International Journal of Pure and Applied Mathematics
Volume 120 No. 4 2018, 511-519
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v120i4.1
ijpam.eu

EDGE TRIMAGIC TOTAL LABELING OF CYCLE RELATED GRAPHS

C. Jayasekaran[1 §], S. Robinson Chellathurai[2], J. Little Flower[3]

[1]Department of Mathematics
Pioneer Kumaraswamy College
Nagercoil - 629003, TamilNadu, INDIA

[2]Department of Mathematics
Scott Christian College
Nagercoil 629 003, Tamilnadu, INDIA

[3]Department of Mathematics
Arignar Anna College
Aralvaimozhi - 629 301, TamilNadu, INDIA

Abstract: An edge trimagic total labeling of a graph $G = (V, E)$ with p vertices and q edges is a bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p+q\}$ if for each edge $uv \in E(G)$, the value of $f(u) + f(uv) + f(v)$ is either k_1 or k_2 or k_3. In this paper, we prove that the Closed helm CH_n, Antiprism A_n and Square graph of C_n are edge trimagic total and super edge trimagic total labeling.

AMS Subject Classification: 05C78
Key Words: edge trimagic total labeling, bijection, closed helm, antiprism, square graph

Received: September 3, 2017
Revised: January 22, 2019
Published: January 26, 2018

§Correspondence author

1. Introduction

All graphs in this paper are finite, planar and undirected. Throughout this paper $V(G)$ and $E(G)$ denote set of vertices and edges. Graph labeling was first introduced in the mid sixties. A labeling of a graph is a map of integers to vertices or sometimes edges in a graph based upon certain criteria.

Magic labeling was introduced by Sedlacek [13]. Kotzing and Rosa [9], defined edge magic of a graph G with a bijection $f : V \cup E \to \{1, 2, \dots, p+q\}$ such that, for each edge $uv \in E(G), f(u) + f(uv) + f(v)$ is a magic constant. Edge bimagic labeling of graphs was introduced by J. Baskar Babujee [2] in 2004, defined by a graph G with a bijection $f : V \cup E \to \{1, 2, \dots, p+q\}$ such that for each edge $uv \in E(G)$, the value of $f(u) + f(uv) + f(v)$ is either k_1 or k_2.

In 2013, Jayasekaran et al. [5] introduced the edge trimagic total labeling of graphs. An edge trimagic total labeling of a (p, q) graph G is a bijection $f : V \cup E \to \{1, 2, \dots, p+q\}$ such that for each edge $uv \in E(G)$, the value of $f(u) + f(uv) + f(v)$ is equal to any of the distinct constants k_1 or k_2 or k_3. An edge trimagic total labeling is called a super edge trimagic total labeling of a graph G, if the vertices are labeled with the smallest possible integers i.e. $1, 2, \dots, p$. In [10, 11, 12] proved that some classes of graphs are edge trimagic total and super edge trimagic total. We proved that umbrella, dumbbell, circular ladder, Mobius ladder, book and dragon graphs are edge trimagic total and super edge trimagic total [6, 7].

A helm $H_n, n \geq 3$ is the graph obtained from the wheel W_n by adding a pendant edge at each vertex on the rim of the wheel W_n. A closed helm CH_n [4] is the graph obtained by taking a helm H_n and adding edges between the pendant vertices. The antiprism [8] on $2n$ vertices has vertex set $\{x_{1,1}, x_{1,2}, \dots, x_{1,n}, x_{2,1}, x_{2,2}, \dots, x_{2,n}\}$ and edge set $\{x_{j,i}, x_{j,i+1}\} \cup \{x_{1,i}, x_{2,i}\} \cup \{x_{1,i}, x_{2,i-1}\}$ (subscripts are taken modulo n) and is denoted by A_n [1]. The square G^2 [3] of a graph G has $V(G^2) = V(G)$ with u, v adjacent in G^2 whenever $d(u, v) \leq 2$ in G.

In this paper, we prove that the closed helm, antiprism and square graph of C_n admits to be edge trimagic total and super edge trimagic total graphs. For more references, we use dynamic survey of graph labeling by Gallian [8].

2. Main Result

Theorem 2.1. *The Closed helm CH_n admits an edge trimagic total labeling for every positive odd integer $n \geq 3$.*

Proof. Let $V = \{w, u_i, v_i/1 \leq i \leq n\}$ be the vertex set and $E = \{wv_i, v_iu_i/1 \leq i \leq n\} \cup \{v_iv_{i+1}, u_iu_{i+1}/1 \leq i \leq n-1\} \cup \{v_1v_n, u_1u_n\}$ be the edge set of the closed helm CH_n. Then CH_n has $2n+1$ vertices and $4n$ edges.

Define a bijection $f : V \cup E \to \{1, 2, \ldots, 6n+1\}$ such that $f(w) = 1$,

$$f(v_i) = \begin{cases} \frac{i+1}{2} + 1, 1 \leq i \leq n \text{ and } \ i \ \text{ is odd} \\ \frac{n+i-1}{2} + 2, 1 \leq i \leq n \text{ and } \ i \ \text{ is even} \end{cases}$$

$$f(u_i) = \begin{cases} n + \frac{n+i}{2} + 1, 1 \leq i \leq n \text{ and } \ i \ \text{ is odd} \\ n + 1 + \frac{i}{2}, 1 \leq i \leq n \text{ and } \ i \ \text{ is even} \end{cases}$$

$$f(wv_i) = \begin{cases} 4n - \frac{i+1}{2} + 2, 1 \leq i \leq n \text{ and } \ i \ \text{ is odd} \\ 3n + \frac{n+1-i}{2} + 1, 1 \leq i \leq n \text{ and } \ i \ \text{ is even} \end{cases}$$

$f(v_iu_i) = 5n - i + 2, 1 \leq i \leq n; f(v_iv_{i+1}) = 6n + 1 - i, 1 \leq i \leq n-1; f(v_1v_n) = 6n + 1; f(u_iu_{i+1}) = 3n + 2 - i, 1 \leq i \leq n-1$ and $f(u_1u_n) = 2n + 2$.

To prove this labeling is an edge trimagic total labeling.

Consider the edges $wv_i, 1 \leq i \leq n$.

For odd $i, f(w) + f(wv_i) + f(v_i) = 1 + 4n - \frac{i+2}{2} + 2 + \frac{i+1}{2} + 1 = 4n + 4 = \lambda_1$.

For even $i, f(w) + f(wv_i) + f(v_i) = 1 + 3n + \frac{n-i+1}{2} + 1 + \frac{n+i-1}{2} + 2 = 4n + 4 = \lambda_1$.

Consider the edges $v_iv_{i+1}, 1 \leq i \leq n-1$. For odd $i, f(v_i) + f(v_iv_{i+1}) + f(v_{i+1}) = \frac{i+1}{2} + 1 + 6n + 1 - i + \frac{n+i}{2} + 2 = \frac{13n+9}{2} = \lambda_2$.

For even $i, f(v_i) + f(v_iv_{i+1}) + f(v_{i+1}) = \frac{n+i+1}{2} + 2 + 6n + 1 - i + \frac{i+2}{2} + 1 = \frac{13n+9}{2} = \lambda_2$.

For the edge $v_1v_n, f(v_1) + f(v_1v_n) + f(v_n) = 2 + 6n + 1 + \frac{n+3}{2} = \frac{13n+9}{2} = \lambda_2$.

Consider the edges $v_iu_i, 1 \leq i \leq n$.

For odd $i, f(v_i) + f(v_iu_i) + f(u_i) = \frac{i+1}{2} + 1 + 5n - i + 2 + n + \frac{n+i}{2} + 1 = \frac{13n+9}{2} = \lambda_2$.

For even $i, f(v_i) + f(v_iu_i) + f(u_i) = \frac{n+i-1}{2} + 2 + 5n - i + 2 + n + 1 + \frac{i}{2} = \frac{13n+9}{2} = \lambda_2$.

Consider the edges $u_iu_{i+1}, 1 \leq i \leq n-1$. For odd $i, f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = n + \frac{n+i}{2} + 1 + 3n + 2 - i + n + \frac{i+3}{2} = \frac{11n+9}{2} = \lambda_3$.

For even i, $f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = n+1+\frac{i}{2}+3n+2-i+n+\frac{n+i+1}{2}+1 = \frac{11n+9}{2} = \lambda_3$.

For the edge u_1u_n, $f(u_1)+f(u_1u_n)+f(u_n) = n+\frac{n+3}{2}+2n+2+2n+1 = \frac{11n+9}{2} = \lambda_3$.
Hence for each edge $uv \in E$, $f(u)+f(uv)+f(v)$ yields any one of the constants $\lambda_1 = 4n+4, \lambda_2 = \frac{13n+9}{2}$ and $\lambda_3 = \frac{11n+9}{2}$. Therefore, the closed helm CH_n admits an edge trimagic total labeling for every positive odd integer $n \geq 3$. □

Corollary 2.2. The Closed helm CH_n admits super edge trimagic total labeling for every positive odd integer $n \geq 3$.

Proof. We have proved that the Closed helm CH_n has an edge trimagic total labeling for every positive odd integer $n \geq 3$ with $2n+1$ vertices. The labeling given in Theorem 2.1 is as follows: $f(w) = 1$

$$f(v_i) = \begin{cases} \frac{i+1}{2}+1, 1 \leq i \leq n \text{ and } i \text{ is odd} \\ \frac{n+i-1}{2}+2, 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f(u_i) = \begin{cases} n+\frac{n+i}{2}+1, 1 \leq i \leq n \text{ and } i \text{ is odd} \\ n+1+\frac{i}{2}, 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

Hence the $2n+1$ vertices get labels $1, 2, \ldots, 2n+1$. Therefore, the Closed helm CH_n admits super edge trimagic total labeling for every positive odd integer $n \geq 3$. □

Example 2.3. An edge trimagic total labeling of the CH_9 is as shown in figure 1.

Theorem 2.4. *The antiprism A_n admits an edge trimagic total labeling for all $n \geq 3$.*

Proof. Let $V = \{u_i, v_i,$ where $u_i = x_{1i}$ and $v_i = x_{2i}/1 \leq i \leq n\}$ be the vertex set and $E = \{u_iu_{i+1}, v_iv_{i+1}/1 \leq i \leq n-1\} \cup \{u_1u_n, v_1v_n\} \cup \{u_iv_i, /1 \leq i \leq n\} \cup \{u_iv_{i+1}, /1 \leq i \leq n-1\} \cup \{u_nv_1\}$ be the edge set of A_n. Then A_n has $2n$ vertices and $4n$ edges.
Define a bijection $f : V \cup E \to \{1, 2, \ldots, 6n\}$ such that $f(v_i) = i, 1 \leq i \leq n$; $f(u_i) = n+i, 1 \leq i \leq n$; $f(u_iu_{i+1}) = 4n-2i-1, 1 \leq i \leq n-1$; $f(u_1u_n) =$

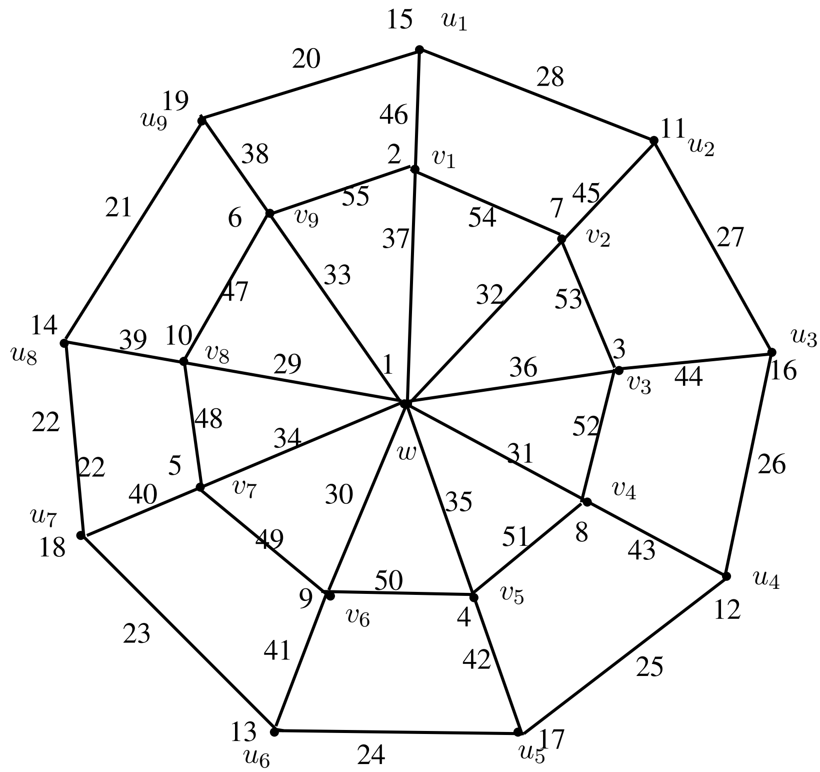

Figure 1: CH_9 with $\lambda_1 = 40, \lambda_2 = 63$ and $\lambda_3 = 54$

$4n; f(v_iv_{i+1}) = 4n - 2i, 1 \leq i \leq n-1; f(v_1v_n) = 6n; f(u_iv_i) = 6n - 2i + 1, 1 \leq i \leq n; f(u_iv_{i+1}) = 6n - 2i, 1 \leq i \leq n-1$ and $f(u_nv_1) = 4n - 1$.

To prove this labeling is an edge trimagic total labeling.

For the edges $u_iu_{i+1}, 1 \leq i \leq n-1, f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = n + i + 4n - 2i - 1 + n + i + 1 = 6n = \lambda_1$.

For the edge $u_nv_1, f(u_n) + f(u_nv_1) + f(v_1) = 2n + 4n - 1 + 1 = 6n = \lambda_1$.

For the edges $v_iv_{i+1}, f(v_i) + f(v_iv_{i+1}) + f(v_{i+1}) = i + 4n - 2i + i + 1 = 4n + 1 = \lambda_2$.

For the edge $u_1u_n, f(u_1) + f(u_1u_n) + f(u_n) = n + 1 + 4n + 2n = 7n + 1 = \lambda_3$.

For the edge $v_1v_n, f(v_1) + f(v_1v_n) + f(v_n) = 1 + 6n + n = 7n + 1 = \lambda_3$.

For the edges $u_iv_i 1 \leq i \leq n, f(u_i) + f(u_iv_i) + f(v_i) = n + i + 6n - 2i + 1 + i = 7n + 1 = \lambda_3$.

For the edges $u_iv_{i+1}, 1 \leq i \leq n-1, f(u_i) + f(u_iv_{i+1}) + f(v_{i+1}) = n + i + 6n - 2i + i + 1 = 7n + 1 = \lambda_3$.

Hence for each edge $uv \in E, f(u) + f(uv) + f(v)$ yields any one of the magic constants $\lambda_1 = 6n, \lambda_2 = 4n + 1$ and $\lambda_3 = 7n + 1$. Therefore, the antiprism A_n admits an edge trimagic total labeling for all $n \geq 3$. □

Corollary 2.5. The antiprism A_n admits super edge trimagic total labeling for all $n \geq 3$.

Proof. We have proved that the antiprism A_n has an edge trimagic total labeling for all n with $2n$ vertices. The labeling given in Theorem 2.4 is as follows: $f(v_i) = i, 1 \leq i \leq n; f(u_i) = n + i, 1 \leq i \leq n$. Hence the $2n$ vertices get labels $1, 2, \ldots, 2n$. Therefore, the antiprism A_n admits super edge trimagic total labeling for all $n \geq 3$. □

Example 2.6. An edge trimagic total labeling of A_5 and A_6 are as shown in figure 2 and figure 3 respectively.

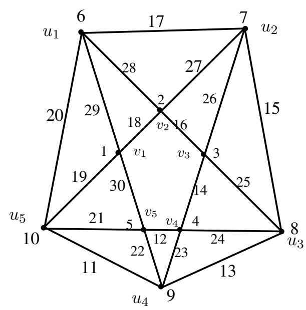

Figure 2: A_5 with $\lambda_1 = 30, \lambda_2 = 21$ and $\lambda_3 = 36$

Theorem 2.7. *The square graph C_n^2 admits an edge trimagic total labeling for even $n \geq 4$.*

Proof. Let $v_1 v_2 \ldots v_n v_1$ be a cycle C_n. For $1 \leq i \leq n - 2$, join v_i and v_{i+2} and join v_{n-1} with v_1 and v_n with v_2. The resultant graph is C_n^2 with vertex set $V = \{v_i / 1 \leq i \leq n\}$ and the edge set $E = \{v_i v_{i+1}, v_i v_{i+2} / 1 \leq i \leq n-2\} \cup \{v_{n-1} v_n, v_n v_1, v_{n-1} v_1, v_n v_2\}$. Clearly the square graph C_n^2 has n vertices and $2n$ edges.

Define a bijection $f : V \cup E \to \{1, 2, \ldots, 3n\}$ such that $f(v_i) = i, 1 \leq i \leq n$;

$$f(v_i v_{i+1}) = \begin{cases} 2n - 2i + 4, 1 \leq i \leq \frac{n}{2} \\ 4n - 2i + 1, \frac{n}{2} + 1 \leq i \leq n - 1 \end{cases}$$

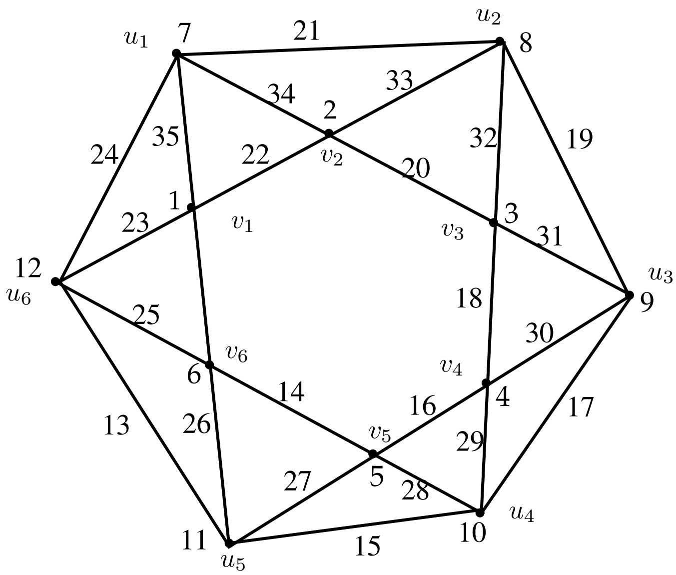

Figure 3: A_6 with $\lambda_1 = 36, \lambda_2 = 25$ and $\lambda_3 = 43$

$$f(v_iv_{i+2}) = \begin{cases} 2n-2i+3, 1 \leq i \leq \frac{n}{2}-1 \\ 4n-2i, \frac{n}{2} \leq i \leq n-2 \end{cases}$$

$f(v_1v_n) = n+2; f(v_1v_{n-1}) = n+3$ and $f(v_2v_n) = n+1$.

To prove this labeling is an edge trimagic total labeling.

For the edges $v_iv_{i+1}, f(v_i) + f(v_iv_{i+1}) + f(v_{i+1}) = i + 2n - 2i + 4 + i + 1 = n + 5 = \lambda_1$.

For the edges $v_iv_{i+1}, \frac{n}{2} \leq i \leq n-1, f(v_i) + f(v_iv_{i+1}) + f(v_{i+1}) = i + 4n - 2i + 1 + i + 1 = 4n + 2 = \lambda_2$.

For the edge $v_1v_n, f(v_1) + f(v_1v_n) + f(v_n) = 1 + n + 2 + n = 2n + 3 = \lambda_3$. For the edges $v_iv_{i+2}, 1 \leq i \leq \frac{n}{2}-1, f(v_i) + f(v_iv_{i+2}) + f(v_{i+2}) = i + 2n - 2i + 3 + i + 2 = 2n + 5 = \lambda_1$.

For the edges $v_iv_{i+2}, \frac{n}{2} \leq i \leq n-2, f(v_i) + f(v_iv_{i+2}) + f(v_{i+2}) = i + 4n - 2i + i + 2 = 4n + 2 = \lambda_2$.

For the edge $v_1v_{n-1}, f(v_1) + f(v_1v_{n-1}) + f(v_{n-1}) = 1 + n + 3 + n - 1 = 2n + 3 = \lambda_3$.

For the edge $v_2v_n, f(v_2) + f(v_2v_n) + f(v_n) = 2 + n + 1 + n = 2n + 3 = \lambda_3$. Hence for each edge $uv \in E, f(u) + f(uv) + f(v)$ yields any one of the magic constants $\lambda_1 = 2n + 5, \lambda_2 = 4n + 2$ and $\lambda_3 = 2n + 3$. Therefore, the graph C_n^2

admits an edge trimagic total labeling for even $n \geq 4$. □

Corollary 2.8. The square graph C_n^2 admits super edge trimagic labeling for even $n \geq 4$.

Proof. We proved that the square graph C_n^2 admits an edge trimagic total graph for even $n \geq 4$ with n vertices. The labeling given in above Theorem 2.7 is as follows: $f(v_i) = i, 1 \leq i \leq n$. Hence the n vertices get labels $1, 2, \ldots, n$. Therefore, the square graph C_n^2 admits a super edge trimagic total labeling for even $n \geq 4$. □

Example 2.9. An edge trimagic labeling of the square graph C_{10}^2 is as shown in figure 4.

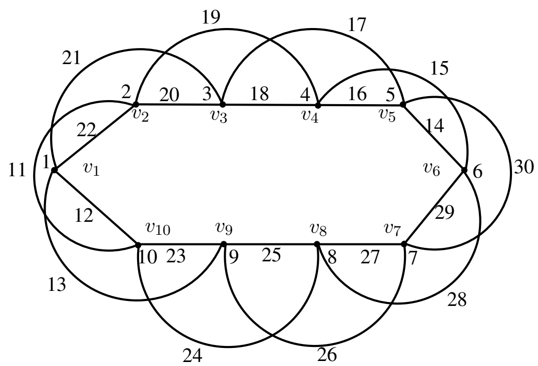

Figure 4: C_{10}^2 with $\lambda_1 = 25$, $\lambda_2 = 42$,and $\lambda_3 = 23$

3. Conclusion

In this paper, we proved that Closed Helm CH_n (odd n), Antiprism A_n and Square graph of cycle C_n (even) are edge trimagic total and super edge trimagic total labeling.

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