u_iu_{i+2}/1 \leq i \leq n-2\}$ be the edge set of the square graph P_n^2. Then the square graph P_n^2 has n vertices and 2n $-$ 3 edges.

Case 1: n is odd.
Define a bijection $f : V \cup E \to \{1, 2, 3, ..., 3n-3\}$ such that

$$f(u_i) = \begin{cases} \frac{i+1}{2} & , \quad 1 \leq i \leq n,\ i \text{ is odd} \\ \frac{n+i+1}{2} & , \quad 1 \leq i \leq n,\ i \text{ is even} \end{cases}$$

$f(u_iu_{i+1}) = 2n-2+i$, $1 \leq i \leq n-1$ and $f(u_iu_{i+2}) = n+i$, $1 \leq i \leq n-2$.
Now, we prove this labeling is an edge trimagic graceful.
Consider the edges u_iu_{i+1}, $1 \leq i \leq n-1$.

For odd i, $| f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) | = | \frac{i+1}{2} - (2n-2+i) + \frac{n+i}{2} + 1 | = | \frac{7-3n}{2} | = c_1$ (say).

For even i, $| f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) | = | \frac{n+i+1}{2} - (2n-2+i) + \frac{i+2}{2} | = | \frac{7-3n}{2} | = c_1$.

Consider the edges u_iu_{i+2}, $1 \leq i \leq n-2$.

For odd i, $| f(u_i) - f(u_iu_{i+2}) + f(u_{i+2}) | = | \frac{i+1}{2} - (n+i) + \frac{i+3}{2} | = |2-n| = c_2$ (say).

For even i, $| f(u_i) - f(u_iu_{i+2}) + f(u_{i+2}) | = | \frac{n+i+1}{2} - (n+i) + \frac{n+i+3}{2} | = 2 = c_3$ (say).

Hence, for each edge $uv \in E\ (P_n^2)$, $| f(u) - f(uv) + f(v) |$ will form any one of the constants $c_1 = | \frac{7-3n}{2} |$, $c_2 = | 2-n |$ and $c_3 = 2$. Therefore, the square graph P_n^2 admits an edge trimagic graceful labeling for odd n.

Case 2: n is even.
Define a bijection $f : V \cup E \to \{1, 2, 3, ..., 3n-3\}$ such that

$$f(u_i) = \begin{cases} \frac{i+1}{2} & , \quad 1 \leq i \leq n,\ i \text{ is odd} \\ \frac{n+i}{2} & , \quad 1 \leq i \leq n,\ i \text{ is even} \end{cases}$$

$f(u_iu_{i+1}) = 2n-2+i$, $1 \leq i \leq n-1$ and $f(u_iu_{i+2}) = n+i$, $1 \leq i \leq n-2$.
Now we prove this labeling is an edge trimagic graceful.
Consider the edges u_iu_{i+1}, $1 \leq i \leq n-1$.

For odd i, $| f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) | = | \frac{i+1}{2} - (2n-2+i) + \frac{n+i+1}{2} | = | \frac{6-3n}{2} | = c_1$ (say).

$$\text{For even } i, \mid f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \mid \;=\; \mid \tfrac{n+i}{2} - (2n-2+i) + \tfrac{i+2}{2} \mid \;=\; \mid \tfrac{6-3n}{2} \mid = c_1.$$

Consider the edges u_iu_{i+2}, $1 \le i \le n-2$.

$$\text{For odd } i, \mid f(u_i) - f(u_iu_{i+2}) + f(u_{i+2}) \mid \;=\; \mid \tfrac{i+1}{2} - (n+i) + \tfrac{i+3}{2} \mid \;=\; \mid 2-n \mid = c_2 \text{ (say)}.$$

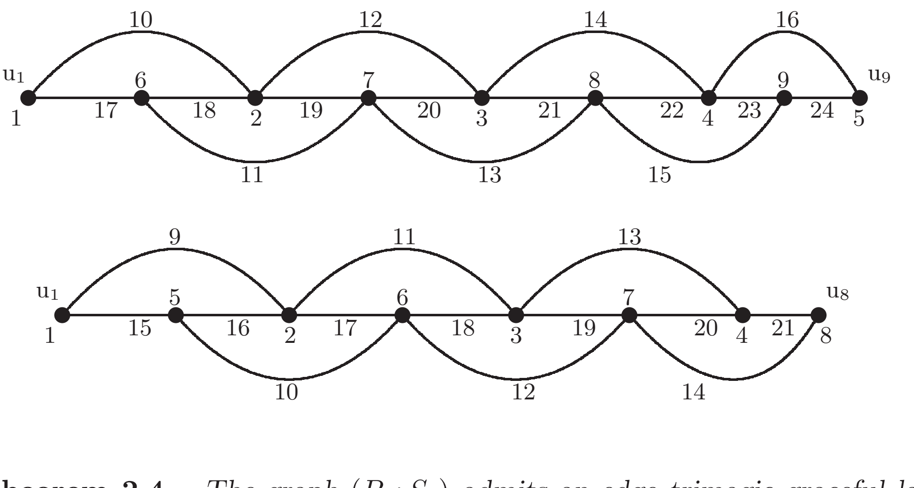
$$\text{For even } i, \mid f(u_i) - f(u_iu_{i+2}) + f(u_{i+2}) \mid \;=\; \mid \tfrac{n+i}{2} - (n+i) + \tfrac{n+i+2}{2} \mid \;=\; \mid n-(n-1) \mid = 1 = c_3.$$

Hence, for each edge $uv \in E\ (P_n^2)$, $\mid f(u) - f(uv) + f(v) \mid$ will form any one of the constants $c_1 = \mid \frac{6-3n}{2} \mid$, $c_2 = \mid 2-n \mid$ and $c_3 = 1$. Therefore, the square graph P_n^2 admits an edge trimagic graceful labeling for even n.
The theorem follows from case 1 and case 2. □

Corollary 2.2. *The square graph P_n^2 admits a super edge trimagic graceful labeling.*

Proof. We proved the square graph admits an edge trimagic graceful labeling. The labeling given in the proof of theorem 2.1, the vertices get labels 1, 2, ..., n. Since the square graph P_n^2 has n vertices and these n vertices have labels 1, 2, ..., n for odd and even n, the square graph P_n^2 is a super edge trimagic graceful graph. □

Example 2.3. A super edge trimagic graceful labeling of P_9^2 and P_8^2 are given in figure 1 and figure 2, respectively.



Theorem 2.4. *The graph $(P_n; S_1)$ admits an edge trimagic graceful labeling.*

Proof. Let $V(P_n;S_1) = \{v_1, v_2, .., v_n\} \cup \{u_1, u_2, ..., u_n\} \cup \{w_1, w_2, ..., w_n\}$ be the vertex set and $E(P_n;S_1) = \{u_iv_i/1 \leq i \leq n\} \cup \{v_iw_i/1 \leq i \leq n\} \cup \{u_iu_{i+1}/1 \leq i \leq n-1\}$ be the edge set. Then $(P_n;S_1)$ has 3n vertices and 3n − 1 edges.

Case 1: n is odd

Define a bijection $f : V \cup E \to \{1, 2, 3, ..., 6n-1\}$ such that

$$f(u_i) = \begin{cases} \frac{i+1}{2} & , \quad 1 \leq i \leq n,\ i \text{ is odd} \\ \frac{n+i+1}{2} & , \quad 1 \leq i \leq n,\ i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} n + \frac{i+1}{2} & , \quad 1 \leq i \leq n,\ i \text{ is odd} \\ n + \frac{n+i+1}{2} & , \quad 1 \leq i \leq n,\ i \text{ is even} \end{cases}$$

$$f(w_i) = \begin{cases} 2n + \frac{i+1}{2} & , \quad 1 \leq i \leq n,\ i \text{ is odd} \\ 2n + \frac{n+i+1}{2} & , \quad 1 \leq i \leq n,\ i \text{ is even} \end{cases}$$

$$f(u_iv_i) = \begin{cases} 4n + i - 1 & , \quad 1 \leq i \leq n,\ i \text{ is odd} \\ 5n + i - 1 & , \quad 1 \leq i \leq n,\ i \text{ is even} \end{cases}$$

$$f(v_iw_i) = \begin{cases} 4n + i & , \quad 1 \leq i \leq n,\ i \text{ is odd} \\ 5n + i & , \quad 1 \leq i \leq n,\ i \text{ is even} \end{cases}$$

and $f(u_iu_{i+1})$ = 3n + i, $1 \leq i \leq n-1$

Now, we prove this labeling is an edge trimagic graceful.

Consider the edges u_iu_{i+1}, $1 \leq i \leq n-1$.

$$\text{For odd } i, \mid f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \mid \; = \; \mid \tfrac{i+1}{2} - (3n+i) + \tfrac{n+i+2}{2} \mid \; = \; \mid \tfrac{3-5n}{2} \mid = c_1 \text{ (say)}.$$

$$\text{For even } i, \mid f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \mid \; = \; \mid \tfrac{n+i+1}{2} - (3n+i) + \tfrac{i+2}{2} \mid \; = \; \mid \tfrac{3-5n}{2} \mid = c_1.$$

Consider the edges u_iv_i, $1 \leq i \leq n-1$.

$$\text{For odd } i, \mid f(u_i) - f(u_iv_i) + f(v_i) \mid \; = \; \mid \tfrac{i+1}{2} - (4n+i-1) + n + \tfrac{i+1}{2} \mid \; = \; \mid 2 - 3n \mid = c_2 \text{ (say)}.$$

$$\text{For even } i, \mid f(u_i) - f(u_iv_i) + f(v_i) \mid \; = \; \mid \tfrac{n+i+1}{2} - (5n+i-1) + n + \tfrac{n+i+1}{2} \mid \; = \; \mid 2 - 3n \mid = c_2.$$

Consider the edges v_iw_i, $1 \leq i \leq n-1$.

$$\text{For odd } i, \mid f(v_i) - f(v_iw_i) + f(w_i) \mid \; = \; \mid n + \tfrac{i+1}{2} - (4n+i) + 2n + \tfrac{i+1}{2} \mid \; = \; \mid 1 - n \mid = c_3 \text{ (say)}.$$

$$\text{For even } i, \mid f(v_i) - f(v_iw_i) + f(w_i) \mid \; = \; \mid n + \tfrac{n+i+1}{2} - (5n+i) + 2n + \tfrac{n+i+1}{2} \mid \; = \; \mid 1 - n \mid = c_3.$$

Hence, for each edge $uv \in E$, $(P_n;S_1)$, $\mid f(u) - f(uv) + f(v) \mid$ will form any one of the constants $c_1 = \mid \frac{3-5n}{2} \mid$, $c_2 = \mid 2 - 3n \mid$ and $c_3 = \mid 1 - n \mid$. Therefore, the graph $(P_n;S_1)$ admits an edge trimagic graceful labeling for odd n.

Case 2: n is even

Define a bijection $f : V \cup E \to \{1, 2, 3, ..., 6n-1\}$ such that

$$f(u_i) = \begin{cases} \frac{i+1}{2} & , \quad 1 \leq i \leq n, \ i \text{ is odd} \\ \frac{n+i}{2} & , \quad 1 \leq i \leq n, \ i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} n + \frac{i+1}{2} & , \quad 1 \leq i \leq n, \ i \text{ is odd} \\ n + \frac{n+i}{2} & , \quad 1 \leq i \leq n, \ i \text{ is even} \end{cases}$$

$$f(w_i) = \begin{cases} 2n + \frac{i+1}{2} & , \quad 1 \leq i \leq n, \ i \text{ is odd} \\ 2n + \frac{n+i}{2} & , \quad 1 \leq i \leq n, \ i \text{ is even} \end{cases}$$

$$f(u_iv_i) = \begin{cases} 4n + i - 1 & , \quad 1 \leq i \leq n, \ i \text{ is odd} \\ 5n + i - 1 & , \quad 1 \leq i \leq n, \ i \text{ is even} \end{cases}$$

$$f(v_iw_i) = \begin{cases} 4n + i & , \quad 1 \leq i \leq n, \ i \text{ is odd} \\ 5n + i - 2 & , \quad 1 \leq i \leq n, \ i \text{ is even} \end{cases}$$

and $f(u_iu_{i+1})$ = 3n+i, $1 \leq i \leq n-1$.

Now, we prove this labeling is an edge trimagic graceful.

Consider the edges u_iu_{i+1}, $1 \leq i \leq n-1$.

$$\text{For odd } i, \mid f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \mid = \mid \tfrac{i+1}{2} - (3n+i) + \tfrac{n+i+1}{2} \mid = \mid \tfrac{2-5n}{2} \mid = c_1 \text{ (say)}.$$

$$\text{For even } i, \mid f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \mid = \mid \tfrac{n+i}{2} - (3n+i) + \tfrac{i+2}{2} \mid = \mid \tfrac{2-5n}{2} \mid = c_1.$$

Consider the edges u_iv_i, $1 \leq i \leq n-1$.

$$\text{For odd } i, \mid f(u_i) - f(u_iv_i) + f(v_i) \mid = \mid \tfrac{i+1}{2} - (4n+i-1) + n + \tfrac{i+1}{2} \mid = \mid 2 - 3n \mid = c_2 \text{ (say)}.$$

$$\text{For even } i, \mid f(u_i) - f(u_iv_i) + f(v_i) \mid = \mid \tfrac{n+i}{2} - (5n+i-2) + n + \tfrac{n+i}{2} \mid = \mid 2 - 3n \mid = c_2.$$

Consider the edges v_iw_i, $1 \leq i \leq n-1$.

$$\text{For odd } i, \mid f(v_i) - f(v_iw_i) + f(w_i) \mid = \mid n + \tfrac{i+1}{2} - (4n+i) + 2n + \tfrac{i+1}{2} \mid = \mid 1 - n \mid = c_3 \text{ (say)}.$$

$$\text{For even } i, \mid f(v_i) - f(v_iw_i) + f(w_i) \mid = \mid n + \tfrac{n+i}{2} - (5n+i-1) + 2n + \tfrac{n+i}{2} \mid = \mid 1 - n \mid = c_3.$$

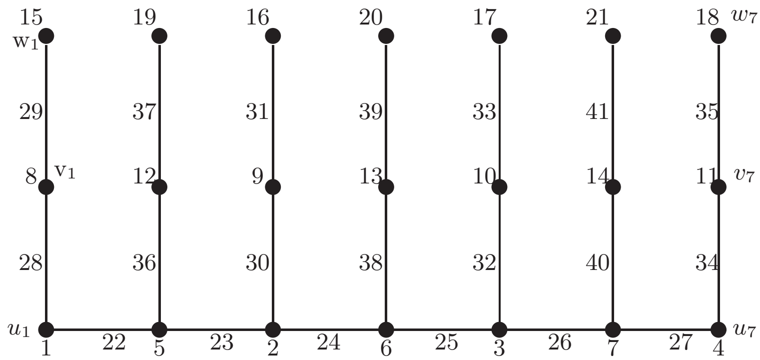
Hence, for each edge $uv \in E(P_n; S_1)$, $\mid f(u) - f(uv) + f(v) \mid$ will form any one of the constants $c_1 = \mid \frac{2-5n}{2} \mid$, $c_2 = \mid 2 - 3n \mid$ and $c_3 = \mid 1 - n \mid$. Therefore, the graph $(P_n; S_1)$ admits an edge trimagic graceful labeling for even n.

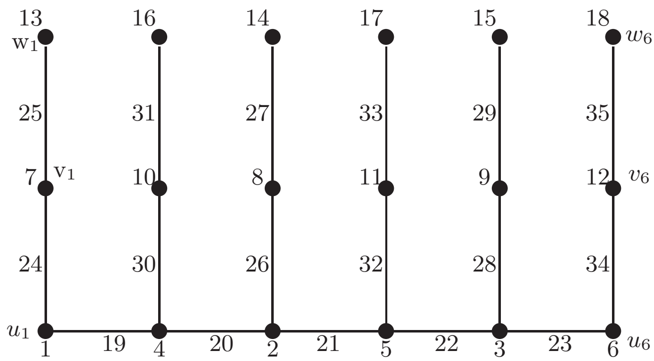
The theorem follows from case 1 and case 2. □

Corollary 2.5. *The graph $(P_n; S_1)$ admits a super edge trimagic graceful labeling.*

Proof. We proved that the graph $(P_n; S_1)$ admits an edge trimagic graceful labeling. The labeling given in the proof of theorem 2.4, the vertices get labels 1, 2,..., 3n. Since the graph $(P_n; S_1)$ has 3n vertices and these 3n vertices have labels 1, 2, ..., 3n for both odd and even n, $(P_n; S_1)$ is a super edge trimagic graceful graph. □

Example 2.6. An edge trimagic graceful labeling of $(P_7; S_1)$ and $(P_6; S_1)$ are given in figure 3 and figure 4, respectively.





Theorem 2.7. *The comb graph $P_n \odot K_1$ admits an edge trimagic graceful labeling for all n.*

Proof. Let $V(P_n \odot K_1) = \{u_i, v_i / 1 \leq i \leq n\}$ be the vertex set and $E(P_n \odot K_1) = \{u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{u_i v_i / 1 \leq i \leq n\}$ be the edge set of the graph $P_n \odot K_1$. Then $P_n \odot K_1$ has 2n vertices and 2n − 1 edges.

Case 1: n is odd

Define a bijection $f : V \cup E \to \{1, 2, 3, ..., 4n-1\}$ such that

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & 1 \le i \le n-1,\ i \text{ is odd} \\ \frac{n+i+1}{2}, & 1 \le i \le n-1,\ i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} \frac{2n+i+1}{2}, & 1 \le i \le n,\ i \text{ is odd} \\ \frac{3n+i+1}{2}, & 1 \le i \le n,\ i \text{ is even} \end{cases}$$

$f(u_iu_{i+1}) = 2n + i$, $1 \le i \le n-1$ and $f(u_iv_i) = 3n + i - 1$, $1 \le i \le n$.

Now, we prove this labeling is an edge trimagic graceful.

Consider the edges u_iu_{i+1}, $1 \le i \le n-1$.

For odd i, $\mid f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \mid = \mid \frac{i+1}{2} - (2n+i) + \frac{n+i+2}{2} \mid = \mid \frac{3-3n}{2} \mid = c_1$ (say).

For even i, $\mid f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \mid = \mid \frac{n+i+1}{2} - (2n+i) + \frac{i+2}{2} \mid = \mid \frac{3-3n}{2} \mid = c_1$.

Consider the edges u_iv_i, $1 \le i \le n$.

For odd i, $\mid f(u_i) - f(u_iv_{i)}) + f(v_i) \mid = \mid \frac{i+1}{2} - (3n+i-1) + \frac{2n+i+1}{2} \mid = \mid 2-2n \mid = c_2$ (say).

For even i, $\mid f(u_i) - f(u_iv_i) + f(v_i) \mid = \mid \frac{n+i+1}{2} - (3n+i-1) + \frac{3n+i+1}{2} \mid = \mid 2-n \mid = c_3$ (say).

Hence, for each edge $uv \in E(P_n \odot K_1)$, $\mid f(u) - f(uv) + f(v) \mid$ will form any one of the constants $c_1 = \mid \frac{3-3n}{2} \mid$, $c_2 = \mid 2-2n \mid$ and $c_3 = \mid 2-n \mid$. Therefore, the graph $P_n \odot K_1$ admits an edge trimagic graceful labeling for odd n.

Case 2: n is even

Define a bijection $f : V \cup E \to \{1, 2, 3, ..., 4n-1\}$ such that

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & 1 \le i \le n-1,\ i \text{ is odd} \\ \frac{n+i}{2}, & 1 \le i \le n-1,\ i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} \frac{2n+i+1}{2}, & 1 \le i \le n,\ i \text{ is odd} \\ \frac{2n+i+4}{2}, & 1 \le i \le n,\ i \text{ is even} \end{cases}$$

$f(u_iu_{i+1}) = 2n+i$, $1 \le i \le n-1$ and $f(u_iv_i) = 3n + i - 1$, $1 \le i \le n$.

Now, we prove this labeling is an edge trimagic graceful.

Consider the edges u_iu_{i+1}, $1 \le i \le n-1$.

For odd i, $\mid f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \mid = \mid \frac{i+1}{2} - (2n+i) + \frac{n+i+1}{2} \mid = \mid \frac{2-3n}{2} \mid = c_1$ (say).

For even i, $\mid f(u_i) - f(u_iu_{i+1}) + f(u_{i+1}) \mid = \mid \frac{n+i}{2} - (2n+i) + \frac{i+2}{2} \mid = \mid \frac{2-3n}{2} \mid = c_1$.

Consider the edges u_iv_i, $1 \le i \le n$.

For odd i, $\mid f(u_i) - f(u_iv_{i)}) + f(v_i) \mid = \mid \frac{i+1}{2} - (3n+i-1) + \frac{2n+i+1}{2} \mid = \mid 2-2n \mid = c_2$ (say).

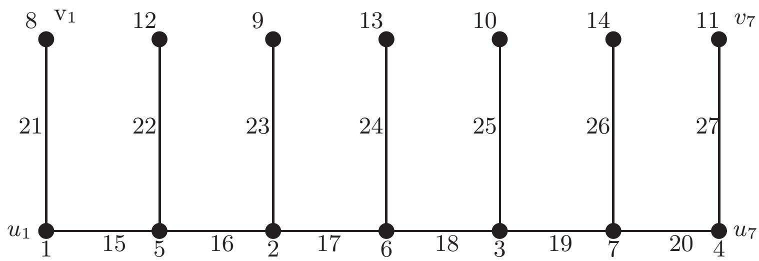
For even i, $\mid f(u_i) - f(u_iv_i) + f(v_i) \mid = \mid \frac{n+i}{2} - (3n+i-1) + \frac{2n+i+4}{2} \mid = \mid 1-n \mid = c_3$ (say).

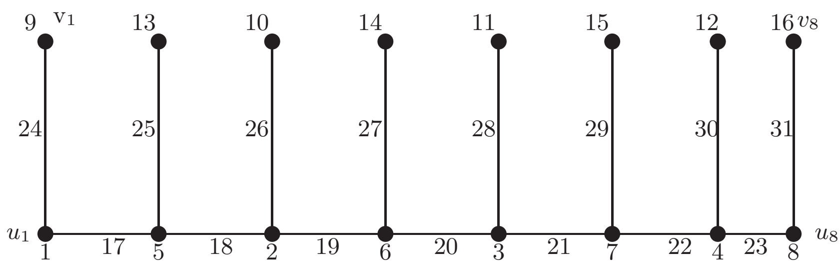
Hence, for each edge $uv \in E\ (P_n \odot K_1)$, $\mid f(u) - f(uv) + f(v) \mid$ will form any one of the constants $c_1 = \mid \frac{2-3n}{2} \mid$, $c_2 = \mid 2 - 2n \mid$ and $c_3 = \mid 1 - n \mid$. Therefore, the graph $P_n \odot K_1$ admits an edge trimagic graceful labeling for even n. The theorem follows from case 1 and case 2.

Corollary 2.8. *The comb graph $P_n \odot K_1$ admits a super edge trimagic graceful labeling.*

Proof. We proved that the comb graph $P_n \odot K_1$ admits an edge trimagic graceful labeling. The labeling given in the proof of theorem 2.7, the vertices get labels 1, 2, ..., 2n. Since the $P_n \odot K_1$ has 2n vertices and these 2n vertices have labels 1, 2, ..., 2n for both odd and even n, $P_n \odot K_1$ is a super edge trimagic graceful graph. □

Example 2.9. An edge trimagic graceful labeling of $P_7 \odot K_1$ and $P_8 \odot K_1$ are given in figure 5, and figure 6 respectively.





3. CONCLUSION

In this paper, we proved that the square graph P_n^2, $(P_n; S_1)$ and $P_n \odot K_1$ are edge trimagic graceful and super edge trimagic graceful. In future, we can construct many trimagic graceful graphs using these ideas.

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